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Original Research Paper

Mathematics

MATHEMATICAL MODEL OF THE SHOCK TREATMENT IN CARDIOLOGY

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ABSTRACT Chaotic solutions of the driven Duffing equation are very important and interesting for researchers in different areas. In many cases chaos has negative effect on complicated processes As we know, chaotic solution means that a small change in one state of a deterministic nonlinear system can result in large differences in a later state, meaning there is sensitive dependence on initial conditions. It happens in electric circuits, economics, meteorology and in the electrophysiology of the heart. Therefore the aim of the present paper to give a new insight on the chaos mitigation by shock.. It was found in the present work by means of the mathematical modeling that the shock-term indeed can mitigate chaos produced in the driven Duffing equation.

KEYWORDS : Chaos Mitigation, Hartbeat, Shock, Duffing Equation

1.INTRODUCTION AND BACKGROUND

The chaotic behavior has been widely studied for a long time [2]-[7] .For many researches it looks like as an undesirable phenomena which can damaged important dynamical systems in different areas. There is a special interest to chaos in Mathematical biology and electrophysiology Contrary to conventional assumptions, doctors and researchers from the American Association for the Advancement of Science, told , that a healthy heart must beat somewhat chaotically rather than in a perfectly predictable pattern. Other organs and physiological systems, including the brain, nervous system and endocrine system, may also be partly governed by a need for chaos. But for many heart diseases the appearance of strange or additional chaotic oscillations, can be very dangerous and may lead to serious , irreversible consequences, even to sudden death. Therefore, the problem of avoiding additional chaos in the heartbeats is very important .

2.MATHEMATICAL MODELS

Two models are described as the two sets of two independent ordinary differential equations given as follows

2.1 The forced form of the Duffing equation

The model is described by two equations (1)- (2) :

$$\frac{\partial x}{\partial t} = y \qquad (1)$$

$$\frac{dy}{dt} = x - x^3 - y + F_0 \cos(t) \qquad (2)$$

2.2 The forced form of the Duffing equation with external shock

The model is described by the equations (3)- (6) :

$$\frac{\partial x}{\partial t} = y \qquad (3)$$

$$\frac{dy}{dt} = x - x^3 - y + F_0 \cos(t) - Sh\,t \qquad (4)$$

In the given equations (1)-(2) and (3)-(4),dimensionless dependent variables $,xy$ respectively are the positions and velocities to be simulated, t is the independent time of the simulation that is defined as $0<t<200$ and $q=0Fcos(t)$- external periodic driving. Also these equations are subjected to the initial conditions for the unknown functions : in the both systems position $x(0) = 1.5$ and velocity $y(0) = 0$.

Note that equation (4) is the same as equation (2) ,ut it is expanded by the shock-term *Sh(t)* defined as follows:

$Sh(t)=0$,if $0<t<150$ (5)

and

$Sh(t)=7.29$, if $150<t<200$ (6)

In the equations (5)- (6) *Sh(t)* is the time-dependent function similar to the pacemaker in cardiology.

3.1 Test solution for the phase-portrait for Duffing equations with forcing parameter =0.8 without Shock .

Solution of the system of the equations (1)-(2) for the phase portrait subjected to the imposed initial conditions with the parameter =0.8 and with no shock was obtained in the [1].Its chaotic behavior is presented in the Fig.1.

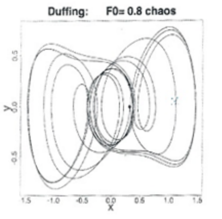

Figure 1: Test solution for the phase portrait in the position –velocity variables for Duffing equations when $0F=0.8$ without Shock .

So, the dynamics of the equations (3)-(6) equations served two contradictory purposes: repulsion of trajectories within the data set and attraction beyond it. Such a complex system is called a "strange attractor".

3.2 Runge-Kutta (RK) solution for Duffing equations with forcing parameter =*0.8* without external Shock.

Also the Duffing equations (1) - (2) were solved numerically using the RK method. Results of the simulation are presented in the Fig.2 which show chaotic behavior similar to that presented in the Fig.1

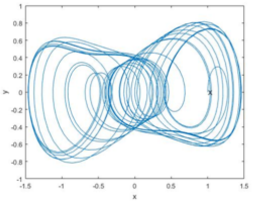

Figure.2 RK solution for the phase portrait in the position x–velocity V variables for Duffing equation when $0F=0.8$ without the Shock-function.

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The oblique cross presents the Initial Conditions (IC) for the Duffing problem.

Note that both Test and RK solutions present the "butterflies" consisting from two "wings" connected by the transient area. Such "butterflies" often appear in the chaotic modeling. An example is familiar Lorenz "butterfly" [10].depicted below in the Fig.3 .

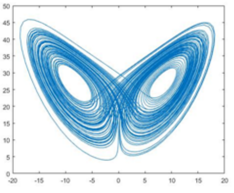

Figure.3." Lorenz butterfly"

It can be concluded that both the known and present RK simulations give essentially the same results. Thus method Runge-Kutta can be used to solve the another Duffing problem (3)-(4) taking into consideration both driving parameter $0F$=0.8 and Shock-function SH(t) defined by the equations (5) –(6).

4. RK solution for the phase-portrait of the Duffing equations containing the parameter =0.8 and the Shock defined by the equations (5)-(6).

The system of the equations (3) - (4) was solved numerically for the phase portrait given in the Fig.4 using the RK method tested previously .

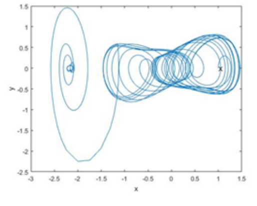

Figure4. Present RK solution for the phase portrait in the position –velocity variables for Duffing equation when $0F$=0.8 and Shock-function is defined by the equations (5)-(6) The oblique cross corresponds to the IC.

It should be concluded from this simulation that if Shock-function is applied to the Duffing equation then area occupied by the solution in the phase space retains the shape of the "butterfly" consisting of two "wings" connected by a transitional zone. Also there is evident the chaotic attractor in the phase-space of the considering model.

5.Present RK solutions for the velocities vs time and spatial variable vs time graphs obtained from the Duffing equations containing driving and the Shock-function.

Present RK solution was obtained for the velocity and spatial variables vs time for the Duffing equations (3)-(4) with forcing and external shock and Shock. Results are presented in the in Fig.5. and Fig.6.

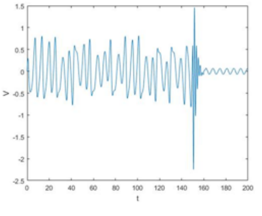

Figure5. Present RK solution for the velocity V –variable vs time t for Duffing equation for$0F$=0.8 and Shock-function defined by the equations (5)-(6).

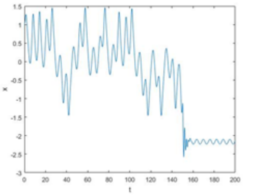

Figure.6. Present Runge-Kutta solution for the x-position –variable for Duffing with $0F$=0.8 and Shock-function defined by the equations (5)-(6).

CONCLUSIONS

1. It should be concluded from the present simulation (see Fig.4) that if the Shock-function is applied to the Duffing equations then area occupied by the solution in the phase space retains the shape of a " butterfly " consisting of two "wings" connected by a transitional zone. This shape is similar to the shape of the Lorens "butterfly".
2. Also there the attractor was obtained .See the phase-space (Fig.4).
3. In the Fig.5 with a time t less than 150 there is a chaotic solution for V because the Shock- function has no effect. When a time t changes from 150 to 200 there are regular oscillations of solution for V without any signs of chaos because driving is suppressed by the Shock-function.
4. Similarly in the Fig.6 with a time t less than 150 there is a chaotic solution for x because the Shock- function has no effect. When a time t changes from 150 to 200 there are regular oscillations of solution for x without any signs of chaos because driving term is suppressed by the Shock-function.

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