VOLUME - 10, ISSUE - 09, SEPTEMBER - 2021 • PRINT ISSN No. 2277 - 8160 • DOI : 10.36106/gjra

Original Research Paper

Mathematics

An Algebraic Solution towards Multi Variable Transportation Problem

S. D. Manjarekar* Department of Mathematics, L.V.H. ASC College, Nashik – 003, India. *Corresponding Author

A. P. Bhadane Department of Mathematics, L.V.H. ASC College, Nashik – 003, India.

ABSTRACT In this paper, we have consider the cost of transportation not as of one variable but as multivariable which includes goods loading charges, vehicle operator charges, on road transportation cost including toll and taxes, maintenance cost of vehicle and goods unloading charges. Then we have solved this multivariable transportation problem using algebraic approach and compare the result with North – West Corner method.

KEYWORDS : Transportation Problem, Field, Prime Number

INTRODUCTION

Though one can think of transportation problem which were generally arises during the second world war but it has its own roots from the evolution of right from the 400 B. C. or from 3500 B. C. when wheel was invented in the middle east of Asia [5].

There are different methods to find out the transportation cost of the given problem [5] which were dependent of one variable only. However, this cost is nothing but the combination of various dependent variable mainly one can think of, loading and unloading charges, vehicle operator charges, maintenance of cost of vehicle. Even the actual transportation cost includes the tolls and taxes on the road. This means the transportation cost must be multi – valued and not a single valued. Because sometimes even if short route may cause the increase in maintenance cost of vehicle due to bad weather or road. Hence, we have to consider the problem of transportation not only of single variable / single valued but also multi variable / multi valued.

The paper mainly consist of four parts. The second part consist of basic definitions which were required for the calculations were given. In third part algorithm of proposed method multi variable transportation problem were given. In the fourth part, algebraic solution of given transportation problem were explained and compared the result with North – West Corner method along with conclusion.

BASIC DEFINITIONS

[I] Transportation:

Let there be 'm' origins $O_1, O_2, \ldots, O_m$ having a_i ($a_i > 0$, i = 1, 2, 3, ..., m) units of availability respectively and 'n' destinations $D_1, D_2, \ldots, D_n$ with b_j ($b_j > 0$, j = 1, 2, 3, ...n) units of requirements. If C_{ij} $(x_1, x_2, x_3, x_4, x_5)$ are the cost of transporting one unit of the commodity from i^{th} origin to j^{th} destination and X_{ij} be the units of transporting from i^{th} to j^{th} destination. The objective is to determine Xij which minimizes the total transporting cost (Z) satisfying all the availability constraints and the requirement constraints [5]. Where the variable are given as ,

$x_1 =$ Loading Charges
$x_2 =$ Vehicle Operator Charges
$x_3 =$ Road transportation cost including Tolls and Taxes
$x_4 =$ Maintenance cost of vehicle
$x_5 =$ Unloading charges

Where, $x_1, x_2, x_4, x_5 > 0$ and $x_3 \geq 0$.

[II] Mathematical Formulation:

If $\sum a_{ij} = S$ is the total availability of origins and $\sum b_{ij} = D$ is the total requirements of destinations and if $S = D$ then the given transportation problem is balanced [5].

Minimize $Z = \sum\sum C_{ij}(x_1, x_2, x_3, x_4, x_5)X_{ij}$

Subject to

$\sum X_{ij} = a_i$, $i = 1, 2, 3, \ldots, m$ (Availability constraints)

$\sum X_{ij} = b_j$, $j = 1, 2, 3, \ldots, n$ (Requirement constraints)

and $X_{ij} \geq 0$ for all I and j (Non Negative constraints)

In case if $\sum a_{ij} \neq \sum b_{ij}$ then it becomes unbalanced so that we have to make some manipulation to make balanced i.e. $\sum a_{ij} = \sum b_{ij}$.

[III] IBFS Methods:

To find the initial basic feasible solution of a transportation problem, there are some standard methods which are mentioned below [5]:

1] Vogel's Approximation Method
2] Least Cost Method
3] North – West Corner Method

[IV] Finite Field:

A commutative ring Z_n is a field if and only if n is a prime number. We take a prime number $n = 4k + 1$. Since, it can be written as sum of two perfect squares [4].

ALGORITHM OF PROPOSED METHOD:

The algebraic method can be summarized into following steps applied for balanced transportation problem [1,2,3,7,8].

Step I] Check the given transportation problem were balanced or not. If balanced, then go to next step.

Step II] Take $\sum_{k=1}^{5} x_k$ for each C_{ij} and rename it as C_{ij}.

Step III]

Case A: Find out the minimum odd prime number of the form $p = 4K + 1, k > 0$ among the prime entries in the given table excluding demand row and supply column.

Case B: If there is no odd prime number of the type $p = 4K + 1$ among the entries then, take its previous immediate prime number of the type $p = 4K + 1$ among the smallest values of the entries in the table excluding demand row and supply column.

Case C: If the least number is less than $p = 5$, then take $p = 5$ and apply the process.

Step IV]: Write the penalties over the rows and columns by taking addition (modulo p).

Step V] Select the row or column with the smallest penalty and allocate as much as possible in the cell that has least cost in the selected rows or column and satisfies the given condition. If there is tie in the values of penalties, one can take any one of them where the minimum allocation can be made.

Step VI] any row or column with zero supply or demand should not be used in computing future penalties.

Step VI] Repeat steps from III] to VI] until the available supply at various sources and demand at various destinations is satisfied.

NUMERICAL EXAMPLES:

A) Consider the following example to find out the minimum transportation cost

	Distribution Centers				
	D_1	D_2	D_3	D_4	Supply
S_1	(1, 2, 0, 5, 2)	(2, 4, 3, 2, 1)	(1, 3, 1, 5, 1)	(2, 3, 5, 1, 2)	6
S_2	(2, 1, 0, 3, 4)	(1, 3, 0, 2, 2)	(2, 1, 0, 1, 1)	(1, 3, 2, 1, 2)	2
S_3	(3, 1, 1, 1, 2)	(1, 2, 4, 2, 3)	(2, 3, 1, 2, 3)	(1, 3, 1, 2, 1)	9
Demand	7	5	3	2	

Solution:

In the above example as the demand and supply are same the said transportation problem is balanced problem.

Step I] Take $\sum_{k=1}^{5} x_k$ for each C_{ij} and renaming it as C_{ij}. We get the table as;

	Distribution Centers				
	D_1	D_2	D_3	D_4	Supply
S_1	10	12	11	13	6
S_2	10	8	5	9	1
S_3	8	12	11	8	10
Demand	7	5	3	2	

Step II] Smallest prime number of the form $p = 4K + 1$ is in the cell $C_{2,3}$ were a prime number. Thus, we take the addition of the entries of rows and columns over Z_5 and apply the above algorithm. We get;

	Distribution Centers					Penalty	Penalty	Penalty	Penalty	Penalty
	D_1	D_2	D_3	D_4	Supply					
S_1	10	12 [5]	11 [1]	13	6	1	3	3	3	3
S_2	10	8	5 [1]	9	1	2	3	3	---	---
S_3	8 [7]	12	11 [1]	8 [2]	10	4	1	3	3	---
Demand	7	5	3	2						
Penalty	3	2	2	0						
Penalty	3	2	2	---						
Penalty	---	2	2	---						
Penalty	---	4	2	---						
Penalty	---	2	---	---						

Total Cost: 5 * 12 + 1 * 11 + 1 * 5 + 7 * 8 + 1 * 11 + 2 * 8 = 60 + 11 + 5 + 56 + 11 + 16 = 159 /-

The same problem has solution 179 /- by using North – West Corner Method.

CONCLUSIONS

In this paper, we have developed the new algorithm for finding the solution towards the initial basic feasible solution of multi – variable transportation problem. The above method is suitable towards finding the initial basic feasible solution of given multi – variable transportation problem also it is better alternative iterative method than North – West Corner. Thus the proposed method is important tool for the decision makers when they are handling various types of transportation / logistic problems.

REFERENCES:

[1] Ansari S. I., Bhadane A. P., (2012) "New modified approach to find initial basic feasible solution to transportation problem using statistical technique", vol. 9(7), Antartica Journal of Mathematics.
[2] Bhadane A. P., Manjarekar S. D., (2020) "APB's Statistical Quartile method for IBFS of a Transportation problem and comparison with Least Cost Method", vol. 7(4), IJRAR, pp. 132 – 135.
[3] Bhadane A. P., Manjarekar S. D., (2020) "APB's method for IBFS of a transportation problem and comparison with Least Cost Method", vol. 6(8), IJREAM.
[4] Gallian J. A., (2012) "Contemporary Abstract Algebra", Brooks / Cole Publication Co.
[5] Sharma J. K., (2013) "Operations Research: Theory and Applications", Trinity Press.
[6] Sharma N. M., Bhadane A. P., (2016) "An Alternative method to north – west corner method for solving transportation problem", IJREAM.
[7] Manjarekar S. D., Bhadane A. P., (2020) "A New algebraic method for the initial basic feasible solution of a transportation problem and comparison with least cost method", vol. 7(4), IJRAR, pp.994 – 997.
[8] Manjarekar S. D., Bhadane A. P., (2020) "A New algebraic method for IBFS of a transportation problem and comparison with NWC Method", vol. 9(12), IJSR, pp. 89 – 90.