VOLUME-7, ISSUE-4, APRIL-2018 • PRINT ISSN No 2277 - 8160

Original Research Paper

Engineering

A METHOD OF SOLUTION TO FUZZY ASSIGNMENT PROBLEM USING GENETIC ALGORITHM

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ABSTRACT We present in this paper a genetic algorithmic approach to a fuzzy assignment problem. The fuzzy assignment problem can be described as a problem assigning n number of jobs to n number of machines with cost(time) as imprecise number by assigning each job to exactly one person. The fuzzy assignment problem is converted into crisp assignment problem using Yager's ranking method. Genetic algorithm approach is used to solve the problem. Numerical example shows the efficiency of this approach.

KEYWORDS : Fuzzy assignment problem, Trapezoidal fuzzy number, Yager's Robust Ranking method, Genetic Algorithm.

INTRODUCTION

A special type of linear programming problem such as assignment problem is a problem with the objective to assign n number of jobs to n number of persons at a minimum cost (time) or maximum profit(sale). The parameters of AP are imprecise number as an alternative of fixed real numbers because time/cost for doing a job by a facility (machine/person) might differ due to different reasons.

The concept of fuzzy sets to deal with ambiguity, pensiveness in real life has been introduced by Zadeh [25].In recent years, fuzzy assignment problems have received greater attention. Chen [2] projected a fuzzy assignment model that did not consider the differences of individuals, and also proved some theorems. Wang [23] solved an analogous model by graph theory.A flexible assignment problem, with the integration of fuzzy theory, multiple criteria decision-making and constraint-directed methodology has proposed by Dubois and Fortemps [5]. An example of fuzzy assignment problem was demonstrated and solved by them. Muruganandam et al.[18] proposed two objective assignment problem using fuzzy numbers. Lin and Wen [13] wished-for an efficient algorithm based on the labelling method for solving fuzzy assignment problems. The algorithm begins with primal feasibility and proceeds to obtain dual feasibility while maintaining complementary slackness until the primal optimal solution is found. Feng and Yang [6] investigated a two objective-cardinality assignment problem. Solution of generalized fuzzy assignment problem with the restriction on cost under fuzzy environment has proposed by Supriya Kar, Kajla Basu, Sathi Mukherjee[21].

Mukherjee and Basu[17] projected a new method for solving fuzzy assignment problems. Hadi Basirzadeh, Vahid Morovati, Abbas Sayadi [10] have developed a quick method to calculate the super-efficient point in multi-objective assignment problems. They also provided how to obtain the best non-dominated point for the multiobjective assignment problems which is more efficient and is so quick,simple while compared with other similar methods.

In this paper, a method of solution is proposed to find the fuzzy assignment problem by genetic algorithm approach. We organize this paper as follows: In section 1, we introduce the fuzzy assignment problem. In section 2, we construct the mathematical model for the problem. In section 3, we give the methodology for the problem. In section 4, we propose a solution algorithm to solve the problem. In section 5, a numerical example is given to show the efficiency of the algorithm and finally we give a conclusion for our problem.

2. MATHEMATICAL MODEL

2.1 The general assignment problem

Suppose there are n people and n jobs. Each job must be done by exactly one person, also each person can do, at most, one job. The problem is to assign the people to the jobs so as to minimize the total cost of completing all of the jobs. The general assignment problem can be mathematically stated as follows:

$$\text{Minimize } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} = 1 \text{ for } i = 1,2,...,n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for } j = 1,2,...,n$$

$$x_{ij} = 0 \text{ or } 1$$

2.2 Fuzzy assignment problem

$$\text{Minimize } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}\, x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for } i = 1,2,...,n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for } j = 1,2,...,n$$

$$x_{ij} = 0 \text{ or } 1$$

3. METHODOLOGY

3.1 Trapezoidal fuzzy number

A fuzzy number A is a Trapezoidal fuzzy number denoted by (a1, a2, a3,a4) and its membership function $\mu_A(x)$ is given below:

$$\mu_A(x)=\begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4, \\ 0, & \text{otherwise} \end{cases}$$

3.2 Yager's Ranking Approach

A number of ranking approaches have been proposed for comparing fuzzy numbers. Dominance of fuzzy numbers can be explained by many ranking methods [1, 3, 4, 8]. In this paper, Yager's ranking approach [24]is used for ranking of fuzzy numbers and which satisfies the properties of compensation,linearity and additivity.This approach involves relatively simple computational and is easily understandable.

Given a convex fuzzy number $\tilde{c}$, the Yager's Ranking Index is defined by

$Y(\tilde{c}) = \int_0^1 0.5\,(c_{\alpha L} + c_{\alpha U})\,d\alpha$, where $(c_{\alpha L}, c_{\alpha U})$ is the α-level cut of the fuzzy numbers $\tilde{c}$.

3.3. Genetic algorithm approach:

Genetic algorithms (GA) are computerized search and optimization algorithms based on the mechanics of natural genetics and natural selection. They were first envisioned by John Holland and were subsequently developed by various researchers. Each potential solution is encoded in the form of a string and a population of strings is created which is further processed by three operators: Reproduction, Crossover, and Mutation. Reproduction is a process in which individual strings are copied according to their fitness function (Here the fitness function is taken to be the total cost function). Crossover is the process of swapping the content of two strings at some point(s) with a probability. Finally, Mutation is the process of flipping the value at a particular location in a string with a very low probability.

A more comprehensive treatment of GA can be found in [9,14]. Sahu, Anshuman, and Rudrajit Tapadar[20]has solved the assignment problem using Genetic Algorithm and Simulated Annealing. Toroslu, Ismail H., and Yilmaz Arslanoglu[22] has introduced a new method to solve Genetic algorithm for the personnel assignment problem with multiple objectives. Liu. L., Gao X [15] has proposed Fuzzy Weighted Equilibrium Multi-Job Assignment Problem and Genetic Algorithm. Majumdar and Bhunia [16] proposed an elitist genetic algorithm to solve generalized assignment problem with imprecise cost/time. Ratli et.al [19] has proposed a hybrid genetic algorithm to solve bi-criteria assignment problem.

4. SOL UTION ALGORITHM:

GENETIC ALGORITHM:

Input: parameters (jobs, workers)

Output: to find the optimized minimum cost – by assigning job to worker or vice versa.

Let job=j, worker=i; where i=1 to n, j=1 to m.

```
Begin
Init Psize=I;
Select matrix R_ij and R_i' in it; R_ij
forC=k;k=randvalue(1ton);   //Cmentionsthechromosomes
(Rearrange)r1inRijusingk;//r1mentionsthefirstrow
Convertr1tobinaryvalues.
endfor
Inita=Intmin;
    if(a<<Ri')
    X←best(a);a
else
    Updatea;
    endif
BeginCrossover
LetPcross(R) ∈  (0,1)//crossoverprobability
SelectparentPfromRij;
fori=1;i≤n;l++
generaterandvalue(r) ∈ R
if®<Pcross)
selectPi
endif
resultC(P1',P2')  //selectedparentsforcrossover
End Crossover
Begin mutation
LetPcross(R) ∈  (0,1)//mutationprobability
N(Msite)=P(M)*Psize*Stringlength;
generaterandvaluetoselectchromosometomutated//rand.no.to
selectabitinthestring
swap(0,1)        //binaryvalues
End mutation
Select highestvaluesofbits(bmax) Ri'
If(bmax=bi(max))
(bmax)=Costlow
RepeatiflooptillCbestisobtained       //fitnessvalueisobtained
    endif
    ContinueforallinPsize
End
```

5.NUMERICALEXAMPLE

Three persons are available to do three different jobs. The cost that each person takes to do each job is represented by triangular fuzzy numbersandareshowninfollowingTable.

Jobs

Persons
$$\begin{pmatrix} (1,2,3,4) & (3,5,6,7) & (4,5,6,7) \\ (3,4,5,6) & (5,7,9,10) & (4,6,9,10) \\ (2,3,4,5) & (3,6,8,9) & (1,3,5,7) \end{pmatrix}$$

Find an optimal assignment of persons to jobs that will minimize the totalcost.

SOLUTION:

Fuzzy numbers are defuzzified into crisp numbers by Yager's ranking technique[24], is a robust ranking technique which satisfies compensation,linearity and additivity properties and provides result whichareconsistentwithhumanintuition.

Given a convex fuzzy number $\tilde{c}$, the Yager's Ranking Index is defined by $Y(\tilde{c}) = \int_0^1 0.5\,(c_{\alpha L} + c_{\alpha U})\,d\alpha$, where $(c_{\alpha L}, c_{\alpha U})$ is the α-levelcutofthefuzzynumbers.

Now we calculate y(2,3,4,5) by applying the Yager's ranking method. Themembershipfunctionofthetriangularfuzzynumber(2,3,4,5)is

$$\mu_A(x) = \begin{cases} \dfrac{x-2}{3-2}, & 2 \le x \le 3, \\ \dfrac{5-x}{5-4}, & 4 \le x \le 5, \\ 0 & \text{otherwise} \end{cases}$$

The α-cut of the fuzzy number (2,3,4,5) is $(c_{\alpha L}, c_{\alpha U}) = (2+\alpha, 5-\alpha)$.

$$Y(2,3,5) = \int_0^1 0.5\,(2+\alpha, 5-\alpha)\,d\alpha$$

$$= 0.5 \int_0^1 (2+\alpha+5-\alpha)\,d\alpha$$

$$= 0.5 \int_0^1 7\,d\alpha$$

$= 3.5 \sim 4$

Similarly, Y(1,2,3,4)=3, Y(3,5,6,7)=5, Y(4,5,6,7)=6, Y(3,4,5,6)=5, Y(5,7,9,10)=8, Y(4,6,9,10)=7,
Y(3,6,8,9)=7, Y(1,3,5,7)=4.
Nowthefuzzycostsaredefuzziedascrispcosts.Theproblembecomes

Jobs

Persons
$$\begin{pmatrix} 3 & 5 & 6 \\ 5 & 8 & 7 \\ 4 & 7 & 4 \end{pmatrix}$$

GENETICALGORITHMAPPROACH:

GenerationI:

INITIALIZATION:

S.No	String	Cost
1	(1 2 3)	(3+8+4)=15
2	(2 3 1)	16
3	(1 3 2)	17
4	(2 3 1)	16
5	(3 2 1)	18
6	(2 3 1)	16
7	(3 1 2)	18
8	(2 1 3)	14
9	(3 2 1)	18
10	(2 1 3)	14

CROSSOVER & MUTATION:

Decision	Crossover	After crossover	Mutation	After mutation	Cost
No	(1 2 3)	(1 2 3)	(1 2 3)	(1 2 3)	15
Yes	(2 3 1)	(2 3 1)	(2 3 1)	(2 3 1)	16
Yes	(1 3 2)	(2 3 1)	(2 3 1)	(2 1 3)	17
Yes	(2 3 1)	(1 3 2)	(1 3 2)	(1 3 2)	16
Yes	(3 2 1)	(2 3 1)	(2 3 1)	(1 3 2)	17
Yes	(2 3 1)	(3 2 1)	(3 2 1)	(3 2 1)	18
Yes	(3 1 2)	(3 1 2)	(3 1 2)	(3 1 2)	18
Yes	(2 1 3)	(2 1 3)	(2 1 3)	(2 1 3)	14*
Yes	(3 2 1)	(2 1 3)	(2 1 3)	(2 1 3)	14*
Yes	(2 1 3)	(3 2 1)	(3 2 1)	(3 2 1)	18

Minimum cost

Generation 2:

S.No	String	Cost
1	(1 2 3)	15
2	(2 3 1)	16
3	(2 1 3)	17
4	(1 3 2)	16
5	(1 3 2)	17
6	(3 2 1)	18
7	(3 1 2)	18
8	(2 1 3)	14*
9	(2 1 3)	14*
10	(3 2 1)	18

CROSSOVER& MUTATION:

Decision	Crossover	After crossover	Mutation	After mutation	Cost
Yes	(1 2 3)	(1 2 3)	(1 2 3)	(2 1 3)	14*
No	(2 3 1)	(2 3 1)	(2 3 1)	(2 3 1)	16
Yes	(2 1 3)	(3 1 2)	(3 1 2)	(1 3 2)	17
Y	(1 3 2)	(1 2 3)	(1 2 3)	(1 2 3)	15
N	(1 3 2)	(1 3 2)	(1 3 2)	(1 3 2)	17
Y	(3 2 1)	(3 2 1)	(3 2 1)	(3 2 1)	18
Y	(3 1 2)	(3 1 2)	(3 1 2)	(3 1 2)	18
Y	(2 1 3)	(2 1 3)	(2 1 3)	(2 1 3)	14*
Y	(2 1 3)	(2 3 1)	(2 3 1)	(2 3 1)	16
Y	(3 2 1)	(1 2 3)	(1 2 3)	(1 2 3)	15

Proceeding this way we get

Generation 11:

S.No	String	Decision	After crossover	After Mutation	Cost
1	(2 3 1)	Y	(2 3 1)	(3 2 1)	18
2	(2 3 1)	Y	(2 3 1)	(2 3 1)	16
3	(2 3 1)	Y	(2 3 1)	(2 3 1)	16
4	(1 2 3)	N	(1 2 3)	(1 2 3)	15
5	(1 3 2)	N	(1 3 2)	(1 3 2)	17
6	(3 2 1)	Y	(3 2 1)	(3 2 1)	18
7	(3 2 1)	Y	(3 2 1)	(3 2 1)	18
8	(2 1 3)	N	(2 1 3)	(2 1 3)	14*
9	(2 3 1)	Y	(1 3 2)	(1 2 3)	15
10	(1 3 2)	Y	(2 3 1)	(2 3 1)	16

Optimum Assignment = A $\leftrightarrow$ 2, B $\leftrightarrow$ 1, C $\leftrightarrow$ 3

Optimum (minimum) cost=14

Solution by Hungarian Method:

$$\begin{pmatrix} 3 & (5) & 6 \\ (5) & 8 & 7 \\ 4 & 7 & (4) \end{pmatrix}$$

Optimum Assignment = A $\rightarrow$ 2, B $\rightarrow$ 1, C $\rightarrow$ 3

Optimum (minimum) assignment Cost=5+5+4=14.

6. CONCLUSION:

Assignment problem is of great use in decision-making, e.g. resource allocation problem, such as assigning personnel to jobs, tasks to machines, etc. As the usual case, in real world, the managers hope not only to promote the quality of each job, task, etc., but also to minimize the total cost used. In this paper, we model a fuzzy Assignment problem in order to overcome such an uncertain environment in the real world application and propose a Yager's ranking approach for defuzzyfication. Genetic algorithm is used to solve the defuzzified assignment problem. Computational experience shows that our proposed method performs satisfactorily.

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