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GENERALIZED HYPERBOLIC APPROXIMATION OF WAVE PROPAGATION IN LAYER AT CHANGING COEFFICIENTS

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ABSTRACT We investigate the contribution of the sixth-order hyperbolic approximation to the phase and group velocities of the propagation of harmonic waves in an elastic layer . The analysis is based on the generalized equation obtained as a special case of 4-dimensional space resulting from n-dimensional Euclidean space. The influence of the Poisson's ratio on the solvability of approximations is investigated.

KEYWORDS : wave propagation, generalized equation, phase velocity, group velocity, hyperbolic approximation

INTRODUCTION

The problem of elastodynamics for the layer under the assumption of small layer thickness has been considered in the works of Cauchy (1828) [1] and Poisson (Poisson, 1829) [2]. An analytical generalization of the Cauchy-Poisson method to an n-dimensional Euclidean space was obtained in [Selezov, 2000] [3], and a generalized hyperbolic approximation of the sixth order was constructed there as a special case. In work (Selezov, 2018) [4], some restrictions and justifications were introduced, so that hyperbolic approximations of any order can be obtained. The Timoshenko's equation (Timoshenko, 1921) [5], as phenomenological one, includes an undeterminated correcting coefficient. This equation is derived analytically in [4] as the fourth-order approximation from n-dimensional Euclidean space.This paper presents the analysis of wave propagation in the elastic layer based on the generalized hyperbolic equation. The effect of the Poisson's ratio on the principal part of differential operator is investigated.

The generalized equation

Corresponding to the 4-dimensional space (three spatial coordinates, and one time) infinite system includes the expansion in the series of planar displacements along the transverse coordinate and the recurrence relations resulting from the equations of motion. Keeping the terms (differential operators) to the order of at least 8, we have obtained a hyperbolic approximation as an approximation of the 6th order in the form of a generalized differential equation (Selezov, 2018) [4]

$$\left\{\left[\left(\frac{\partial^2}{\partial t^2}+a_1\nabla^2\nabla^2\right)_K-a_2\frac{\partial^2}{\partial t^2}\nabla^2+a_3\frac{\partial^4}{\partial t^4}\right]_{TM}-\right.$$
$$-b_1\nabla^2\nabla^2\nabla^2+b_2\frac{\partial^2}{\partial t^2}\nabla^2\nabla^2-b_3\frac{\partial^4}{\partial t^4}\nabla^2+$$
$$\left.+b_4\frac{\partial^6}{\partial t^6}\right\}_{TMC}w_0=\left\{\left[1-d_1\nabla^2+d_2\frac{\partial^2}{\partial t^2}\right]_{TM}+d_3\nabla^2\nabla^2-\right.$$
$$\left.-d_4\frac{\partial^2}{\partial t^2}\nabla^2+d_5\frac{\partial^4}{\partial t^4}\right\}_{TMC}\left(q^+-q^-\right) \qquad (1)$$

where $w_0 = u_3$ is the transverse component of the displacement vector, $(q^+ - q^-)$ - transverse load, $\nabla^2=\vec{\nabla}\cdot\vec{\nabla}$ is the planar Laplacian, a_k, b_r, d_m -constant coefficients for each fixed Poisson's ratio.

The dimensionless form

Dimensionless variables are introduced in (1) and everywhere on later by formulas, taking as typical values the thickness 2 h (m), the shear wave velocity (m/s), the density (kg/m3) (Selezov et al., 2018) [6]

$$u_k^*=\frac{1}{2h}u_k,\ (x_1^*,x_2^*)=\frac{1}{2h}(x_1,x_2),$$
$$t^*=\frac{c_s}{2h}t\ ,\ q^*=\frac{1}{G}q\ ,\ h^*=\frac{1}{2},\ c_s^2=G/\rho\ ,$$

Where $G=\frac{E}{2(1+\nu)}$ is the shear modulus, E Yong's modulus, ν Poisson's ratio. In studying the propagation of waves the dimensionless variables are introduced:

$l^*=\frac{1}{2h}l$ wave length, $c^*=\frac{c}{c_s}$ phase velocity. The asterics are omitted on later.

The dispersion equation

The dispersion equation corresponding to equation (1) is of the form

$$b_4c^6-\left[a_3\left(\frac{l}{2\pi}\right)^2+b_3\right]c^4+\left[\left(\frac{l}{2\pi}\right)^4+a_2\left(\frac{l}{2\pi}\right)^2+b_2\right]c^2-\left[a_1\left(\frac{l}{2\pi}\right)^2+b_1\right]=0.$$

Where the coefficients are as follows:

$$a_1=\frac{1}{6(1-\nu)}\ ,\quad a_2=\frac{2-\nu}{6(1-\nu)}\ ,\quad a_3=\frac{7-8\nu}{48(1-\nu)}$$
$$b_1=\frac{1}{120(1-\nu)},\qquad b_2=\frac{4\nu^2-16\nu+11}{480(1-\nu)^2},$$
$$b_3=\frac{16\nu^2-37\nu+19}{5760(1-\nu)^2},\qquad b_4=\frac{64\nu^2-104\nu+41}{7680(1-\nu)^2} \qquad (3)$$

Dispersion equation of sixth order in terms $c^*(l^*)$ has the form:

$$b_4c^6-\left(\frac{a_3}{l^2}+b_3\right)c^4+\left(\frac{a_0}{l^2}+\frac{a_2}{l^2}+b_2\right)c^2-$$
$$\left(\frac{a_1}{l^2}+b_1\right)=0 \qquad (4)$$

$$a_1=\frac{a_1}{(2\pi)^2},\quad a_2=\frac{a_2}{(2\pi)^2},\quad a_0=\frac{1}{(2\pi)^4},\quad a_3=\frac{a_3}{(2\pi)^2};$$
$$a_3=\frac{a_3}{(2\pi)^2}=\frac{0.13690}{39.47835}=0.00347\ ,$$

$$a_0=\frac{1}{(2\pi)^4}=\frac{1}{15558.5401}=0.00064$$

$$a_2=\frac{a_2}{(2\pi)^2}=\frac{0.40476}{39.47835}=0.01025,$$

$$,a_1=\frac{a_1}{(2\pi)^2}=\frac{0.23809}{39.47835}=0.00603$$ all of them are for $\upsilon=0.3$

Phase velocity

The phase velocity $c(l)$, depending on the wavelength l , determines the solvability of the problem in the class of traveling waves exp [$l(kx-\varpi t)$]. Here it is investigated in the interval $l\in$ [0,10) ccording to equation (4) for $v = 0.3$. The results of calculations are shown at Fig. 1. In the considered interval, the equation has one real root for $v = 0.3$. The dashed line represents the phase velocity in the 4th order approximation.

l	*c*
0	0,661149
1	0,701455
2	0,709528
3	0,646317
4	0,566528
5	0,495198
6	0,436082
7	0,387737
8	0,348031
9	0,315106
10	0,287501

Table1: *l* versus *c*

The values of the coefficients k_i of the equation $c^6\text{-}k_1c^4\text{+}k_2c^2\text{-}k_3\text{=}0$ were investigated for the values ν considered in the interval $0\leq l\leq 10$.

From the analysis of the generalized equation, it is established that in the approximation of 6 order there is one real root and two complex

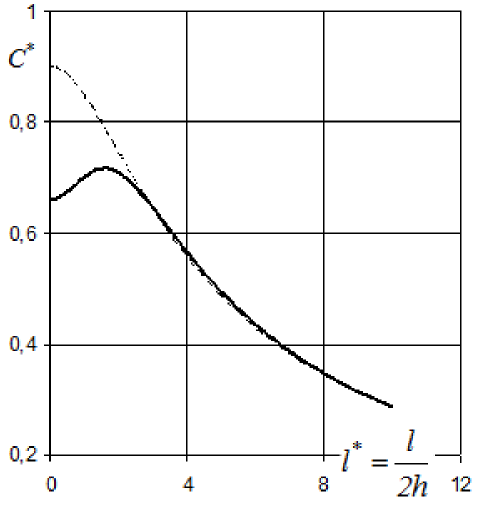

Figure 1: Phase velocity changes as a function of the wave length for v=0.3.

V	0,38	0,4	0,45	0,48
a1	0,268817	0,277778	0,30303	0,320513
a2	0,435484	0,444444	0,469697	0,487179
a3	0,133065	0,131944	0,128788	0,126603
b1	0,013441	0,013889	0,015152	0,016026
b2	0,029795	0,030324	0,031749	0,03268
b3	0,003275	0,00326	0,003208	0,003163
b3	0,003632	0,003487	0,003082	0,002805

Table2: The influence of the Poisson coefficients versus other parameters.

However, there is a region of Poisson's coefficients, where all three roots 0,35<*V*<0,5 are real.

In the 4-order approximation, the dispersion equation has the form

$$\frac{a_3}{c^4}-\left(\frac{a_0}{l^2}+a_2\right)c^2+a_1=0 \qquad (5)$$

	0,24	0,28	0,3	0,34
a1	0,219298	0,231481	0,238095	0,252525
a2	0,385965	0,398148	0,404762	0,419192
a3	0,139254	0,137731	0,136905	0,135101
b1	0,010965	0,011574	0,011905	0,012626
b2	0,026656	0,027463	0,027891	0,028803
b3	0,003319	0,003314	0,003309	0,003296
b3	0,004447	0,004244	0,004135	0,003897

Table3 : Poisson coefficients versus other parameters.

The group velocity $c_g(l)$ is defined in the interval $l\in$[0,10) by formula

$$c_g = c - l\frac{dc}{dl} \qquad (6)$$

Computer calculations were also performed using formulas (5) and (6).

Conclusion

The problem of wave propagation in an elastic layer is solved on the basis of a generalized equation. The influence of Poisson's ratio is investigated, the existence of complex roots is shown. It is established from computer calculations that there are both real and complex roots, which significantly affects the solvability of the problem.

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