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# FINGERO – IMBIBITION IN DOUBLE PHASE FLOW THROUGH POROUS MEDIA BY USING ADOMAIN DECOMPOSITION METHOD


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**ABSTRACT** Here a theoretical study has been done for the phenomena of Fingero – Imbibition in double phase flow through porous homogeneous, cracked and slightly dipping porous media involving Adomian decomposition method.


**KEYWORDS** : Adomian decomposition method, fingering, Immiscible Fluid.

**INTRODUCTION:**

Nonlinear partial differential equations can be found in wide variety scientific and engineering applications. Many important mathematical models can be expressed in terms of nonlinear partial differential equations. The most general form of nonlinear partial differential equation is given by:

$$F(u,u_t,u_x,u_y,x,y,t)=0 \qquad (1a)$$

With initial and boundary conditions

$$u(x,y,0)=\emptyset(x,y),\forall x,y\in\Omega,\Omega\in R^2 \qquad (1b)$$
$$u(x,y,t)=f(x,y,t),\forall x,y\in\partial\Omega \qquad (1c)$$

Where Ω is the solution region and ∂Ω is the boundary of Ω.

In recent years, much research has been focused on the numerical solution of nonlinear partial equations by using numerical methods and developing these methods (Al-Saif, 2007; Leveque, 2006; Rossler & Husner, 1997; Wescot & Rizwan-Uddin, 2001). In the numerical methods, which are commonly used for solving these kind of equations large size or difficult of computations is appeared and usually the round-off error causes the loss of

The Adomian decomposition method which needs less computation was employed to solve many problems (Celik et al., 2006; Javidi & Golbabai, 2007). Therefore, we applied the Adomian decomposition method to solve some models of nonlinear partial equation, this study reveals that the Adomian decomposition method is very efficient for nonlinear models, and it results give evidence that high accuracy can be achieved.

**Mathematical Methodology:**

The principle of the Adomian decomposition method (ADM) when applied to a general nonlinear equation is in the following form(Celik et al., 2006; Seng et al., 1996):

$$Lu+Ru+Nu=g \qquad (2)$$

The linear terms decomposed into *Lu+Ru*, while the nonlinear terms are represented by *Nu*, where *L* is an easily invertible linear operator, R is the remaining linear part. By inverse operator L, with $L^{-1}(\cdot)\int_0^t = (\cdot)dt$. Equation (2) can be hence as;

$$u=L^{-1})(g)-L^{-1})(Ru)-L^{-1})(Nu) \qquad (3)$$

The decomposition method represents the solution of equation (3) as the following infinite series:

$$u=\sum_{n=0}^{\infty}u_n \qquad (4)$$

The nonlinear operator *Nu*=Ψ(*u*) is decomposed as:

$$Nu=\sum_{n=0}^{\infty}A_n \qquad (5)$$

Where *An* are Adomian's polynomials, which are defined as (Seng et al., 1996):

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}[\psi(\textstyle\sum_{i=1}^{n}\lambda^i u_i)]_{\lambda=0} \quad ,n=0,1,2,\ldots \qquad (6)$$

Substituting equations (4) and (5) into equation (3), we have

$$u=\sum_{n=0}^{\infty}u_n=u_0-L^{-1}(R(\textstyle\sum_{n=0}^{\infty}u_n))-L^{-1}(\sum_{n=0}^{\infty}A_n) \qquad (7)$$

Consequently, it can be written as:

$$\begin{aligned}
u_0 &= \varphi+L^{-1}(g)\\
u_1 &= -L^{-1}(R(u_0))-L^{-1}(A_0)\\
u_2 &= -L^{-1}(R(u_1))-L^{-1}(A_1)\\
&\;\;\vdots\\
u_n &= -L^{-1}(R(u_{n-1}))-L^{-1}(A_{n-1})
\end{aligned} \qquad (8)$$

Where φ is the initial condition,

Hence all the terms of *u* are calculated and the general solution obtained according to

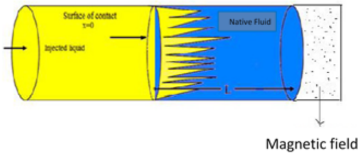
ADM as $u=\sum_{n=0}^{\infty}u_n$. The convergent of this series has been proved in (Seng et al., 1996). *n* = 0

However, for some problems (Celik et al., 2006) this series can't be determined, so we use an approximation of the solution from truncated series

$$U_M=\sum_{n=0}^{M}u_n \; with \; \lim_{M\to\infty}U_M=u$$

**Statement of the Problem:**

We consider here a finite cylindrical mass of porous medium of length L(=1) saturated with native liquid (o) , is completely besiged by an impermeable surface except for one end of the cylinder which is labeled as the imbibition face (x=0) and this end is exposed to an adjacent formation of 'injected' liquid (w) which involves a thin layer of suitable magnetic fluid. It is assumed that the later fluid is preferentially wetting and less viscous. This arrangement gives rise to a displacement process in which the injection of the fluid (w) is initiated by imbibition and the consequent displacement of native liquid (o) produces protuberances (fingers). This arrangement describes a one – dimensional phenomenon of Fingero – Imbibition.



**Mathematical Modeling:**

The capillary pressure 'P_c' is defined as the pressure discontinuity between the phases across their common interface and is a function of phase saturation. For definiteness, here we have assumed a continuous linear function of the form |8|

$$Pc=\beta Sw \quad (9)$$

$$\text{And } Pc=Po-Pw \quad (10)$$

Where β is a capillary constant. The case $Pc=_0$ has been discussed by number of authors by considering β to be small enough, therefore, we have considered βas a one of the perturbation parameter.

An analytical relationship between relative permeability and phase saturation has been given by Scheidegger and Johnson |9|, which are

$$kw=Sw \quad (11)$$

$$\text{And } k_o=1-Sw=S_o \quad (12)$$

Where $kw$ and $k_0$ are the fictions relative permeabilities of water and oil respectively. $S$ w and $S_0$ are the saturation of injected and native phase, viz., water and oil respectively.

Assuming that the flow of two immiscible phases is governed by Darcy's law, we may write the seepage velocity of an injected and native liquid as

$$V_w=-\left(\frac{kw}{\delta_w}\right)k\,\frac{\partial p_w}{\partial x} \quad (13)$$

$$V_0=-\left(\frac{k_0}{\delta_0}\right)K\,\frac{\partial p_0}{\partial x} \quad (14)$$

Neglecting the variation into phase density, the equation of continuity for injected phase is given by

$$P\,\frac{\partial S_w}{\partial t}+\frac{\partial V_w}{\partial x}=0 \quad (15)$$

Where P is the porosity of the medium.

Following Scheidegger |10|, the analytical condition governing imbibitions is

$$Vw+V_0=0 \quad (16)$$

Substituting the value of Vw and $V_0$ from equations (13, 14) into equation (16), we get

$$K\left(\frac{k_0}{\delta_0}\right)\frac{\partial p_0}{\partial x}+K\left(\frac{k_w}{\delta_w}\right)\frac{\partial p_w}{\partial x}=0 \quad (17)$$

Now equation (10) implies that

$$\frac{\partial p_c}{\partial x}=\frac{\partial p_0}{\partial x}-\frac{\partial p_w}{\partial x} \quad (18)$$

Combining (17, 18), we get

$$K[\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_0}{\delta_0}\right)]\,\frac{\partial p_w}{\partial x}+K\left(\frac{k_0}{\delta_0}\right)\frac{\partial p_c}{\partial x}=0 \quad (19)$$

Equations (19) implies

$$\frac{\partial p_w}{\partial x}=-\frac{\left(\frac{k_0}{\delta_0}\right)}{\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_0}{\delta_0}\right)}\,\frac{\partial p_c}{\partial x} \quad (20)$$

Substituting the equation (20) into equation (13), we get,

$$V_w=-K\left(\frac{k_w}{\delta_w}\right)\left[-\frac{\left(\frac{k_0}{\delta_0}\right)}{\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_0}{\delta_0}\right)}\,\frac{\partial p_c}{\partial x}\right] \quad (21)$$

Substituting the value of V_w from (21) into (15), we obtain

$$P\,\frac{\partial S_w}{\partial t}+K\,\frac{\left(\frac{k_w}{\delta_w}\right)\left(\frac{k_0}{\delta_0}\right)}{\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_0}{\delta_0}\right)}\,\frac{\partial p_c}{\partial x}=0 \quad (22)$$

That is

$$P\,\frac{\partial S_w}{\partial t}+K\,\frac{\left(\frac{k_w}{\delta_w}\right)\left(\frac{k_0}{\delta_0}\right)}{\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_0}{\delta_0}\right)}\,\frac{dp_c}{dS_w}\,\frac{\partial S_w}{\partial x}=0 \quad (23)$$

Equation (23) is a desired non-linear differential equation describing the phenomena of finger imbibitions for the flow of two immiscible phases through homogeneous porous media.

In this section we examine the behavior of finger-imbibition phenomena in homogeneous porous medium in particular the present investigation involved water and less viscous oil, therefore, according to scheidegger |10|, we use the following approximation,

$$\frac{\left(\frac{k_w}{\delta_w}\right)\left(\frac{k_0}{\delta_0}\right)}{\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_0}{\delta_0}\right)}\approx\frac{k_0}{\delta_0} \quad (24)$$

Substituting (24) into (23), we obtain

$$P\,\frac{\partial S_w}{\partial t}+\frac{\partial}{\partial x}\left[K\left(\frac{k_0}{\delta_0}\right)\frac{dp_c}{dS_w}\,\frac{\partial S_w}{\partial x}\right]=0 \quad (25)$$

Substituting the value of k_0 and p_c from equation (11) and (10) into (25), we get

$$P\,\frac{\partial S_w}{\partial t}+\left(\frac{K\beta}{\delta_0}\right)\frac{\partial}{\partial x}\left[(1-S_w)\,\frac{\partial S_w}{\partial x}\right]=0 \quad (26)$$

Now getting

$$\frac{x}{L}=X,\quad T=\sigma t\quad \text{and } 1-S_w=S \quad (27)$$

Where $\sigma=\left(\frac{K\beta}{\delta_0\,pL^2}\right)$

**Solution of the Problem:**

Therefore using (27) into (26) reduces to

$$\frac{\partial S}{\partial T}+S\,\frac{\partial^2 S}{\partial x^2}+\left(\frac{\partial S}{\partial x}\right)^2=0$$

With the initial Condition

$\theta\,(x,T)=S$

**Solution:** In this problem, we have

$$N(S)=\Psi(S_w)=\left(\frac{\partial S}{\partial x}\right)^2$$
$$g(X,t)=-S\,\frac{\partial^2 S}{\partial x^2}$$
$$R(S_w)=0$$
$$L(S_w)=\frac{\partial S}{\partial t}$$

And $\phi=\theta(x,T)=S$

By using equation (6) Adomain's polynomials can be derived as follows:

$$\left.\begin{aligned}A_0&=\left(\frac{\partial S_0}{\partial x}\right)^2\\ A_1&=2\,\frac{\partial S_0}{\partial x}\frac{\partial S_1}{\partial x}\end{aligned}\right\}$$

$$\begin{aligned}A_2&=\left(\frac{\partial S_1}{\partial x}\right)^2+2\,\frac{\partial S_0}{\partial x}\frac{\partial S_2}{\partial x}\\ A_3&=2\,\frac{\partial S_1}{\partial x}\frac{\partial S_2}{\partial x}+2\,\frac{\partial S_0}{\partial x}\frac{\partial S_3}{\partial x}\\ A_4&=\left(\frac{\partial S_2}{\partial x}\right)^2+2\,\frac{\partial S_1}{\partial x}\frac{\partial S_3}{\partial x}+2\,\frac{\partial S_0}{\partial x}\frac{\partial S_4}{\partial x}\\ &\vdots\end{aligned} \quad (28)$$

And so on. The rest of the polynomials can be constructed in similar manner.

By using Equation (8), we have

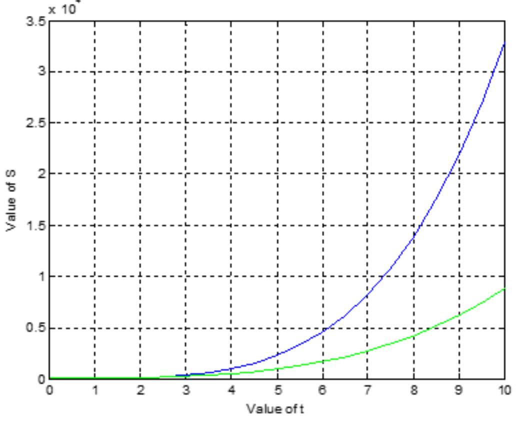
$$S_{(0)} = S - S\frac{\partial^2 S}{\partial x^2}T$$
$$S_{(1)} = -\frac{\partial S}{\partial x}T + S\frac{\partial^3 S}{\partial x^3}\frac{T^2}{2}$$
$$S_{(2)} = T^2\frac{\partial S}{\partial x}\frac{\partial^3 S}{\partial x^3} - S\frac{T^3}{3}\frac{\partial S}{\partial x}\frac{\partial^4 S}{\partial x^4} - \frac{2}{3}ST^3\frac{\partial^2 S}{\partial x^2}\frac{\partial^3 S}{\partial x^3} + S^2\frac{T^4}{4}\frac{\partial^3 S}{\partial x^3}\frac{\partial^4 S}{\partial x^4}$$
$$\vdots$$

Substituting these individual terms in equation (4) obtain

$$\theta(x,t) = S + \frac{\partial S}{\partial x}\left(-T + T^2\frac{\partial^2 S}{\partial x^2} - S\frac{T^3}{3}\frac{\partial^4 S}{\partial x^4} + \cdots\right) + \frac{\partial^2 S}{\partial x^2}\left(-ST + \frac{2}{3}ST^3\frac{\partial^3 S}{\partial x^3} - \cdots\right) + \frac{\partial^3 S}{\partial x^3}\left(S\frac{T^2}{2} + S^2\frac{T^4}{4}\frac{\partial^4 S}{\partial x^4}\cdots\right) + \cdots$$

**Table1:**

| T | S=1 | $\theta$ (x,t) | S=1 | $\theta$ (x,t) |
|---|---|---|---|---|
| 0 | S=1, $\frac{\partial S}{\partial x}=1$, $\frac{\partial^2 S}{\partial x^2}=2$, $\frac{\partial^3 S}{\partial x^3}=3$, $\frac{\partial^4 S}{\partial x^4}=4$ | 0.0001 | S=1, $\frac{\partial S}{\partial x}=4$, $\frac{\partial^2 S}{\partial x^2}=3$, $\frac{\partial^3 S}{\partial x^3}=2$, $\frac{\partial^4 S}{\partial x^4}=1$ | 0.0010 |
| 1 | | 0.0007 | | 0.0102 |
| 2 | | 0.0078 | | 0.0683 |
| 3 | | 0.0338 | | 0.2095 |
| 4 | | 0.0984 | | 0.4797 |
| 5 | | 0.2282 | | 0.9368 |
| 6 | | 0.4573 | | 1.6510 |
| 7 | | 0.8269 | | 2.7042 |
| 8 | | 1.3854 | | 4.1903 |
| 9 | | 2.1885 | | 6.2155 |
| 10 | | 3.2988 | | 8.8977 |




**CONCLUSION:**

From figure, it is clear that as time (T) increases saturation (S) increases. Keeping differentiation is constant, for different values of (T) the saturation (S) increases parabolically as time (T) increases. It is clear that the curves are of parabolic type.

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