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SOLVING FUZZY INTEGER LINEAR FRACTIONAL PROGRAMMING PROBLEM

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ABSTRACT A new method is proposed to solve fuzzy integer linear fractional programming problem. A linear fractional programming problem whose objective function, technological coefficients and resources are triangular fuzzy numbers is considered and it is transformed into a crisp linear fractional programming problem by means of Robust ranking method. The proposed method is applied to find the integer solution of the problem. A numerical example is given to illustrate the efficiency of the proposed method.

KEYWORDS : Linear fractional programming problem, Triangular fuzzy number, Robust ranking technique.

1. INTRODUCTION

The decision makers in the sectors like financial and corporate planning, production planning, marketing and media selection, university planning and student admissions, health care and hospital planning, etc. often face problems to take decisions that optimize department/equity ratio, profit/cost, inventory/sales, actual cost/standard cost, output/employee, student/cost, nurse/patient ratio etc. The above problems can be solved efficiently through linear fractional programming problems. Fractional programming has grown rapidly in the last four decades and it continues to find new applications in different areas of social life and economy. Linear fractional programming problem (LFPP) is one which optimizes one or more ratios of two linear functions and the decision variables must persuade certain conditions. The constraints are in the structure of linear equalities or linear inequalities.

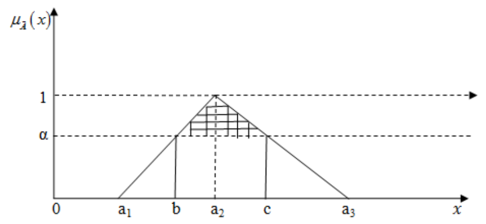
Charnes & Cooper (1962) transformed the linear fractional programming problem into the linear programming problem by adding a new constraint and a new variable. The optimum solution is obtained by simplex method. The concept of Decision making in fuzzy environment was first proposed by Bellman & Zadeh (1970). Zimmermann (1976) first introduced fuzzy linear programming as conventional LP and he considered LPP with a fuzzy goal and fuzzy constraints. Arsham & Kahn (1990) presented a complete algorithm for linear fractional programs.Tantawy (2008) suggested a feasible direction approach and a duality approach to solve a linear fractional programming problem. Babul Hasan & Acharjee (2011) introduced a computer oriented technique for solving linear fractional programming problem by converting it to a single linear programming problem. Dheyab (2012) proposed a complementary development method to solve Fuzzy Linear Fractional Programming Problems. Andrew Odior (2012) proposed a new method to solve linear fractional programming problems. Arefin & Islam (2013) presented additive algorithm for solving 0-1 integer linear fractional programming problem by taking all the coefficients of the numerator of the objective function are of same sign.

The paper is organized as follows. Preliminary concepts are given in Section 2. Section 3 includes the mathematical formulation of the linear fractional programming problem and fuzzy linear fractional programming problem. Section 4 gives the robust ranking technique and the proposed algorithm. Numerical example is given in Section 5 and Section 6 concludes the paper.

2. PRELIMINARIES

2.1 α-cut of a Fuzzy Set

The set of all elements of a fuzzy set $\tilde{A}$ with membership grades greater than or equal to α. The α - level cut or α- cut of the fuzzy set $\tilde{A}$ is defined by $A_\alpha = \{x / \mu_A(x) \geq \alpha, \forall x \in X\}$



2.2 Triangular Fuzzy Number

A triangular fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3)$ such that with membership function $\mu_{\tilde{A}}(x)$ is defined by

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

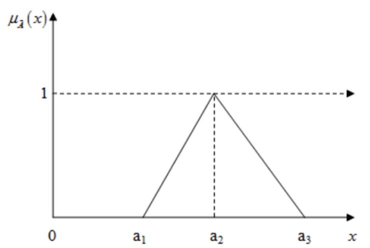


Figure 2. Membership function of a triangular fuzzy number $\tilde{A}$

3. MATHEMATICAL FORMULATION

3.1 Linear Fractional Programming Problem

A linear fractional programming can mathematically be represented as follows.

Maximize or Minimize

$$Z = \frac{c^T x + \alpha}{d^T x + \beta} \tag{3.1}$$

Subject to the constraints

$$Ax(\leq, =, \geq) b$$

$$x \geq 0 \tag{3.2}$$

where x is $n \times 1$ vector of decision variables, and c, d are $n \times 1$ vectors, A is $m \times n$ constraint matrix, b is $m \times 1$ vector, α and β are scalars.

The linear fractional programming problem (LFPP) becomes linear programming problem (LPP) if $d = 0$ and $\beta = 1$ in (3.1).

3.2 Fuzzy Linear Fractional Programming Problem

Owing to the existence of haziness in real world situations, it would be more comfortable to interpret the coefficients of the objective functions and the constraints as fuzzy numbers.

A fuzzy linear fractional programming can mathematically be represented as follows.

Maximize or Minimize

$$Z = \frac{c^T x + \alpha}{d^T x + \beta} \qquad (3.3)$$

Subject to the constraints

$$\tilde{A}x (\le, =, \ge) \tilde{b}$$
$$x \ge 0 \qquad (3.4)$$

Here x is $n \times 1$ vector of decision variables, and $\tilde{c}, \tilde{d}$ are $n \times 1$ fuzzy vectors, A is $m \times n$ constraint fuzzy matrix, b is $m \times 1$ fuzzy vector, $\tilde{\alpha}$ and $\tilde{\beta}$ are fuzzy scalars.

4. METHODOLOGY

4.1 ROBUST RANKING TECHNIQUE

Robust ranking technique is used to defuzzify the fuzzy numbers. If $\tilde{a} = (a_1, a_2, a_3)$ is a triangular fuzzy number, then the Robust Ranking index is defined by $R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U) d\alpha$, where (a_α^L, a_α^U) is the α – cut of the fuzzy number α and is defined by $(a_\alpha^L, a_\alpha^U) = ((a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha)$

4.2 PROPOSED ALGORITHM

Step 1: The fuzzy integer linear fractional programming is converted into crisp linear fractional programming problem by using robust ranking technique as below.

Maximize or Minimize

$$Z = \frac{c^T x + \alpha}{d^T x + \beta}$$

Subject to the constraints

$$Ax (\le, =, \ge) b$$
$$x \ge 0 \ , \ x \text{ is integer.}$$

Step 2: Convert the problem into its standard form by introducing slack and surplus variables for the inequality constraints. If the objective function is to be minimized, then it can be written as Maximize Z = - Minimize (-Z)

Let $S = \{x \in R^n / Ax = b, x \ge 0, x \in Z\}$. It is assumed that

(i) S is non empty and bounded.

(ii) $d^T x + \beta > 0$ for all $x \in S$.

Step 3: For every real number μ, define the subsidiary integer programming problem $P(\mu)$ as follows.

Maximize $Z = (c^T x + \alpha) - \mu(d^T x + \beta)$

Subject to

$Ax = b$

$x \ge 0$, and x is an integer.

Step4: Let X_0 be any feasible solution of P. Set $i = 0$.

Step5: Set $\mu_{i+1} = \dfrac{c^T x + \alpha}{d^T x + \beta}$. Solve the integer programming problem $P(\mu_{i+1})$. If X_i is be an optimal solution of $P(\mu_{i+1})$, then X_i is an optimal solution of P. Otherwise, let X_{i+1} be an optimal solution of $P(\mu_{i+1})$. Set $i = i + 1$ and repeat step 4.

5. NUMERICAL EXAMPLE

Consider the following Integer Fuzzy Linear Fractional Programming Problem Maximize

$$Z = \frac{-(0,2,4)x_1 + (1,3,5)x_2 + (1,5,9)x_3 + (1,2,3)}{(1,1,1)x_1 + (-1,2,5)x_2 + (0,1,2)x_3 + (-2,2,6)}$$

Subject to the constraints

$$\begin{aligned} &(-1,2,5)x_1 + (2,3,4)x_2 + (0,1,2)x_3 \le (-3,2,7) \\ &(1,1,1)x_1 - (-5,2,9)x_2 \ge (2,2,2) \\ &(0,1,2)x_1 + (-1,1,3)x_3 \le (7,8,9) \\ &x_1, x_2, x_3 \ge 0 \text{ and integers.} \end{aligned} \qquad (5.1)$$

Now, the FLFPP is converted into the following crisp LFPP using Robust ranking technique.

The α -cut of fuzzy number $(1,3,5)$ is

$$(a_\alpha^L, a_\alpha^U) = (2\alpha + 1, 5 - 2\alpha)$$

$$R(1,3,5) = \int_0^1 0.5(2\alpha + 1 + 5 - 2\alpha) d\alpha = 3$$

Proceeding similarly, the problem (5.1) can be written as the following crisp LFPP

$$\text{Maximize } Z = \frac{-2x_1 + 3x_2 + 5x_3 + 2}{x_1 + 2x_2 + x_3 + 2} = P_1$$

Subject to the constraints

$$2x_1 + 3x_2 + x_3 \le 2$$
$$x_1 - 2x_2 \ge 2$$
$$x_1 + x_3 \le 8$$
$$x_1, x_2, x_3 \ge 0 \text{ and integers} \qquad (5.2)$$

Change the inequality constraints into equality constraints by introducing slack and surplus variables. The problem (5.2) becomes

$$\text{Maximize } Z = \frac{-2x_1 + 3x_2 + 5x_3 + 2}{x_1 + 2x_2 + x_3 + 2} = P_1 \qquad (5.3)$$

Subject to the constraints

$$2x_1 + 3x_2 + x_3 + x_4 = 2 \qquad (5.4)$$
$$x_1 - 2x_2 - x_5 = 2 \qquad (5.5)$$
$$x_1 + x_3 + x_6 = 8 \qquad (5.6)$$
$$x_1, x_2, x_3 \ge 0 \text{ and integers} \qquad (5.7)$$
$$x_4, x_5, x_6 \ge 0 \qquad (5.8)$$

Let

$$P(\mu) = -2x_1 + 3x_2 + 5x_3 + 2 - \mu(x_1 + 2x_2 + x_3 + 2)$$

On solving the problem (3), $X_0 = (0,0,0,2,-2,8)$ is the feasible solution of P_1.

Set $\mu_1 = 1$.

Consider the problem $P(\mu_1)$

Maximize $Z = -3x_1 + x_2 + 4x_3$

Subject to (5.4), (5.5), (5.6), (5.7) and (5.8) (5.9)

$X_1 = (2,0,6,2,0,0)$ solves $P(\mu_1)$.

$$x_1 + x_3 + x_6 = 8 \qquad (5.6)$$

$$x_1, x_2, x_3 \geq 0 \text{ and integers} \qquad (5.7)$$

$$x_4, x_5, x_6 \geq 0 \qquad (5.8)$$

Let

$$P(\mu) = -2x_1 + 3x_2 + 5x_3 + 2 - \mu\left(x_1 + 2x_2 + x_3 + 2\right)$$

On solving the problem (3), $X_0 = \left(0,0,0,2,-2,8\right)$ is the feasible solution of P_1.

Set $\mu_1 = 1$.

Consider the problem $P\left(\mu_1\right)$

Maximize $Z = -3x_1 + x_2 + 4x_3$

Subject to (5.4), (5.5), (5.6), (5.7) and (5.8) (5.9)

$X_1 = \left(2,0,6,2,0,0\right)$ solves $P\left(\mu_1\right)$.

Set $\mu_2 = \dfrac{14}{5}$

Consider the problem $P\left(\mu_2\right)$

Maximize

$$Z = -\left(24/5\right)x_1 - \left(13/5\right)x_2 + \left(11/5\right)x_3 - \left(18/5\right.$$

Subject to (5.4), (5.5), (5.6), (5.7) and (5.8)

$X_2 = \left(2,0,6,2,0,0\right)$ solves $P\left(\mu_2\right)$.

Hence the process terminates and $X_2 = \left(2,0,6,2,0,0\right)$ is an optimum solution of P_1

6. CONCLUSION

A new method is presented to solve fuzzy integer linear fractional programming problem. An integer linear fractional programming problem is considered and the coefficients of the problem are taken as triangular fuzzy numbers. The fuzzy integer linear fractional programming problem is transformed into a crisp integer linear fractional programming problem by using Robust ranking technique. The proposed algorithm is applied to find the optimal integer solution of the problem. The numerical example illustrated the simplicity and effectiveness of the suggested method.

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