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Original Research Paper

Mathematics

DIFFUSIONAL GLUEING IN SOLUTIONS OF THE PERIODICALLY FORCED FITZHUGH-NAGUMO EQUATIONS

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ABSTRACT The FitzHugh-Nagumo(FHN) equations are widely used in medicine,physiology,chemistry and other fields in the form of two-coupled partial differential equations for activator and inhibitor variables.In this paper we use an extended FHN model ,which considers diffusion and external periodic forcing. In this work we discovered an interesting phenomenon:the diffusion acts as a glue. The glue combines multiple waves and turns them into a uniform structure. The results of numerical analysis are presented.

KEYWORDS : diffusion,FitzHugh-Nagumo equations,PDE(s) solution,Runge-Kutta.

INTRODUCTION

In the early 50s of the previous century a mathematical model was developed by Hodgkin and Huxley [5] consisting of four non-linear partial differential equations for variables (V,m,n,h) that dscribe the giant This model which received abbreviated name HH model ,proved to be too difficult for an analytical solution. Therefore, later FitzHugh and Nagumo created a simplified version, briefly called the FHN model,which contains only two variables [3,6], in contrast with four variables of the HH model. These equations provide approximate solutions for problems arising in many fields, like medicone, physiology and others. Often, an external periodic forcing defined by the formula (1) is introduced into the FHN equations [7] as follows:

$$F = A\sin(\varpi t) \qquad (1)$$

Here A is the amplitude,T-period and $\omega=2\pi/T$-angular frequency of oscillations, respectively.

MATHEMATICAL MODEL

In dimensionless variables FHN model is described by the following dimensionless equations [10]:

$$\frac{\partial u}{\partial t} = u - u^3 - v + Du_{xx} + F(t) \qquad (2)$$

$$\frac{\partial v}{\partial t} = \varepsilon(u - \gamma_1 v - \gamma_2) + \delta v_{xx} \qquad (3)$$

Here u and v represent an activator or membrane potential Here u and v are an activator or membrane potential and inhibitor or regenerating voltage. For these equations, the initial conditions for the u and v functions are written as:

$$u(x,0) = -0.8 + 2(\exp(x/10))^2 \qquad (4)$$

$$v(x,0) = -0.1 \qquad (5)$$

Equations (4)-(5) are depicted graphically below in the Fig.1 : which shows that the initial conditions for the activator and inhibitor are represented by a Gaussian-like curve a straight line, respectively. The independent variable x is defined in the interval [-50<x<50] .

No-flow boundary conditions are applied for the variables u and v at the ends of the interval.

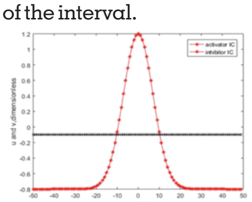

Fig.1 Initial conditions for the activator u and inhibitor v vs x-coordinate

Also it should be noted that the parameters in the equations (2)-(3) are: $\gamma_1 = 2.0, \chi_2 = 0.0, \varepsilon = 0.003, \delta = 2.5, D = 1$.

No-flow boundary conditions are applied for the variables u and v at the ends of the interval $|x_1, x_2|$.

A NOTICE ON THE DIFFUSION COOEFFICIENT

The parameter of main interest in the present model is the diffusion coefficient D for the activator which is assumed to be equal to 1.For example,in [9] it is assumed that the diffusion coefficient D is represented as a function of the x-coordinate localized relative to the center at the point $0x$ as:

$$D(x) = 1 + 9\exp(-(x - x_0)^2 / 200)$$

At the same time ,at the point $0x$ itself ,the diffusion coefficient is 10 times greater than the mentioned value D=1.Accordingly the question of the impact on the tissue of an extremely large diffusion coefficient D could be of interest with relation to the real biological tissues.

THE PURPOSE OF THE PAPER

The purpose of this paper is to present a new insight on the effect of the elevated diffusivity on the processes described by tht FHN model. One very interesting phenomenon was discovered: in FHN model the diffusion acts as a glue. The glue combines multiple waves and turns them into a uniform structure. The numerical simulations demon-strate the tendency of diffusion coefficient to "glue" the solu-tions of the FHN equations

COMPARISON OF THE RESULTS

The test solution for the FHN equationa is given in [10]. It was obtained using the Moving Mesh Method and is presented in the Fig.2.This solution consists of two steep wave fronts spreadings in the directions of of two boundaries and then bouncing back.

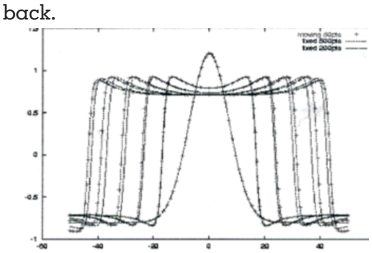

Fig.2 Test solution u(x) for the activator ,obtained using the Moving Mesh Method with no forcing when D=1 at the moments t=0.0,40,80,120,160,200.

In the present work ,the test problem from [10] was solved numerically by the fourth order Runge-Kutta method in combination with the Method of Lines (RK4-MOL). This approach, implemented in Matlab, is currently used to solve partial equations in many works described in [1],[2],[4],[11] and others. In fact, this method transforms the partial differential equations into a system of the ordinary differential equations, whicjh is then solved numerically or analytically. The present results of the application of the RK4-MOL to the test problem are shown in the Fig. 3.It can be concluded that test and present solution are essentially consistent

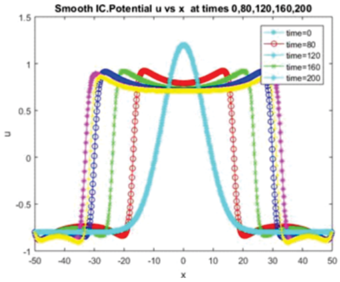

Fig.3 Present solution for the activator u(x) obtained using RK4-MOL approach with no forcing when D=1 at the moments t=0.0,40,80,120,160,200.

CASES STUDY AND DISCUSSION

Equations (2)-(3) include the diffusion term and forcing (1).They were solved numerically for the time moments t=0.,40,80,120,160,200 by means of the combined RK4-MOL techniques for the small D=1,intermediate D=2 and D=4 and large D=10 which was mentioned in [9].

Case 1 .Small D=1

It is obtained by numerical modeling that for case 3.1 depicted in the Fig.4 the solution demonstrates three waves.

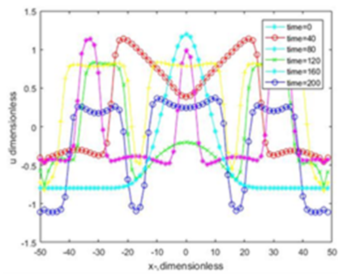

Fig.4.Solution for activator u vs x when D=1 with forcing

Case 2 Intermediate D=2

It is obtained by numerical modeling that for case 3.2 depicted in the Fig.5 the solution demonstrates two waves.

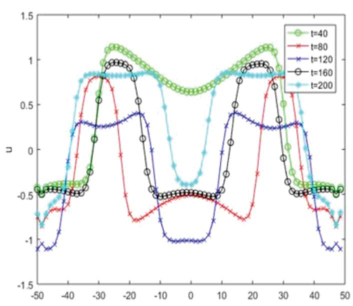

Fig.5.Solution for activator *u* vs *x* when *D*=2 .

Case 3 .Intermediate D=4

It is obtained by numerical modeling that for the Case 3.3 with forcing the solution demonstrates one wave with concave upper part as is obvious from the Fig.6.

It is obtained by numerical modeling that for the Case 3.3 with forcing the solution demonstrates one wave with concave upper part as is obvious from the Fig.6.

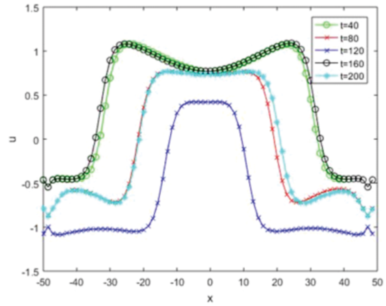

Fig.6.Solution for activator when D=4 contains one wave with the concave up- per part.

Case 4 Large D=10

In this case solution consists from the one wave with no concavity in its upper part.

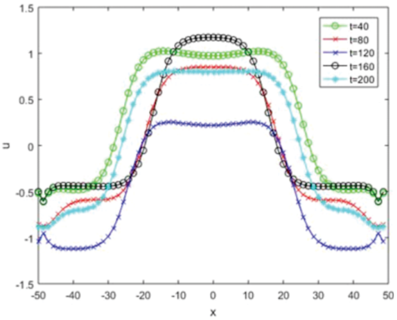

Fig.7.Solution for activator when D=10 with forcing contains one wave with no concavity in its upper part.

CONCLUSIONS

According to calculations on the FHN model (1)-(2)-(3) the following conclusions were obtained .The presence of the diffusion coefficient D and external periodical forcing can lead to the group of waves analogues to addition a portion of "glue".The diffusion coefficient D combines different separated waves .In numerical experiments performed using RK4-MOLA techniques three waves were observed for the D=1,two waves foe D=2 and single wave was obtained when D=10.Thus due to the "gluing" the number of waves decreased. For D=6 and D=10 the concavity of the wave profile was smoothed.

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