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Original Research Paper

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WAVE PROPAGATIONS IN A SYSTEM WITH FULL ABSORPTION

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ABSTRACT We consider the initial-boundary value (IBV) problem based on a new generalized model. The Laplace transform and the numerical inversion are used in the case of arbitrary coefficients. The results of calculations for certain values of the coefficients characterizing real tissues are presented. Earlier, an approximate analytical solution of the problem for the generalized hyperbolic equation modeling the spread of a medical preparation injected into a biotissue was constructed.

KEYWORDS : generalized hyperbolic equation, Laplace transform, initial-boundary value (IBV) problem, numerical inversion

INTRODUCTION

The extended equation describing a spread of the medicine in the biotissue as the generalization of the parabolic operator to hyperbolic once has been proposed in (Selezov & Kryvonos, 2017) [1]. As a result a perturbation propagates with the finite velocity. Methods dealing with the numerical inversion of the Laplace transform has been presented in (Selezov et al, 2018) [2]. We use the method to construct solution as the expansion by odd arcs of sinus function (Papoulis, 1957) [3] and (Deutsch, 1967) [4].

2. Initial-boundary value problem

As characteristic values,we take the length l_{1q} [m], velocity $c_{tq} = \sqrt{k_q / \eta_q}$ (m²/s) and the initial concentration C_0 [kg/m³].

The IBV- problem in one-dimensional case for a function $C^*(x^*, t^*)$ is formulated for equation:

$$\frac{\partial^2 C^*}{\partial x^{*2}} - \frac{1}{c_t^{*2}} \frac{\partial^2 C^*}{\partial t^{*2}} - \gamma^* \frac{\partial C^*}{\partial t^*} = 0,$$

$$x^* \in (0, \infty), \quad t^* \in (0, T^*_1) \tag{1}$$

with the boundary conditions of incomplete absorption

$$C^*(x^*, t^*)\Big|_{x^*=0} = H(t^*) \tag{2}$$

$$C^*(x^*, t^*)\Big|_{x^*=l_1^*} = \alpha H\left(t^* - l_1^* / c_t^*\right) \tag{3}$$

and the zero initial conditions for the function and its derivative with respect to t

$$C_t^*(x^*, t^*)\Big|_{t^*=0} = 0 \tag{4}$$

where $H(t)$ is the Heaviside function , α is the quantity which characterizes incomplete absorbtion , $\alpha \in [0,1]$, $\alpha = 0$ corresponds to the free propagation of the pulse without its locking, $\alpha = 1$ corresponds to the full absorbtion at the end of the interval l_{1q}^*.

We apply the Laplace transform for the problem (1)-(4)

$$C^{*L}(x^*, p^*) = \int_0^\infty C^*(x^*, t^*) e^{-p^* t^*} dt^* \tag{5}$$

A similar problem has been considered in (Ditkin & Prudnikov, 1961) [5] , solution of which is possible only for some relations between the coefficients. Therefore it is of interest to construct approximate solutions using the numerical inversion of the Laplace transform In the image space of the Laplace transform (5) we obtain from (1) (4)

$$\frac{d^2 C^{*L}}{dx^{*2}} - (p^{*2} + p^* \gamma^*) C^{*L} = 0 \tag{6}$$

$$C^{*L}(x^*, p^*)\Big|_{x^*=0} = \frac{1}{p^*} \tag{7}$$

The solution of the equation (1) is written as

$$C^{*L}(x^*, p^*) = A_1 e^{\lambda x^*} + A_2 e^{-\lambda x^*} \tag{8}$$

Where $\lambda^* = \sqrt{(p^{*2} + p^* \gamma^*)}$ (9)

and the arbitrary constants A_1 and A_2 are

$$A_1 = -\frac{1}{p^*} \frac{e^{-\lambda^* l_1^*} - \alpha e^{-p^* l_1^* / c_t^*}}{e^{\lambda^* l_1^*} - e^{-\lambda^* l_1^*}}, \tag{10}$$

$$A_2 = \frac{1}{p^*} \frac{e^{\lambda^* l_1^*} - \alpha e^{-p^* l_1^* / c_t^*}}{e^{\lambda^* l_1^*} - e^{-\lambda^* l_1^*}} \tag{11}$$

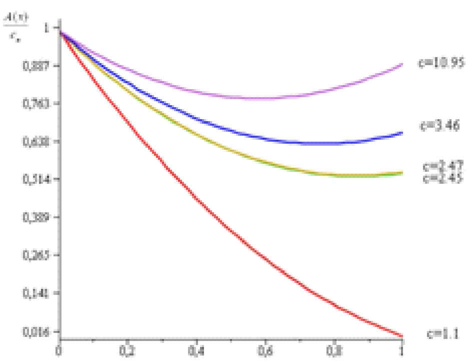
The relations (8)-(11) present the exact solution in the image space of the Laplace transform . In such a way the IBV- problem (1)-(4) defined on the finite interval $x^* \in [0, l_1^*]$ is agrees well with an input perturbation in the system at $x^* = 0$ and its propagation to the end of the interval $x^* = l_1^*$ when full absorption is occurred if $\alpha = 1$. A perturbation in the input $x^* = 0$ at the initial time is the Heaviside function with amplitude A =1, which decreases along the interval from 1 to 0 when perturbation propagates along the x-axis. It is possible to introduce the value of the amplitude A(x) and investigate its dependence on x.A correcting parameter σ for the numerical inversion of the Laplace transform solution (Selezov, 2018) [6] is accepted as = 0.0119 with its forward-backward variations of the 20% order.

Results of computations

The exact solution of the IBV problem defined at the Laplace transform space was implemented in the computational program for the numerical inversion of the Laplace transform. A lot of the computations has been performed and some results are depicted in the Fig.1,2.

4.Changes of diffusion coefficient

We consider three diffusion coefficients : $k = 10^{-5}\ m^2/s$, $k = 10^{-4}\ m^2/s$ and k=10-3 m^2/s when the relaxation time is accepted to be the constant $\eta = 0.833\square 10^{-5}\ s$. In this case we obtain respectively three values of the velocity $c_m = 1.1$, 3.46 and 10.95.



σ

Fig 1. The changes in the perturbation amplitudes along x-axis for the diffusion coefficients 1.1, 3.46, 10.95.

5. Change in relaxation time

Consider three values for the relaxation time = 0.833, 2 and 4 and we get the follows values for velocity 2: 0.707, 2.236, 7.07 and for =4: 0.5, 1.58, 5.0.For the η = 0.833 results are given in the Fig.1 and for the value = 4 - in the Fig.2.

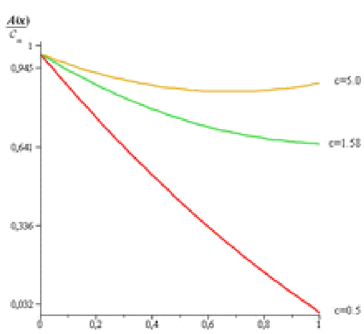


Fig.2.Change in the perturbation amplitude along x-axis.

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