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STEADY STATE ANALYSIS IN N FAILURE OF K PHASE (N,K) REPLACEMENT QUEUEING SYSTEM IN MATRIX METHOD

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ABSTRACT We consider the N- groups of bulk arrival queueing system and arriving customer are independent. The replacement queueing process is beginning with the failure node and the variable cost proportional to the number of failure channels. Due to the nature of problem some service channels are provided the delay service obviously. So, we distributed to the service cost in entire system. If we kept the delay process the group maintenance are decreases on the other hand length of the customer ratio has increase as well as processing cost also. But most of the time queue parameters are uncertain. In this paper we develop the group of m-failure channel and customers are independent, but retained the queue start with failure stages at the level of m and analyze average length of queue replacement size and cost. Here, we consider the queue parameters are triangular fuzzy number. A numerical example is included

KEYWORDS : Group maintenance policy; N-failure customer in K – phase distribution, Matrix-geometric method; Fuzzy membership function

Introduction:

We consider the group of bulk arrival queueing system and each group consists of k-phase that phase consider as erlang distribution. Generally the system replacement is preparable to after losing the service or due to inconvenience of service station. This type of model is mostly used in industrial, production, etc., because replacement is starting from the renovation process up to the facility of receive the delay collections of system. In this case the replacement costs are fixed. Many researchers have been devoted to optimal replacement procedures of the group replacement models. Mostly analysis system in the number of repaired system until due to the scheduled time, simultaneous the failed systems are fixed. Hunter [1965] proposed the unique optimal policy of the increasing repair failure rate. Extended this work Okumoto(1983) obtained the optimal cost analysis of failure system and the service channels are considered in exponential distribution. [9]Assaf (1987) has proposed two models (i) Well known for the failure phases and resulted that the how to simulate the queueing system (ii) Depart from the above assumption and require an inspection to determine the number of failure system. Benmerzouga(1990) discussed the m-failure system analyze the optimal cost in nonnegative repair time. Nakagawa has minimized mean cost rate of the schedule replacement time. Ritchken(1990) have considered the (m,T) group replacement model which requires the scheduled time or when exactly m system have failed, whichever comes first. So, we replaced the failure system in new units. The service cost are sustained the variable replacement cost and the idle time penalties. The analysis of the group replacement model is also related that the single as well as multiservice channel with minimal repair and replacement options. Generally, the cost rates are depending only on the number of jobs waiting for service at any time. It means that we can obtain the production loss of the traditional group replacement models. The above problem has described in the category of queueing system with untrustworthy servers and analyzed the balance equation Markovian system. Netus (1981) motivated the problem analyzing the steady state behavior of M/G/1 and G/M/q queueing system. Netus and Lucantoni (1979) considered group of arrival in c<N repair system. When the servers are fall down we immediate replace another server or otherwise the increase the queue size as well as breakdown to the entire system. This type of system only performed in the group replacement model and calculated the length of waiting in erlang distribution. But due to some insufficient availability of the service resources or some natural calamities the queues parameters are not certain. So, we haven't obtained the exact optimal cost. In this case fuzzy set theory is most helpful to analyze the exact optimal cost value. L.A Zadeh has very first introduced the fuzzy set theory. Later many author proved some more results in fuzzy queueing theory Julia Rose Mary, Gokilavani , Ritha. & Sreelekha Menon . In this paper, we analyze the optimal replacement queueing cost in fuzzy environment with the help of Neuts and Lucantoni algoritm and the parameters are consider as a triangular fuzzy number.

Problem Description and Formulation: Let us consider the $FM/FE_k/1$ queueing service station in which idle fraction of server's time is optimized by shifting the server to another service station while is in cyclic. The k-type Erlang distribution made up of K independent and identical exponential stages each with mean $1/k\mu$. In this paper we follow the general discipline and they are interested in the optimal management policy of the $FM/FE_k/1$queueing system. Here we follow the parallel service channel. The failure system is characterized by m-group maintenance process. For particularly N< m (number replacement system performed as long as the number of interruption servers), suppose the service has performed by a group of repairmen denoted by p and repair rates denoted by k_w and it is equal to $\dfrac{kp}{N-w}$. After that renovation is completed, all replacement servers becomes operating on same time. Already this type of models are discussed Neuts and Lucantoni (1979) where the failed servers start repair at the failure instant if any repairman is available.

The inconsiderable generator $\overline{Q}$ takes the form as:

$$\overline{Q}=\begin{bmatrix} R_{00} & R_{01} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ R_{10} & R_{11} & R_{12} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & R_{N-2,0} & R_{N-2,1} & R_{N-2,2} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & R_{N-1,0} & R_{N-1,1} & R_{N-1,2} & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & A_2 & A_1 & A_0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_2 & A_1 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_2 & \cdots \\ . & . & . & . & . & . & . & . & . & . \end{bmatrix} \text{---------} (1),$$

Where $\Delta(\lambda)=diag(\lambda_0,\lambda_1,\ldots,\lambda_N)$, $\Delta(\mu)=diag(0,\mu,2\mu,\ldots,N\mu)$, $R_{00}=Q-\Delta(\lambda)$, $R_{01}=R_{12}=R_{22}=\ldots=R_{N-1,2}=A_0=\Delta(\lambda)$, -------------**(2)**

$R_{z0}=diag\{\mu\min(z,w),0\le w\le N\}$, $for\ 1\le z\le N$, $R_{z1}=Q-\Delta(\lambda)-T_{z0}$ $for\ 1\le z\le N-1$, $A_2=\Delta(\mu)$, $A_1=Q-\Delta(\lambda)-A_2$, Where the matrix Q is the generator which describes the replacement process: $Q_{w,w-1}=wf$ $for\ N-m+1\le w\le N$, $Q_{w,w-1}=0$ $for\ 1\le w\le N-m$,

$Q_{w,b}=0$ $for\ w-b>1$ $When\ 0\le w\le N$ $and\ 0\le b<N$

$Q_{w,N}=k_w$ $for\ 0\le w\le N-m$, $Q_{w,b}=0$ $for\ b-w>0$ $and\ 0\le b<N$ -------- **(3)**

$Q_{00}=-k_0$, $Q_{w,w}=-Q_{w,w-1}-Q_{w,N}$ $for\ 1\le w\le N-m$,

$Q_{w,w}=-Q_{w,w-1}$ $for\ N-m+1\le w\le N$.

Here m is the number of group failure system, we have

$$Q=\begin{bmatrix} -t_0 & 0 & . & 0 & 0 & 0 & . & 0 & 0 & t_0 \\ 0 & -t_1 & . & 0 & 0 & 0 & . & 0 & 0 & t_1 \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & . & 0 & t_{N-m} & 0 & . & 0 & 0 & t_{N-m} \\ 0 & 0 & . & 0 & (N-m+1)f & -(N-m+1)f & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & . & 0 & 0 & 0 & . & (N-1)f & -(N-1)f & 0 \\ 0 & 0 & . & 0 & 0 & 0 & . & 0 & Nf & -Nf \end{bmatrix}$$ -------- (4)

But, Neuts and Lucantoni has discussed only for certain cases as well as queueing parameters are consider as Poisson and exponential distribution. Now we extended this work based k-phase erlang distribution and together with the cost analysis in fuzzy environment.

Stability and Steady State Analysis

To analysis the stability of the queueing system, it follows that the above propagation model it's described the queue is: $\pi\lambda<\pi\mu$,where π denotes the steady state probability of Q in (4) and $\bar{\lambda}=[\lambda_0,\lambda_1,\lambda_2\ldots\ldots\lambda_N]$ $\bar{\mu}=[\mu_0,\mu_1,\mu_2\ldots\ldots\mu_N]$ The steady state condition taken from $\pi\lambda<\mu\sum_{w=1}^{N}W\pi_w$ -- (1) the matrix method can be applied for steady state analysis (i) N=m=1 (ii) genetic algorithm of $N\ge 2$

Case: 1 If N=m=1, the explicit solution for $\begin{bmatrix} -kp & kp \\ f & -f \end{bmatrix}$ ------- (2), using the steady state condition of transition probability matrix, we obtain $\pi_0=f/(kp+f)$ and $\pi_1=kp/(kp+f)$ ------- (3)

Case: 2 The queue size is constant or $\bar{Q}$ is +ve persistent iff $\pi\lambda<\mu\sum_{w=1}^{N}W\pi_w\to f\lambda_0+kp\lambda_1<kp\mu$

that is $\pi\lambda<kp\mu$ -------- (4)

Case: 3 The insignificant optimal solution of the equation is $\sum_{k=0}^{\infty}\varphi^k A^k=0$ and it is follows that

$$\Delta(\lambda)+\varphi[Q-\Delta(\lambda+\mu)]+\varphi^2\Delta(\mu)=0 \Rightarrow \begin{cases} \varphi_{00}(kp+\lambda_0)-f\varphi_{01}-\lambda_0=0 \\ \mu(\varphi_{00}\varphi_{01}+\varphi_{01}\varphi_{11})+kp\,\varphi_{00}-\varphi_{01}(f+\lambda_1+\mu)=0 \\ \varphi_{10}(kp+\lambda_0)-f\varphi_{11}=0 \\ \mu(\varphi_{10}\varphi_{01}+\varphi_{11}^2)+kp\,\varphi_{10}-\varphi_{11}(f+\lambda_1+\mu)+\lambda_1=0 \end{cases}$$ ------------- (5)

Where $\varphi\mu=\lambda$ can be used to facilitate the solution of the quadratic system (5),

But $$\varphi=\begin{bmatrix} \frac{\lambda_0(f+\mu)}{\mu(kp+\lambda_0)} & \frac{\lambda_0}{\mu} \\ \frac{f\lambda_1}{\mu(kp+\lambda_0)} & \frac{\lambda_1}{\mu} \end{bmatrix}$$ ----------- (6)

To find y_0 using the geometric equation $y_x=y_0\varphi^x$, Therefore $\sum_{x=0}^{\infty} y_x = \pi \Rightarrow y_0(I - \varphi)^{-1} = \pi \Rightarrow y_0 = \pi(I - \varphi)$ ------- **(7)**

By substituting (5 & 6) into (7), We obtain $y_0 = \pi(I - \varphi) = \left[\frac{f(\mu kp - \lambda_0 f - \lambda_1 kp)}{\mu(kp + \lambda_0)(kp + f)} \quad \frac{\mu \; kp \lambda_0 \; f \; \lambda_1 kp}{\mu(kp + f)} \right]$ ------ (8)

Case: 4 y_0 Satisfies equation $y_0 B[\varphi] = 0$, where $B[\varphi] = \sum_{k=0}^{1} \varphi^k R_{k0}$ ------- (9)

By substituting the equation (2, 2, 6) into (7) ,we obtain $y_0 B[\varphi] = y[Q_0 - (\Delta\bar{\lambda}) + \varphi(\Delta\bar{\mu})]$ = [0, 0]

Case: 5 y_0 also satisfy normalizing equation as follows:

$$y_0(I - \varphi)^{-1} e = \left[\frac{f(\mu kp - \lambda_0 f - \lambda_1 kp)}{\mu(kp + \lambda_0)(kp + f)} \quad \frac{\mu kp - \lambda_1 kp - f\lambda_0}{\mu(kp + f)} \right] \begin{bmatrix} \frac{\mu kp - \lambda_0 f}{\mu(kp + \lambda_0)} & -\frac{\lambda_0}{\mu} \\ \frac{-f\lambda_1}{\mu(kp + \lambda_0)} & \frac{\mu - \lambda_1}{\mu} \end{bmatrix}^{-1} e = 1$$

Case: 6 The steady state distribution is:

$y_x = y_0 \varphi^x$, where $y_0 = \left[\frac{f(\mu kp - \lambda_0 f - \lambda_1 kp)}{\mu(kp + \lambda_0)(kp + f)} \quad \frac{\mu \; kp \lambda_0 \; f \; \lambda_1 kp}{\mu(kp + f)} \right]$, Recall that for any x, $y_x = (y_{x0}, y_{x1})$ where y_{x0} = P[x customers in system, server is not working], and y_{x1} = P[x customers in system, server is working].

Algorithm of Cost Analysis:

In this section propose an algorithm of expected average cost of replacement queueing model in steady state position (long run) and defined as follows:

(i) Expected Number of customer in the system:

Consider $\overline{M} = \sum_{i=0}^{N} M_i$, Where $M_k = \pi_k E(N)_k$ and $E(N)_k$ = E (Number of customer working as k servers) for $0 \le s \le N$.

Using the stability condition of steady state analysis and to derive the $\overline{M}$ as follows:

$\overline{M} = \sum_{x=1}^{\infty} x y_x = \sum_{x=1}^{N-2} x y_x + \sum_{x=N-1}^{\infty} x y_x$. Then we have

$$\sum_{x=N-1}^{\infty} x y_x = y_{N-1} \sum_{x=N-1}^{\infty} x \varphi^{x-N+1} = y_{N-1} \sum_{j=0}^{\infty} (j + N - 1)\varphi^j = y_{N-1} \sum_{j=0}^{\infty} j\varphi^j + y_{N-1} \sum_{j=0}^{\infty} (N-1)\varphi^j$$

$$= y_{N-1}\varphi \sum_{j=1}^{\infty} j\varphi^{j-1} + y_{N-1}(N-1)(I - \varphi)^{-1}$$

$$= y_{N-1}\varphi(I - \varphi)^{-2} + y_{N-1}(N-1)(I - \varphi)^{-1} = y_{N-1}(I - \varphi)^{-2}[(N-1)I - \varphi(N-2)]$$

Therefore, we have $\overline{M} = \sum_{x=1}^{N-2} x y_x + y_{N-1}(I - \varphi)^{-2}[(N-1)I - (N-2)\varphi]$ ------- (10)

(ii) Average holding cost: Consider h denotes the holding cost per customer. The Average holding cost is defined as $h\overline{M}$ --------- (11)

(iii) Expected cycle time: Let γ_{ij} represent the average transition time from the state i to j, and γ_{ij} is the repetition time of the state i. We know that the cyclic time lies between two sequential repairman. But we start with N-k operating servers in the replacement system and it's defined by $\gamma_{N-k,N-k}$. From the Markov renewal processes (Ross, 1970), we achieve the as follows:

$$\gamma_{j,N-k} = -\frac{1}{Q[j,j]} + \sum_{l \neq N-k, l \neq j} \frac{Q[j,l]}{-Q[j,j]} t_{l,N-k}, \quad for\ j = 0, 1, ..., N. \text{----- (12)}$$

Using (Q-4), we get $\gamma_{N-k,N-k}$. Because failure services are idle due to the period. But, the sequential servicing processing cost as:

$$\gamma_{N-k,N-k} = \frac{1}{v_{N-k}} + \sum_{i=N+1-m}^{N} \frac{1}{if} \text{------(13)}$$

(iv) Expected variable cost of repair: We start with the k-group replacement is N-k, the number of repaired servers for each group obtained as: variable repair cost = kv_c

(v) Expected average cost: The average cost is involving (i, ii, iii and iv) it's denoted by S and defined as:

$$Expected\ Average\ Cost = \frac{S}{\gamma_{N-k,N-k}} + h^* \overline{M} + \frac{kv_c}{\gamma_{N-k,N-k}} \quad \text{---------- (14)}$$

Equation (14) is used to the analyze the optimal group replacement parameter.

Proposed Algorithm: Consider the arrival rate λ and service rate μ are fuzzy number and the remaining costs are considered as crisp value. We have $\overline{\lambda} = \{(x, \mu_{\overline{\lambda}}(x)) / x \in X\}$, $\overline{\mu} = \{(y, \mu_{\overline{\mu}}(y)) / y \in Y\}$ we know that the fuzzy set ω is convex $\mu_{\varpi}(\beta x_1 + (1-\beta)x_2) \geq \min\{\mu_{\varpi}(x_1), \mu_{\varpi}(x_2)\}$. It is following the extension of Zadeh's principle the membership function of the objective value is defined by $\mu_{\overline{p}(\overline{\lambda},\overline{\mu})}(Z) = \sup_{\substack{x\in X, y\in Y, h\in H, \\ s\in S, v\in R}} \min\{(\mu_{\overline{\lambda}}(x), \mu_{\overline{\mu}}(y))\}$.

The α - cut of fuzzy arrival and service are defined as follows: $\overline{\lambda_\alpha} = [x_\alpha^l, x_\alpha^u] = [\min\ \{x/\mu_\lambda(x) \geq \alpha\ \},\ \max\ \{x/\mu_\lambda(x) \geq \alpha\}]$ $\overline{\mu_\alpha} = [y_\alpha^l, y_\alpha^u] = [\min\ \{x/\mu_\mu(y) \geq \alpha\ \},\ \max\ \{y/\mu_\mu(y) \geq \alpha\}]$

It represented the queue parameter in different possibility levels of intervals, therefore; FM/FEk/1 can be summarized the family of crisp M/Ek/1 queues in the α -level sets. The Appearance of above two sets is relation between ordinary and fuzzy sets. The bounds of α cut fuzzy interval obtained as $x_\alpha^l = \min(\mu_\lambda^{-1}(\alpha))$, $x_\alpha^u = \max(\mu_\lambda^{-1}(\alpha))$, $y_\alpha^l = \min(\mu_\mu^{-1}(\alpha))$ and $y_\alpha^u = \max(\mu_\mu^{-1}(\alpha))$. From the equation (10, 11, 12, 13 and 14) has converted into fuzzy approach with the help of Zadeh's extension principle. Finally we get different possibility value of α can be collected to approximate the shapes of the expected number of customers in the system. The fuzziness values are converted to crisp value using Robust Ranking Technique and we estimated the queues parameters used in statistical manner.

Numerical Example: Consider an FM/FE$_k$/1 queueing system of k phase replacement service station. Also we may consider the parameters are triangular fuzzy number. The arrival and service costs are $\overline{\lambda}$ = (2 3 4); $\overline{\mu}$ = (7 8 9)) (i) The total number of system is N=8 (ii) The number of replacement group k=4 (iii) the arrival rate $\lambda_0 = \lambda_1 = \quad = \lambda_8 = \lambda = (2,3,4) = 3$ (iv) the service rate $\overline{\mu}$ = (7 8 9) = 8 (v) The service failure rate f = 1 (vi) Repair cost for each service node p=4 (vii) fixed repair cost S=25 (8) variable repair cost per failed machine V_c =2; (9) Holding cost per customer per unit time h=20. The α-cut value of arrival rate, service rate this system is $\overline{\lambda} = [x_\alpha^L, x_\alpha^U] = (\alpha + 2,\ 4 - \alpha)$, $\overline{\mu} = [y_\alpha^L, y_\alpha^U] = (\alpha + 7,\ 9 - \alpha)$, Using our proposed algorithm we obtain the following results:

Table: 1

N=8,c=4, λ_k = 3, $k = 0,..,8$,μ=5, f=1, r=4, S=25, v_c =2, h=20				
	Stability rate $=\pi\lambda/\pi\mu$	Avg. Number of customer	Cycle time $\gamma_{N-k,N-k}$	Expected avg. cost
K=0	0	0	0	0
k=1	0.1173	1.3669	0.1875	97.3382
k=2	0.1380	1.3677	0.3928	95.1730
k=3	0.1632	1.3714	0.3220	80.2655
k=4	0.1948	1.3859	0.5845	64.0276
k=5	0.1360	1.4386	0.8023	68.0116
k=6	0.1932	1.5142	0.9285	73.5132
k=7	0.1829	2.1817	1.5553	79.7286
k=8	0.1703	4.8623	2.2178	97.9885

From the above table, we can see that stability rate, average number of customer in the system, cyclic time are increasing and expected average cost is convex in k.

Conclusion: Here, we consider as k-failure customer an independent and the group replacement model. We presently formulated the matrix method and to calculate the steady state distribution and find the expected fuzzy average cost as a function of the replacement time parameter k. In this system consider optimistic replacement time and no failure servers are allowed during the replacement process. But, we analyze the expected average number of customer in system and holding costs are obtained in fuzzy environment. We presented there exists an optimal group replacement parameter $1 \leq k \leq N$, which can be obtained the minimal average cost for this system. The system performances are analyzed in statistical manner. Finally, we identified unique optimal cost for this system.

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