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ON THE TERNARY CUBIC EQUATION WITH FOUR UNKNOWNS $x^3+y^3=24zw^2$.

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ABSTRACT The sequence of Integral solutions to the cubic equation with four variables are obtained. A few properties among the solutions are also presented.

KEYWORDS : Cubic equation having four unknowns with integral solutions.

Introductions

The Diophantine equation offer an unlimited for research due to their verify [1-2]. In particular, one may refer [3-14] for cubic equation with three unknowns. In [15-19] cubic equations with four unknowns are studied for its non-trivial integral solutions. This communication concern with the problem of obtaining non-zero integral solutions of cubic equation with four variables is given by$x^3+y^3=24zw^2$. A few properties among the solutions and special numbers are presented.

Notations:

$t_{m,n}$ = Polygonal number of rank n with size m

P_n^m = pyramidal number of rank n with size m

P_r^n =pronic number of rank n

S_n= Star number of rank n.

SO_n= Stella octangular number of rank n

j_n=Jacobsthal lucas number of rank n

J_n= Jacobsthal number of rank n

Gno_n= Gnomic number of rank n

Cp_n^m = Centered pyramidal number of rank n with size m

Cp_n^m = Centered tetradecagonal pyramidal number of rank m

II Method of Analysis:

The cubic Diophantine equation with four unknowns to be solved for obtaining non-zero integral solution is

$$x^3+y^3=24zw^2 \quad (1)$$

on substituting the linear transformations

$$x=u+v, y=u-v, z=u \quad (2)$$

in (1) leads to

$$u^2+3v^2=12w^2 \quad (3)$$

We obtain different choices of integral solutions to (1) through solving (3) which are illustrated as follows:

Choice 1:

Assume $w=a^2+3b^2$ (4)

Write $12=(3+i\sqrt{3})(3-i\sqrt{3})$ (5)

Using (4), (5) in (3) and employing factorization it is expressed as

$$(u+i\sqrt{3}v)(u-i\sqrt{3}v) = (3+i\sqrt{3})(3-i\sqrt{3})(a+i\sqrt{3}b)^2(a-i\sqrt{3}b)^2$$

Which is equivalent to the system of equations

$$(u+i\sqrt{3}v)=(3+i\sqrt{3})(a+i\sqrt{3}b)^2 \quad (6a)$$

$$(u-i\sqrt{3}v)=(3-i\sqrt{3})(a-i\sqrt{3}b)^2 \quad (6b)$$

Equating the real and imaginary parts either in (6a) or (6b), we have

$$u=3a^2-9b^2-6ab$$

$$v=a^2-3b^2+6ab$$

In view of (2), the non-zero distinct integral solutions of (1) are

$$x=4a^2-12b^2$$

$$y=2a^2-6b^2-12ab$$

$$z=3a^2-9b^2-6ab$$

along with $w=a^2+3b^2$

Properties:

1) $x(a(a+1),a+2)-2y(a(a+1),a+2)-17Cub_n-3CP_a^{14}-8t_{4,3a}\equiv 0(mod\ 52)$
2) $x(a(a+1),a+2)-4Biq_n-8Cub_n+8t_{4,a}\equiv 48(mod\ 24)$
3) $z(a^2+a,2a^2-1)+324FN_n^4+12P_a^5-6Pr_n-18t_{4,a}+9=0$
4) $y(a+1,a+2)-2z(a+1,a+2)-8Pr_a\equiv 44(mod\ 32)$
5) $x(a,a^2+1)-2y(a,a^2+1)-24CP_n^6\equiv 0(mod\ 24)$
6) $x(6a,a-1)-2y(6a,a-1)-24S_a+24=0$
7) $3w(a(a+1),a+2)-z(a(a+1),a+2)-6Cub_n-36Pr_a\equiv 72(mod\ 48)$
8) $2y(a^2+a,a+3)-x(a^2+a,a+3)+24Cub_n+6t_{4,2a}+72Pr_a=0$
9) $3w(2b^2+1,b)-z(2b^2+1,b)-12Cub_n-3t_{4,2b}-6Pr_b=0$

Note

In (5) , 12 may also be considered as

$$12=(-3+i\sqrt{3})(-3-i\sqrt{3}) \quad (7)$$

For this case, the corresponding integer solutions are given by

$$x=-2a^2+6b^2-12ab$$

$$y=-4a^2+12b^2$$

$$z=-3a^2+9b^2-6ab$$

Choice 2:

Write 12 as

$$12=\frac{(24+i2\sqrt{3})(24-i2\sqrt{3})}{49} \quad (8)$$

Using (4) and (8) in (3) and proceeding as in choice 1, the corresponding integer solutions are given by

$$x=\frac{1}{7}[26a^2-78b^2+36ab]$$

$$y=\frac{1}{7}[22a^2-66b^2-60ab]$$

$$z=\frac{1}{7}[24a^2-72b^2-12ab]$$

As our interest is of finding integral solutions, choose a and b suitably so that the solutions are in integers. In particular, the choice $a=7A, b=7B$ leads to the integer solution to (1) are given by,

$$x=182A^2-546B^2+252AB$$

$$y=154A^2-462B^2-420AB$$

$$z=168A^2-504B^2-84AB$$

$$w=49A^2+147B^2$$

Properties:

1) $y(a(a+1),a+2)-w(a(a+1),a+2)-x(a(a+1,a+2)+77Biq_n+539t_{4,2a}+826Cub_n \equiv 252(mod\ 1596)$
2) $x(a,1)+3w(a,1)-73t_{4,a}-252Pr_a+105=0$
3) $x(a,1)-y(a,1)-w(a,1)+21t_{4,a} \equiv 231(mod\ 672)$
4) $154(a,a(a+1))-182(a,a(a+1))-230496Pp_a=0$
5) $49z(a,2a^2-1)-168(a,2a^2-1)-2370816\,FN_a^4+4116\,So_a+49392=0$
6) $504y(a,2a^2+1)-462z(a,2a^2+1)-518616\,OH_a=0$
7) $z(a,5)-588\,Obl_a+t_{842,a}+j_{10}+J_{13} \equiv 3374(mod\ 1427)$

Note:

In (8), 12 can also be written as

$$12=\frac{(-24+i2\sqrt{3})(-24-i2\sqrt{3})}{49} \quad (9)$$

For this case, the non-zero distinct solutions are illustrated below

For the choice $a=7A,\ b=7B$

$$x=-154A^2+462B^2-420AB$$

$$y=-182A^2+546B^2+252AB$$

$$z=-168A^2+504B^2-84AB$$

$$w=49A^2+147B^2$$

Choice 3:

Equation (3) can also be written as

$$u^2+3v^2=12w^2*1 \quad (10)$$

Write 1 as

$$1=\frac{(1+i\sqrt{3})(1-i\sqrt{3})}{4} \quad (11)$$

Using (4),(8) and (11) in (10) and employing the method of factorization as in choice 1, the corresponding integral solutions are given by

$$x=\frac{1}{7}[22a^2-66b^2-60ab]$$

$$y=\frac{1}{7}[-4a^2+12b^2-96ab]$$

$$z=\frac{1}{7}[9a^2-27b^2-78ab]$$

As our aim is to find integral solutions, choose a and b suitably so that the solutions are in integers. In particular, the choice $a=7A$, $b=7B$ leads to the integer solutions to equation (1) are given by,

$$x=154A^2-462B^2-420AB$$

$$y=-28A^2+84B^2-672AB$$

$$z=63A^2-189B^2-546AB$$

$$w=49A^2+147B^2$$

Properties

1) $x(a-1,1)-154t_{4,a} \equiv 112(mod\ 728)$
2) $z(a^2,a)-63Biq_n+546Cub_n+189t_{4,a}=0$
3) $y(a+1,a)-14t_{4,a}+672Pr_a \equiv 28(mod\ 56)$
4) $y(b+1,b)+616Pr_b+3J_4+j_2 \equiv 6(mod\ 112)$
5) $28x(a,a(a+1))+154y(a,a(a+1))+230496\,Pp_a=0$
6) $147z(a,7a^2-4)+189w(a,7a^2-4)-18522\,Obl_a+240786\,CP_a^{14} \equiv 0(mod\ 18522)$
7) $w(2a,3)-392t_{3,a} \equiv 1323(mod\ 196)$

Note:

In (11), 1 can also be written as

$$1=\frac{(-1+i\sqrt{3})(-1-i\sqrt{3})}{4} \quad (12)$$

For this case of '1' the corresponding, non-zero distinct integer solutions are illustrated below,

For the choice $a=2A,\ b=2B$

$$x=-28A^2+84B^2+672AB$$

$$y=154A^2-462B^2+420AB$$

$$z=63A^2-189B^2+546AB$$

$$w=49A^2+147B^2$$

Choice 4:

'1' can also be written as

$$1 = \frac{(1+i4\sqrt{3})(1-i4\sqrt{3})}{49} \quad (13)$$

Using (4),(5) and (13) in (10) and employing the method of factorization as in choice 1, the corresponding integer solutions are given by

$$x = \frac{1}{7}[4a^2 - 12b^2 - 96ab]$$

$$y = \frac{1}{7}[-22a^2 + 66b^2 - 60ab]$$

$$z = \frac{1}{7}[-9a^2 + 27b^2 - 78ab]$$

As our interest is of finding integer solutions, choose a & b suitably so that the solutions are in integers. By taking $a = 7A,\ b = 7B$ leads to the integer solutions to (1) to be,

$$x = 28A^2 - 84B^2 - 672AB$$

$$y = -154A^2 + 462B^2 - 420AB$$

$$z = -63A^2 + 189B^2 - 546AB$$

$$w = 49A^2 + 147B^2$$

Properties

1) $55x(a, a^2 - 1) + 10y(a, a^2 - 1) + 41160Cub_n \equiv 0(mod\ 41160)$
2) $55x(7a - 5, a) + 10y(7a - 5, a) - 72030t_{4,2a} \equiv 0(mod\ 205800)$
3) $28w(1, b) - 49x(1, b) - 2058t_{4,2b} \equiv 0(mod\ 32928)$
4) $28y\big(a(a + 1), (2a + 1)\big) + 154\big(a(a + 1), (2a + 1)\big) - 691488\ Sqp_a = 0$
5) $x(2a - 1,1) - 112\ Obl_a - j_8 \equiv 360(mod\ 1568)$
6) $z(a^2, a) + 756\ FN_a^4 - 14\ t_{4,3a} - 54\ CP_a^6 = 0$
7) $w(b, b + 1) - 147\ Pr_b \equiv 147(mod\ 196)$

Note:

In (13), '1' can also be written as

$$1 = \frac{(-1+i4\sqrt{3})(-1-i4\sqrt{3})}{49} \quad (14)$$

For the choice, the integer solutions are

$$x = -154A^2 + 462B^2 + 420AB$$

$$y = 28A^2 - 84B^2 + 672AB$$

$$z = -63A^2 + 189B^2 + 546AB$$

$$w = 49A^2 + 147B^2$$

Choice 5:

CASE 1:

Equation (3) can be re-written as

$$u^2 - 9w^2 = 3(w^2 - v^2)$$

Which is written in the form of ratio as,

$$\frac{u+3w}{w-v} = \frac{3(w+v)}{u-3w} = \frac{a}{b} \quad (15)$$

Which is equivalent to the system of equations,

$$bu + av + w(3b - a) = 0$$

$$au - 3bv - w(3a + 3b) = 0$$

Applying the method of cross multiplication we have,

$$u = -3a^2 + 9b^2 - 6ab$$

$$v = -a^2 + 3b^2 + 6ab$$

$$w = -a^2 - 3b^2$$

Substituting the values of u and v we get the non-zero distinct integer solutions to be

$$x = -4a^2 + 12b^2$$

$$y = -2a^2 + 6b^2 - 12ab$$

$$z = -3a^2 + 9b^2 - 6ab$$

$$w = -a^2 - 3b^2$$

Properties:

1) $x(2a, a^2) - 2y(2a, a^2) + w(2a, a^2) + 36FN_a^4 - 96P_a^5 + 55t_{4,a} = 0$
2) $x(a^2, a + 1) - 2w(a^2, a + 1) + 2Biq_a - 18Pr_a - 9Gno_a = 27$
3) $y(a + 1, a) + 2w(a + 1, a) + 12Pr_a + 6t_{4,a} = 0$
4) $x(a^2, 2a) + z(a^2, 2a) + 7Biq_a + 24P_a^5 - 96t_{4,a} = 0$
5) $3w(2a, a^2) + z(2a, a^2) + 12Cub_n + 6t_{4,2a} = 0$
6) $x(2b - 1, b) - 2y(2b - 1, b) - 48Pr_b \equiv 0(mod\ 72)$
7) $4z(a^2, a) - 3x(a^2, a) + 24Cub_n = 0$
8) $x(a^2, a + 1) - 2y(a^2, a + 1) - 24Cub_n - 24t_{4,a} - 12Pr_a = 0$

CASE 2:

(3) can also be written as

$$\frac{u+3w}{3(w-v)} = \frac{w+v}{u-3w} = \frac{a}{b}$$

Which is equivalent to the system of equations,

$$bu + 3av + w(3b - 3a) = 0$$

$$au - bv - w(3a + b) = 0$$

Applying the method of cross multiplication we have,

$$u = -9a^2 + 3b^2 - 6ab$$

$$v = -3a^2 + b^2 + 6ab$$

$$w = -3a^2 - b^2$$

In view of (2), we get the non-zero distinct integer solutions to be

$$x = -12a^2 + 4b^2$$

$$y = -6a^2 + 2b^2 - 12ab$$

$$z = -9a^2 + 3b^2 - 6ab$$

$$w = -3a^2 - b^2$$

Properties

1) $y(a, a^2) - 2z(a, a^2) + 4Biq_n - 3t_{4,2a} = 0$
2) $x(a + 1, a) + 4w(a + 1, a) + 24Pr_a \equiv 24(mod\ 24)$
3) $-12y(a, a) - 12t_{4,4a} = 0$
4) $8x(a, a)$is a perfect square.
5) $y\big(a(a + 1), (a + 2)\big) - 2w\big(a(a + 1), (a + 2)\big) - 4\ t_{4,a} - 8\ Gno_a + 72\ Tet_a \equiv 0(mod\ 24)$
6) $z(a^{2,}, a) + 12\ Biq_a - 36\ FN_a^4 + 6\ Cp_a^6 = 0$

7) $2z(a^2+a, 2a^2-1) - 4\,Biq_a + 48\,Pp_a + 4\,t_{4,a} + 4 = 0$

Now instead of (2), writing the linear transformation as,

$$x = u + v, y = u - v, z = 4u \quad (16)$$

In (1), it leads to

$$u^2 + 3v^2 = 48w^2 \quad (17)$$

Solving (17) in different ways one obtains other different choice of integer solutions to (1) which are illustrated as below:

Choice 7:

Write '48' as

$$48 = \frac{(12+i4\sqrt{3})(12-i4\sqrt{3})}{4} \quad (18)$$

Using (4) and (18) in (17) and proceeding as in choice 1, the corresponding integer solutions are given by,

$$x = 8a^2 - 24b^2$$

$$y = 4a^2 - 12b^2 - 24ab$$

$$z = 24a^2 - 72b^2 - 48ab$$

$$w = a^2 + 3b^2$$

Properties

1) $x(a, 2a-1) + 8w(a, 2a-1) - t_{4,4a} = 0$
2) $6y(6b, b-1) - z(6b, b-1) + 96S_b - 96 = 0$
3) $x(a+1, a(a+2)) + 8w(a+1, a(a+2)) - 16Pr_a \equiv 16(mod\ 16)$
4) $6y(1, a(2a^2+1)) - z(1, a(2a^2+1)) + 96SO_a \equiv 0(mod\ 192)$
5) $x(a,1) - 2y(a,1) \equiv 0(mod\ 48)$
6) $y(2b^2-1, b) - 4w(2b^2-1, b) + 6t_{4,2b} + 24SO_b = 0$
7) $x(a(a+1), a+2) - 2y(a(a+1), a+2) - 48Cub_n - 3t_{4,4a} - 96Pr_a = 0$
8) $z(6a, a-1) + 24w(6a, a-1) - 12t_{4,12a} + 48S_a - 48 = 0$

Note

In (18), '48' may also be considered as

$$48 = \frac{(-12+i4\sqrt{3})(-12-i4\sqrt{3})}{4} \quad (19)$$

For this choice the corresponding non-zero distinct integer solutions are obtained as,

$$x = -4a^2 + 12b^2 - 24ab$$

$$y = -8a^2 + 24b^2$$

$$z = -24a^2 + 72b^2 - 48ab$$

$$w = a^2 + 3b^2$$

Choice 8:

In (17) can also be written as

$$u^2 + 3v^2 = 48w^2 * 1 \quad (20)$$

Using (11) and (18) in (20) and proceeding as in pattern 1, the corresponding integral solutions are given by,

$$x = 4a^2 - 12b^2 - 24ab$$

$$y = -4a^2 + 12b^2 - 24ab$$

$$z = -96ab$$

$$w = a^2 + 3b^2$$

Properties:

1) $x(1, a(2a^2-1)) + 4w(1, a(2a^2-1)) + 24SO_a - 8 = 0$
2) $y(6a(a-1), 1) + 4w(6a(a-1), 1) + 24S_a - 48 = 0$
3) $y(b+1, b+2) + 4w(b+1, b+2) \equiv 48(mod\ 24)$
4) $x(a^2+a, (a+2)(a+3)) + y(a^2+a, (a+2)(a+3)) + 48Biq_n + 288Cub_n + 33t_{4,4a} \equiv 0(mod\ 288)$
5) $x(a,1) - y(a,1) - w(a,1) - 7t_{4,a} + 3J_3 + j_3 + 3 = 0$
1) $x(2b-1, b) + y(2b-1, b) + 48Gno_n = 0$

Note:

Using (4), (12) and (19) in (20), we have the following set of solutions satisfying (1).

$$x = -4a^2 + 12b^2 + 24ab$$

$$y = 4a^2 - 12b^2 + 24ab$$

$$z = 96ab$$

$$w = a^2 + 3b^2$$

Choice 9:

Using (4),(13) and (18) in (20) and employing the method of factorization we have the distinct integer solutions are as follows,

$$x = \frac{1}{7}[8a^2 - 24b^2 - 192ab]$$

$$y = \frac{1}{7}[-44a^2 + 132b^2 - 120ab]$$

$$z = \frac{1}{7}[-72a^2 + 216b^2 - 624ab]$$

As our aim is to find integral solutions choose a & b suitably so that the solutions are integers taking $a = 7A, b = 7B$

$$x = 56A^2 - 168B^2 - 1344AB$$

$$y = -308A^2 + 924B^2 - 840AB$$

$$z = -504A^2 + 1512B^2 - 4368AB$$

$$w = 49A^2 + 147B^2$$

Properties:

1. $z(a^2+a, 2a^2-1) + 9x(a^2+a, 2a^2-1) + 32928Biq_n - 1029t_{4,4a} + 16464SO_a = 0$
2. $7x(b+1, b) + 8w(b+1, b) - 16t_{4,7b} + 9408Pr_b \equiv 784(mod\ 1568)$
3. $11x(a, 2a^2+1) + 2y(a, 2a^2+1) + 32928Cub_n \equiv 0(mod\ 16464)$
4. $11x(a(a+1), a+2) + 2y(a(a+1), a+2) + 16464Cub_n + 1029t_{4,4a} + 32928Pr_a = 0$
5. $y(a, a^2) - 3696\,FN_a^4 - 616\,Biq_a - 840\,Cp_a^6 = 0$
6. $x(a,2) - 56\,Obl_a + j_9 \equiv 161(mod\ 2744)$

Choice 10:

CASE 1:

(17) can also be written as

$u^2 + 3v^2 = 48w^2 * 1$

$$u^2 - 36w^2 = 3(4w^2 - v^2) \quad (21)$$

Which is written in the form of ratio as,

$$\frac{u+6w}{3(2w+v)} = \frac{2w-v}{u-6w} = \frac{a}{b} \quad (22)$$

Which is equivalent to the system of equations,

$$bu - 3av + w(6b - 6a) = 0$$

$$au + bv - w(6a + 2b) = 0$$

Applying the method of cross multiplication we have,

$$u = 18a^2 - 6b^2 + 12ab$$

$$v = -6a^2 + 2b^2 + 12ab$$

$$w = b^2 + 3a^2$$

Substituting the values of u,v in (16), we get the non-zero distinct integer solutions to (1) to be

$$x = 12a^2 - 4b^2 + 24ab$$

$$y = 24a^2 - 8b^2$$

$$x = 72a^2 - 24b^2 + 48ab$$

$$w = 3a^2 + b^2$$

Properties

1. $2x(a, 2a^2 + 1) - y(a, 2a^2 + 1) - 96Cub_n \equiv 0(mod\ 48)$
2. $3y(b^2 + b, b) - z(b^2 + b, b) + 48Cub_n + 3t_{4,4b} = 0$
3. $x(a, a^2 + a) + 4w(a, a^2 + a) - 3t_{4,4a} - 24Cub_n = 0$
4. $y(2b^2 - 1, b) - 8w(2b^2 - 1, b)$is a perfect square
5. $3y(2b - 1, b) - z(2b - 1, b) + 48Gno_b = 0$
6. $2x(b + 1, b) - y(b + 1, b) - 4S_b - 48Pr_b - j_5 \equiv 9(mod\ 48)$

CASE 2:

(17) can also be written as,

$$\frac{u+6w}{2w+v} = \frac{3(2w-v)}{u-6w} = \frac{a}{b}$$

Which is equivalent to the system of equations,

$$bu - av + w(6b - 2a) = 0$$

$$au + 3bv - w(6a + 6b) = 0$$

Applying the method of cross multiplication we have,

$$u = 6a^2 - 18b^2 + 12ab$$

$$v = -2a^2 + 6b^2 + 12ab$$

$$w = a^2 + 3b^2$$

In view of (16), we get the non-zero distinct integer solutions to (1) are

$$x = 4a^2 - 12b^2 + 24ab$$

$$y = 8a^2 - 24b^2$$

$$x = 24a^2 - 72b^2 + 48ab$$

$$w = a^2 + 3b^2$$

Properties:

1. $2x(a, a) + y(a, a) + 512Biq_n = 0$
2. $x(a + 1, a) - 16Pr_a \equiv 4(mod\ 16)$
3. $x(a, a) and\ 4w(a, a)$represents a perfect square
4. $x(a, 1) - 4w(a, 1) + 3J_3 \equiv 17(mod\ 24)$
5. $3y(2a^2 - 1, a + 1) - z(2a^2 - 1, a + 1) + 96Cub_n + 48Pr_a + 3t_{4,4a} - 3J_5 - j_2 - 12 = 0$
6. $y(a^2, a) - 8\ t_{4,a^2} - 24\ CP_a^6 = 0$
7. $x(a^2, a - 1) - 4w(a^2, a - 1) - 36\ OH_a + 48\ Obl_a + 3\ J_3 + j_2 \equiv 10(mod\ 12)$

III. CONCLUSION

One may search for other Choices of solutions and their corresponding properties.

REFERENCES

1. Dickson, L.E.," History of the theory numbers", Vol.2: Diophantine Analysis, New York:Dover, 2005
2. Carmichael,R.D., "The theory of numbers and Diophantine Analysis", New York:Dover, 1959
3. Gopalan. M.A, ManjuSomanath and Vanitha,N., "On Ternary Cubic Diophantine Equation $x^2 + y^2 = 2z^3$ ",Advances in Theoretical and Applied Mathematics Vol.1,No.3 Pp.227-231, 2006
4. Gopalan. M.A, Manju Somanath and Vanitha,N., "On Ternary Cubic Diophantine Equation $x^2 - y^2 = z^3$ ", Acta Ciencia Indica, Vol,XXXIIIM, No.3. Pp.705-707, 2007
5. Gopalan, M.A., and Anbuselvi,R., "Integral solution of ternary cubic Diophantine equation $x^2 + y^2 + 4N = zxy$", Pure and Applied Mathematics Sciences, Vol.LXVII, No. 1-2, March Pp.107-111, 2008
6. Gopalan. M.A, ManjuSomanath and Vanitha,N., "Note on the equation $x^3 + y^3 = a(x^2 - y^2) + b(x + y)$ ", International Journal of Mathematics, Computer Sciences and Information Technologies Vol.No-1, January-June ,pp 135-136, 2008
7. Gopalan. M.A and Pandichelvi. V," Integral Solutions of Ternary Cubic Equation $x^2 - xy + y^2 = (k^2 - 2k + 4)z^3$ ", Pacific-Asian Journal of Mathematics Vol2, No 1-2,91-96, 2008
8. Gopalan.M.A.andKaligaRani.J."Integral solutions of $x^2 - xy + y^2 = (k^2 - 2kz + 4)z^3$ ($\alpha > 1$) and α is square free",ImpactJ.Sci.Tech., Vol.2(4)Pp201-204,2008
9. Gopalan.M.A.,Devibala.S., and Manjusomanath,"Integral solutions of $x^3 + x + y^3 + y = 4(z - 2)(z + 2)$,",Impact J.Sci.Tech., Vol.2(2)Pp65-69,2008
10. Gopalan. M.A, ManjuSomanath and Vanitha,N., "On Ternary Cubic Diophantine Equation $2^{2\alpha-1}(x^2 + y^2) = z^3$ ", ActaCienciaIndica, Vol,XXXIVM, No.3,Pp.135-137, .2008
11. Gopalan.M.A.,KaligaRani.J."Integral Solutions of $x^3 + y^3 + 8k(x + y) = (2k + 1)z^3$", Bulletin of pure and Applied Sciences,Vol.29E,(No.1)Pp95-99,2010
12. Gopalan.M.A.andJanaki.G., "Integral solution of $x^2 - y^2 + xy = (m^2 - 5n^2)z^3$ ",Antartica J.Math.,7(1)Pg.63-67,2010
13. Gopalan.M.A.,andShanmuganantham.P."OntheEquation $x^2 + xy - y^2 = (n^2 + 4n - 1)z^3$" , Bulletin of pure and Applied Sciences',Vol.29E, Pg231-235 Issue2, 2010
14. Gopalan.M.A. andVijayasankar.A.Integral Solutions of Ternary Cubic Equation $x^2 + y^2 - xy + 2(x + y + 2) = z^3$",Antartica J.Math.,Vol.7(No.4)pg.455-460,2010
15. Gopalan. M.A and Pandichelvi.V,"Observation on the cubic equation with four unknowns $x^2 - y^2 = z^3 + w^3$ ", Advances in Mathematics Scientific Developments and Engineering Applications, Narosa Publishing house, Chennai,Pp-177-187,2009
16. Gopalan.M.A., S.Vidhyalakshmi.,andG.Sumathiand"On the homogeneous cubic equation with four unknowns $x^3 + y^3 = 14z^3 - 3w^2(x + y)$" ,Discovery J.Maths Vol,2,No.4,pg17-19,2012
17. Gopalan.M.A., G.Sumathi., and S.Vidhyalakshmi "On the homogeneous cubic equation with four unknowns, $x^3 + y^3 = z^3 + w^2(x + y)$" ,DiophantusJ.Maths, Vol2(2),pg99-103,2013
18. Gopalan.M.A., S.Vidhyalakshmi.,andT.R.UshaRanid"On the homogeneous cubic equation with four unknowns, $x^3 + y^3 = (z + w)^2(z - w)$" ,Sch.J.Eng.Tech.,2014:2(2B):264-26
19. Anbuselvi.R. and Shanmugavadivu.S.A, "Integral solution of the ternary cubic equation with three unknowns $5(x^2 + y^2) - 9xy + x + y + 1 = 28z^3$ ",Vol2(11),Nov 2016, pg 7-10,E-ISSN No:2454-9916