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TOTAL NONSPLIT GEODETIC NUMBER IN GRAPHS

Dr. A. Vijayalekshmi	Associate Professor in Mathematics, S. T. Hindu College, Nagercoil-629 002, Tamilnadu, India.
Aksha S. S Jilu*	M. Phil Scholar, Department of Mathematics, S.T. Hindu College, Nagercoil-629 002, Tamilnadu, India. *Corresponding Author

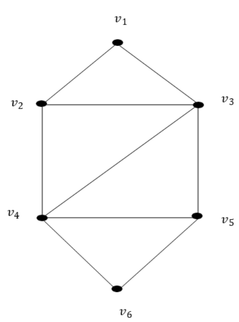
ABSTRACT The nonsplit geodetic set S of a graph G=(V (G), E (G)) is said to be total nonsplit geodetic set if the induced subgraph <S> has no isolated vertices. The minimum cardinality of a total nonsplit geodetic set in G is called total nonsplit geodetic number of G and is denoted by $g_{tns}(G)$. In this paper we obtain a relationship between nonsplit geodetic number and the total nonsplit geodetic number of G. Also we find the total nonsplit geodetic number of some classes of graphs.

KEYWORDS : Geodesic, geodetic set, geodetic number, nonsplit geodetic set, nonsplit geodetic number, total nonsplit geodetic number.

1. INTRODUCTION:

All graphs considered in these paper are finite, undirected, simple graphs. We denote the vertex set and the edge set of a graph G by V(G) and E(G) respectively. For a vertex v of G, theeccentricity e(v) is the distance between v and a vertex farthest from v.The minimum eccentricity among the vertices of G is the radius, rad G, and the maximum eccentricity is the diameter, dim G. A u-v path of length d(u,v) is also referred to as a u-v geodesic. We define I[u,v] to the set of all vertices lying on some u-v geodesic of G and for a nonempty subset S of V(G), I[S]=$\cup_{u,v\in S}$I[u,v]. A set S of vertices of G is called a geodetic set in G if I[S]=V(G). The geodetic number $g(G)$is the minimum cardinality of a geodetic set of G. A set of vertices S in a graph G is a nonsplit geodetic set if S is a geodetic set and the induced subgraph < V(G) – S > is connected. The minimum cardinality of a nonsplit geodetic set, denoted $g_{ns}(G)$ is called the nonsplit geodetic number of G. The nonsplit geodetic set S of a graph G is said to be total nonsplit geodetic set if the induced subgraph <S> has no isolated vertices. The cardinality of a minimum total nonsplit geodetic set in G is called total nonsplit geodetic number of a graph and is denoted by $g_{tns}(G)$. We denote a path on n vertices and a cycle on n vertices by P_n and C_n respectively. A path on n vertices is a tree with two vertices of degree 1 and the other (n-2) vertices are of degree 2. A wheel graph W_n is a graph n vertices constructed by connecting a single vertex to every vertex in an (n-1) cycle. The (m, n)-tadpol graph also called a dragongraph is the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge. The Cartesian product G H of graphs G and H is a graph with the vertex set is the Cartesian product V(G)×V(H) and two vertices (u, u') and (v, v') are adjacent in G H iff either u=v and u' is adjacent to v' in H or u'=v' and u is adjacent to v in G. The n-ladder graph can be defined as P_2 P_n. It is therefore equivalent to the 2×n grid graph. The n-book graph is defined as the Cartesian product S_m+1 P_2, where Sm is a star graph. The fan graph $F_{m,n}$ is defined as the graph join $\bar{K}_m$+P_n where $\bar{K}_m$ is the empty graph on m nodes. A n-barbell graph is the simple graph obtained by connecting two copies of a complete graph K_n by a bridge. We use this idea to develop the concept of total nonsplit geodetic number.

2. Main Results:



Definition 2.1: The geodetic set S is said to be a total nonsplit geodetic set if the subgraph induced by S has no isolated vertices and the induced subgraph < $V(G)$- S > is connected. The minimum cardinality of a total nonsplit geodetic set is called the total nonsplit geodetic number of G and is denoted by $g_{ns}(G)$.

For this graph G, S={v_1, v_3, v_5, v_6} is a total nonsplit geodetic set. Therefore $g_{tns}(G)$=4.

Theorem 2.2. For any path graph G of order n>4, $g_{tns}(G)$=4.

Proof. Let V(G) ={v_1, v_2, … , v_n–1, v_n} be the vertices of the path graph G. Let S={v_1, v_2, v_n–1, v_n} be the total nonsplit geodetic set of G and I[S]=V(G). The induced subgraph<S> has no isolated vertices and the subgraph induced by <V(G) – S>is connected. Therefore S is the minimum total nonsplit geodetic set of G. Hence $g_{tns}(G)$=4. ∎

Theorem 2.3. For the cycle C_n, n≥4 , $g_{tns}(C_n) = |\frac{n}{2}+1|$

Case(I): When n is even

Consider {v_1, v_2, … , v_{2n}, v_n} be the cycle with 2n vertices. Let S={v_1, v_2, … , v_i–1, v_n} be a total nonsplit geodetic set of C_{2n}. For any two antipodal vertices v_1 and v_i and {v_2, … , v_i–1 } arethe internal vertices of $v_1 – v_i$. Since no two vertices of S form a total nonsplit geodetic set, there exists a total nonsplit geodetic set contains $\frac{n}{2}$+1vertices. Therefore Therefore I[S]=V(C_{2n}). Since $v_1 – vi$ is a path, the subgraph induced by S has no isolated vertices and the induced subgraph < V(G) – S > is connected. Therefore S is the minimum total non split geodetic set ofC_{2n}. So $g_{tns}(G)$ (C_{2n})=|S|=[$\frac{n}{2}$+1]

Case(ii): When n is odd.

Consider {v_1, v_2, … , v_{2n+1} , v_i} be a cycle with 2_{n+1} vertices. Let S ={ v_1, v_2, … , v_i, v_{i+1}} be a total nonsplit geodetic set of C_{2n+1}. Then the paths, $v1 – v_{i+1}$ and v_i - v_{i+1} includes all the verticesof C_{2n+1} , |S| = [$\frac{n}{2}$+1] Therefore I[S]=V(C_{2n+1}). The subgraph induced<S> has no isolatedvertices and the induced subgraph< V(G) – S >is connected. Therefore S is the minimum total nonsplit geodetic set of C_{2n+1} , $g_{tns}(C_{2n+1})$ = |S| = [$\frac{n}{2}$+1]. Hence g_{tns} (C_n) = [$\frac{n}{2}$+1]

Theorem 2.4. For any wheel graph Wn of order n≥5, $g_{tns}(W_n)$ = n-2.

Proof. Let W_n =K_1+C_{n-1} (n≥5) with x be the vertex of K_1 and $V(C_{n-1})$={ v_1, v_2, … , $v_{n–1}$}. Let S={ v_1, v_2, … , $v_{n–2}$ } be the total nonsplit geodetic set of W_n. Then any path v_i - v_j, i,j = 1(n-2) include all the vertices of W_n , I[S]=V(W_n). Since $v_1 – v_n$–2 is a path the subgraph induced by S has no isolated vertices. The remaining vertices of W_n are $v_{n–1}$and x is connected. Thus S is the minimum total nonsplit geodetic set of W_n. Therefore $g_{tns}(W_n)$=|S| =n-2. ∎

Teorem2.5. For any tadpole graph G of order $n \geq 1, m \geq 3$,

$$g_{tns}(G)=\begin{cases} g(G)+1 & if\ m\ is\ odd \\ g(G)+2 & if\ m\ is\ even \end{cases}$$

Proof. We prove this theorem by the following cases.

Case(I): When m is odd

Consider $\{v_1, v_2, v_{m+n}\}$ be the vertices of G . Suppose $S=\{v_1, v_2, v_{m+n}\}$ be the geodetic set of G . It is clear that < V(*G*) – *S* >is connected. Thus S is a nonsplit geodetic set. But <*S*> hasisolated vertices , S not a total geodetic set. Let $S' = S \cup \{v_{m+n-1}\}$. Clearly < *S'* >has no isolated vertices. Hence *S'* forms a minimal total nonsplit geodetic set. This follows that $|S'| = |S|+1$ implies that $g_{tns}(G)=g(G)+1$.

Case (ii): When *m* is even.

Consider $\{v_1, v_2, v_{m+n}\}$ be the vertices of a tadpole graph. Now consider $S=\{v_1, v_{m+n}\}$ be the geodetic set of G and it is clear that < V(*G*) – *S* >is connected. Thus S is a nonsplit geodetic set. But <*S*> has an isolated vertices. Therefore consider $S' = S \cup \{v_{2,} v_{m+n-1}\}$ such that <*S'*> has no isolated vertices and < V(*G*) – *S* > is connected. Thus *S'* forms a minimum total nonsplit geodetic set. This follows that $|S'| = |S| +2$ implies that $g_{tns}(G)=g(G)+2$.∎

Theorem 2.6. For any ladder graph G of order n≥3 ,$g_{tns}(G)+g_{tns}(G)$.

Proof. Let G be a ladder graph and let $V(G)=\{v_1, v_2, \ldots, v_n-1, v_n\}$. In the ladder graph the geodetic set $S=\{v_1, v_n\}$ is not a total nonsplit geodetic set because <*S*> has isolated vertices. Therefore let us consider $S' = \{v_1, v_2, \ldots, v_n-1, v_n\}$. Now all the vertices of G lies in the geodetic of vertices of *S'* and < V(*G*) – *S'* > is connected. Thus forms a total nopnsplit geodetic set. Implies $g_{tns}(G)=S'=4=g(G)+g_{ns}(G)$.

Theorem 2.7. Let G=*Kn - e* ,*n*≥3. Then $g_{tns}(G) = 3$.

Proof. Let $\{v_1, v_2, \ldots, v_n-1, v_n\}$ be the vertices of G. Let $S=\{v_1, v_n\}$ be such that degree(v_1)=degree(v_n)=n-2 and I[S]=V(G). Clearly *S* is a nonsplit geodetic set but the induced subgraph <*S*> has isolated vertices. Therefore consider $S'=S\cup\{v_i\}$, where *2≤i≤n-1* such that<*S'*> has no isolated vertices and < V(*G*) – *S'* >is connected. Thus *S'* forms a minimum total nonsplit geodetic set. Therefore $g_{tns}(G)=|S'|=3$. ∎

Theorem 2.8. For any book graph G of order *n*≥2, $g_{tns}(G)= r(G)+n$.

Proof. Let $\{v_1, v_2, \ldots, v_{2n+2}\}$ be the set of all vertices of G and let $S=\{v_1, v_2, v_4, v_6, \ldots, v_{2n}\}$ be the geodetic set. Clearly $|S|=n+1$. Then < V(*G*) – *S*> is connected *S* is a nonsplit geodetic set. But <*S*> has isolated vertices. Let $S'=S\cup\{v_{2n+2}\}$. Then <*S'*> has no isolated vertices. Thus *S'* forms a minimum total nonsplit geodetic set. It follows that *S'* =n+1+1=n+2.Implies $g_{tns}(G)=n+2$. We have for any book graph, r(*G*)=2. Therfore $g_{tns}(G)= r(G)+n$.

Theorem 2.9. For any fan graph G of order $m \geq 1, n \geq 3$, g_{tns}

$$(G)=\begin{cases} n & if\ m=1 \\ m+1 & if\ m \geq 2 \end{cases}$$

Proof. We prove this theorem by the following cases.

Case (I): When *m*=1

Consider $\{v_1, v_2, \ldots, v_{n+1}\}$ be the vertices of G. Now $S=\{v_1, v_3, \ldots, v_n\}$ be the nonsplit geodetic set.Clearly I[S]=V(G). But <*S*> has an isolated vertices. So $S'= S\cup\{v_2, v_4, \ldots, v_{n-1}\}$. Then <*S'*> has no isolated vertices. Thus *S'* forms a total nonsplit geodetic set of G with minimum cardinality. Therefore $g_{tns}(G) = \frac{n}{2}+\frac{n}{2}= n$.

Case (ii): When *m*≥2

Consider $\{v_1, v_2, v_{m+n}\}$ be the vertices of G. Let $S=\{v_1, v_2, \ldots, v_{m+n-1,} v_{m+n}\}$ be the nonsplit geodetic set. Now consider induced subgraph <*S*>. Clearly <*S*> has isolated vertices. Let $S' = S \cup \{v_{m+n-1}\}$.Thus *S'* forms a minimum total nonsplit geodetic set. Therefore $g_{tns}(G)=|S'|=m+1$. ∎

Theorem 2.10. For any graph of order n , $g_{ns}(G) \leq g_{tns}(G)$.

Proof. Let G be any graph with $V(G)=\{v_1, v_2, \ldots, v_n\}$. Let *S*   V(G) be a nonsplit geodetic set of G. If the induced subgraph<*S*>has no isolated vertices, *S* itself is a total nonsplit geodetic set and therefore $g_{ns}(G) = g_{tns}(G)$. Otherwise choose a minimal subset *S'*   V(G)-S such that induced subgraph < *V*(*G*) – (S U *S'*) > is connected and the induced subgraph<S U *S'*> has no isolated vertices. Clearly S U *S'* is total nonsplit geodetic set of G and so $g_{ns}(G)< g_{tns}(G)$. Hence $g_{ns}(G) \leq g_{tns}(G)$

Theorem 2.11. For a n-barbell graph G, $g(G)=g_{ns}(G)=g_{tns}(G)$.

Proof. Let G be a n-barbell graph. Obviously $g(G)=g_{ns}(G)=g_{tns}(G) = 2$ *(n-1)*.

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