VOLUME - 10, ISSUE - 09, SEPTEMBER - 2021 • PRINT ISSN No. 2277 - 8160 • DOI : 10.36106/gjra

Original Research Paper

Mathematics

A STUDY OF DIFFICULTIES FACED BY CHILDREN IN LEARNING LIMIT AT CLASS XI AND ITS REMEDIAL MEASURE

Mohan Sharma

Lecturer in Mathematics, Government Model S.M. Senior Secondary School Indora, District Kangra HP, PIN 176401.

KEYWORDS : Limit, Conceptual Level Difficulty, Computational Level Difficulty, Neighborhood of a Point

1. INTRODUCTION

In mathematics, one of the important goals should be learning with understanding. It can occur by actively building new knowledge from experience and prior knowledge. And also by minimizing the gap between pieces of knowledge that student have. At higher level, mathematical concepts become more abstract to understand and to find their application. The concept of Limit is introduced to the children in class XI and students face various difficulties in grasping the concept. The difficulties faced by the children in learning limit, can be divided into **conceptual level difficulty** and **computational level difficulty**. The concept of limit deals with the behavior of a function in the vicinity of a point without any difference to the value of the function at that point. To understand this concept of limits, the students should have knowledge and understanding of some lower level concepts like functions, real numbers, intervals and **neighborhood of a point** etc. But the gap, in learning of lower level concepts creates difficulty. The meaning of behavior of a function in the neighborhood of a point is not sometimes understood by the students. Some students may not know what is the value of a function? They may not be able to understand to the meaning of limiting value of a function. The clarity of existence of limiting value or its non-existence is another issue which many students are unable to follow. There is nomenclature related difficulty too like what is the meaning of the word limit? What is meaning of left hand limit and right hand limit? What is the difference between them? How it is represented mathematically?

Limit can be understand as, If the values of a function f(x) approaches the value L as x approaches c we say that f has limit L as x approaches c and write it as

$$\lim_{x \to c} f(x) = L$$

And Limit can be defined as the limit of f(x) as x approaches c is the number L if the following criterion holds given any radius $\epsilon > 0$ about L there exists a radius $\delta > 0$ about c such that for all x, $0 < |x - c| < \delta$ implies $|f(x) - L| < \epsilon$

2. TYPES OF DIFFICULTIES OBSERVED

I. Conceptual Level Difficulties:

- In class XI some results of limits are taken without proof the students grasp them with some confusions in their mind like

(i) $\lim_{x \to a} k = k$ where k is real number

(ii) $\lim_{x \to a} x^n = a^n$ for all $n \in N$

(iii) $\lim_{x \to a} f(x) = f(a)$ where $f(x)$ is a real polynomial x.

- Sometimes the students consider the Converse of algebra of limits as true however which is not so. For example

$$\lim_{x \to a} f(x) + \lim_{x \to a} g(x) \neq \lim_{x \to a}$$

- Difficulty in understanding theorems of limits

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = nx^{n-1}$$

In the proof of this theorem we use the concept of factorization, combinations if these concepts are not clear to the student then he just grasp the result. Also the clarity of various types of functions is required like greatest integer function and smallest integer function, trigonometric functions, exponential functions, logarithmic functions etc.

Thus there are large numbers of concepts which are prerequisite to understand limits many difficulties arise from it and the concept of limits is itself complex for a student to understand. The teacher also faces difficulty to make this understand for a student. So this study focuses to find those difficulties so that remedies to them are given to students. Further in this study the field tryout after remedies has been covered in which a survey after remedies to students has been conducted and a comparative study of conceptual level difficulty before and after remedies has been analyzed.

I. Computational level difficulty:

The students have some difficulties in computing limits which are as follows:

a) How to find the value of a function at a point?
b) How to calculate the left and limit and right hand limit of a function at a point?
c) Solving the binomial coefficients?
d) Factorization using identities?
e) Limits of greatest integer and smallest integer function and modulus functions?
f) In long division method and basic operations?
g) Rationalization and using identities?
h) In solving limits of trigonometric, exponential functions?

Thus there are number of calculation based difficulties too which students face while computing limits. How a teacher can help in minimizing such errors. In this study we shall observe that what difference to computational level difficulty is seen after remedies to students?

Also it would be observed in this study that out of these two types of difficulty which type of difficulty is higher among students even after remedies?

3. OBJECTIVES OF THE STUDY

1 To observe that how students perceive the idea of limits
2. To find the conceptual level and computational difficulties in faced by students in learning limits
3. To analyze gathered data and give remedies to those difficulties
4. To observe the improvement in learning of limit after remedies if any

4. METHODOLOGY

4.1 Sample of the Study

A random sample of 60 students of 10+1Non medical class of government Senior Secondary school Indora district Kangra HP was taken. It was a mixed sample of lowest achievers to highest achievers of class including both boys and girls. The sample was collected twice with same students. The students were not aware of survey means they were not declared in advance that we shall conduct a survey of your understanding of limit after topic is delivered to you. At first it was collected when the topic of limits was completed as per the syllabus of their class. The syllabus included intuitive idea of limits, limits of polynomials, rational, trigonometric, exponential and logarithmic functions. Secondly it was collected when remedies to the difficulties faced by the students were applied.

4.2 Tools and Techniques used

The researcher developed a questionnaire for the students to address the objectives of the study. The questionnaire was developed to seek information regarding the:

- Visualization of limit by class XI students
- determining the concept level difficulties in learning limits
- Determining calculation level difficulties in learning limits.
- finding the misconceptions and give remedies accordingly

4.3 Procedure of Data collection

The observations were made at two levels base level and after remedies i.e. at end level. At first level the questionnaire was used to get data from sample students on understanding of limit concept and its computation. The sample was collected in disciplined way where fixed time was given to mark the one choice out of four choices given. It was ensured that students should not communicate with each other so that every student gives his/her own answer and thought.

The second survey was collected after two weeks with *same* questionnaire when students were given remedies to their conceptual level and computational level difficulties which were observed in first sample. The examples taken while giving remedies to problems faced were different than the sample questionnaire. It was ensured that students were not told about the answers of questionnaire. It was collected from same 60 students who participated in first survey in same time frame and setup.

5. Analysis of the Data

The data was analyzed in order to identify the difficulties. These difficulties were sorted as concept level and computational level.

5.1 Visualization of limit by students of class XI

Sr. No.	Different responses by students regards visualization of limit	First survey	Second survey (After remedies)
1.	Limit as boundary that cannot be passed	31.66%	30%
2.	Value of a function at a point	50%	38.33%
3.	See limit as restriction	16.66%	21.66%
4.	find it as behavior of a function near a point	01.66%	10%

It was found in both surveys that higher percentage of students has misconception that limit is the value of a function at point means they visualize the limit like substitution of a point in a function. However the students percentage after remedies declined by 11.67%. Second most prevalent view is that limit is boundary that cannot be passed and this view of students percentage remained nearly the same even after remedies in which concept of limits was again explained with more examples. The percentages of students, who found limit as restriction, increased by 5% after remedies. A very small percentage of students found limit as behavior of a function near a point and after remedies it increased nearly 9%.

5.2 Conceptual Level Difficulty

Q4 ' $\lim_{x\to\frac{5}{2}}[x]$ =' and Q10 'If limit of f(x) and g(x) exist as x approaches to a then $\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ exist is

' involved the limits of functions which include the concepts of greatest integer function and algebra of limits. It means that these concepts were required as prerequisite from the students so that the student having knowledge and understanding of these concepts can solve the questions with accuracy. But it was observed that students showed learning gaps and percentage of students with correct answer is very low in such questions which involve concept of limit with other concepts. See table below

Question	Students %age with correct answer in First Survey	Students %age with correct answer in second Survey (after remedies)
Q4' $\lim_{x\to\frac{5}{2}}[x]$ '	1.66	10
Q10 If limit of f(x) and g(x) exist as x approaches to a then $\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ exist is	6.66	21.66

This question involved the concept of greatest integer function the response of one student was correct so we observe that students have very high difficulty in concept based question specifically when concept of limit is mixed with concept of greatest integer function. In such cases students are not able to build concept image they consider limit parallel as substitution process and 55 students (91.66%) gave c option as answer.

In second survey when they were explained about the concepts of greatest and lowest integer function still they didn't showed much increased accuracy and only 9% increase in correct response. So it can be concluded that in such questions more efforts are required to build proper concept image. Here we can make use of modern computer based technology to help the student to learn better.

In response to question 10, Mathematics requires in depth learning of concepts but sometimes students hurriedly give answer to multiple choice questions without giving second thought to question it can be observed from their response to this question in which only 4 students (6.66%) gave correct answer but after remedial work it increased to 13 students (21.66%) only. Also it was observed that 31 students (51.66%) gave b as answer in first survey. It shows they lack the practice of repetition of question for better understanding.

5.3 Computational Level difficulty

The Q2 and Q7 involved the calculation work mainly. It can be observed from the table below that they have comparatively less difficulty in computational level questions to that of concept based questions. They have shown better accuracy in such questions.

Question	First Survey	Second Survey (After remedial work)
Q2	86.66%	80%
Q7	38.33	36.33%

In response to Q2 it was found that, this question required the direct substitution of value and 52 students did it correctly while 8 students showed operational level difficulty in calculations. It may be due to lack of practice, speedy work or carelessness. This remained nearly same in second survey.

For Q.7 it was found that, students should have knowledge of formula and its application in calculations. Here the knowledge of formula was mandatory for students so the accuracy level sharply came down to 38.33% i.e. 23 student's response was correct thus we find that where combination of computation with formula was required the correct response of students came down.

5.4 Both Conceptual and Computational level Difficulty

Question no. 3, 5, 6 & 9 involved the limits of functions which include the concepts of factorization, exponential, trigonometric functions, squeeze principle the process of rationalization and surds along with computational work which involves the knowledge and understanding of different formulas and basic operations etc.

Question No.	Students %age with correct answer in First Survey	Students %age with correct answer in second Survey (After remedies)
Q3	36.66%	56.66%
Q5	16.66%	23.33%
Q6	11.66%	38.33%
Q9	8.33%	38.33%

Question 3 was answered correctly by 22 students while the responses of other 38 students were incorrect it reflects the learning gap of process of factorization and formula involved. In this question 28 students (46.66%) gave 0 as answer it shows that they have misconception that zero divided by zero give zero means that they are not aware about indeterminate forms. But when these gaps of learning were addressed in remedial teaching so in second survey 20% more students covered those gaps and gave correct response.

In respond to question 5, it was found that questions involved the knowledge of exponential functions along with trigonometric functions and squeeze principle. This combination of different concepts along with computations in limits increased the complexity of questions so only 10 students (16.66%) gave correct answer and 29 students (48.33%) responded that limit of it does not exists. Here it can be observed that students substituted value of x as zero in denominator and gave said answers so initially large number of students consider limit as substitution process.

After explaining these confusions and recalling squeeze principle along with the concepts of exponential, trigonometric functions in second survey the correct response to this question increased only by 7%.

Question no. 6 requires the knowledge of trigonometric formulas and its applications in limits. It was observed that only 11.66% of students gave correct answer and 35 students gave 0 as answer it showed that they have learning gaps of trigonometric formulas and lack of revision. After providing remedies to students regarding trigonometric formulas and its applications, the accuracy level increased by 27%.

In respond to question no. 9, 8.33% of students gave correct answer to this question which involved the process of rationalization and operations on surds. It reflected that students had difficulty in understanding the lower level concepts. When such learning gaps of students were tried to be covered under the remedies they showed comparatively more improvement i.e. 30% for this question. It can be concluded that the learning of students varies from concept to concept and in some cases they have shown less improvement while better improvement in other.

6. CONCLUSION

The data interpretation showed that the conceptual level of understanding of students was much lower than the computational level of understanding as it was observed that in the questions which were having conceptual level difficulty, the accuracy of students was very low but questions which involved computational level difficulties, students showed comparatively better accuracy. In questions involving both levels, the accuracy was in between even when remedies were given to students. The students visualize limits with confusion, isolated facts, calculation work and their knowledge and understanding of concept image of limits was weak even when remedies were given. The main observation was that many students consider the computation of limits parallel to substitution process as they consider limit as value of a function at a point. It may be due to nature of limit that we use words like very close to, tends to or approaches to and ultimately for calculations we substitute that value of limit in function.

The study helped in bringing out some misconceptions which students have about limits.

- Limit is calculated by substitution
- The limit exists for a function.
- When a number is rounded off then they consider it equal and not closer to that number e.g. 1.00010… = 1
- Indeterminate forms like zero divided by zero gives zero.

7. SUGGESTIONS FOR FURTHER RESEARCH

Further research is required to find that why student's response to conceptual level difficulty problems is very weak while in computational level difficulty problems they have shown comparatively better accuracy. How modern technology can help teaching strategies in building concept image of limits and ease the understanding of it for students and teachers. How diagrams and graphical representations can help in better understanding of limits. How we can equip the teacher with more effective tools and pedagogy with information and communication technologies.

REFERENCES

1. NCERT (2017). Mathematics: Class XI, NCERT, New Delhi, India 281-320.
2. Bezuidenhout, J. (2001). Limits and Continuity: Some Conceptions of First - year Students. International Journal of Mathematical Education in Science and Technology 32(4): 487-500.
3. Denbel, GD (2014). Students' Misconceptions of the Limit Concept in a First Calculus Course, Journal of Education and Practice 34(5):24-40
4. Blaisdell, R. (2012) Student understanding in the concept of limit in calculus: University of Maine.https://new.umaine.edu/merg/wp-content/uploads/ sites/440/2012/08/Blaisdell_RUME_proceedings_2012.pdf