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Original Research Paper

Mathematics

COSINE SIMILARITY AND ITS APPLICATION BY PYTHAGOREAN FUZZY SETS

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ABSTRACT A Pythagorean fuzzy set is the successful field which includes membership and non membership functions. It has been extended from intuitionistic fuzzy set. It reaches many application with the support of score, Accuracy, Distance and Similarity measures. In this paper, cosine similarity measure is used with Pythagorean fuzzy set. An algorithm is developed for proposed method. An illustrative example is included. Comparison is also made with Score, Accuracy and Similarity measure function.

KEYWORDS : Pythagorean fuzzy set Cosine Similarity Measure Similarity Measure using Distance function Score function Accuracy function

1. INTRODUCTION

The concept of fuzzy set is introduced by Zadeh [21] in which elements are measured with membership degree lies between 0 and 1. Atanassov [2] initiated the concept of intuitionistic fuzzy set (IFS) in which he considered both the membership and non-membership functions. After that Yager [17] introduced Pythagorean fuzzy set (PFS) as an efficient expansion of the intuitionistic fuzzy sets. Pythagorean fuzzy set is also extended into different forms, such as interval-valued Pythagorean fuzzy set [4], decision making [3,18,19] and hesitant Pythagorean fuzzy set [5,9] and linguistic Pythagorean fuzzy set [6] and some other applications [20].

Peng and Yang [13] developed a pythagorean fuzzy superiority and inferiority ranking method to solve uncertainty multiple attribute group decision-making problem. Similarity measure is a significant means for measuring the uncertain information. Zhang [22] proposed the Pythagorean fuzzy similarity measures in the multi-attribute decision making problems. Peng [12] proposed the many new distance measures and similarity measures for dealing with many issues such as pattern recognition, medical diagnosis and clustering analysis. Wei and Wei [14] presented some Pythagorean fuzzy cosine similarity for the decision making problems.

In this paper we propose a method to calculate the degree of similarity between Pythagorean fuzzy set where extended form of the cosine similarity measure between 2 pythagorean fuzzy set recommended. Then to demonstrate the efficiency of the proposed extended cosine similarity measure between 2 pythagorean fuzzy similarity, numerical example is illustrated.

This paper is organized as follows: In section 2, some basic concepts are briefly introduced. An algorithm is proposed in section 3. In section 4, an example is illustrated for the proposed algorithm and section 5 reveals the result and discussion. Finally conclusion is delivered in section 6.

2. Concepts

In this section some concepts, related to Pythagorean fuzzy sets are introduced.

Definition 2.1

Let X be a fixed set, then a fuzzy sets A in X can be defined as
$A = \{ x, \alpha_A(x) / x \in X\}$
where $\alpha_A : X \to [0,1]$ is called membership degree of $x \in X$.

Definition 2.2

Let X be a fixed set, then the Intuitionistic fuzzy set in X can be defined as:

$I = \{ x, \alpha I(x), \beta I (x) / x \in X\}$

Where $\alpha I(x)$ and $\beta I (x)$ are mapping from X to [0,1] with condition $0 \le \alpha I(x) \le 1, 0 \le \beta I (x) \le 1, 0 \le \alpha I(x) + \beta I (x) \le 1$ for all $x \in X$. Let $\gamma I (x) = 1 - \alpha I(x) - \beta I (x)$ then it is called the intuitionistic fuzzy index of $x \in X$ to the set I, representing the degree of indeterminancy x to I. Also $0 \le \gamma I (x) \le 1$ for every $x \in X$.

Definition 2.3

Let X be a fixed set, then Pythagorean fuzzy set in X can be defined as follows:

$P = \{ x, \alpha p(x), \beta p (x) / x \in X\}$
Where $\alpha p(x)$ and $\beta p (x)$ are mapping from X to [0,1] with condition $0 \le \alpha p(x) \le 1, 0 \le \beta p (x) \le 1$, also $0 \le \alpha p2(x) + \beta p2 (x) \le 1$ for all $x \in X$ and they denote the degree of membership and degree of non membership of element $x \in X$ to the set P respectively. Let $\gamma P (x) =$ then it is called the Pythagorean fuzzy index of element $x \in X$ to the set P representing the degree of indeterminancy x to P. Also $0 \le \gamma P (x) \le 1$ for every $x \in X$.

Definition 2.4

Let R and S be 2 pythagorean fuzzy sets then the operations can be defined as follows:

i) $R \cup S = \{ \langle x, \max(\alpha_R(x), \alpha_S(x)), \min(\beta_R(x), \beta_S(x)) \rangle | x \epsilon X\}$
ii) $R \cap S = \{ \langle x, \min(\alpha_R(x), \alpha_S(x)), \max(\beta_R(x), \beta_S(x)) \rangle | x \epsilon X\}$
iii) $R + S = \{ \langle x, \sqrt{\propto_R^2 (x) + \propto_S^2 (x) - \propto_R^2 (x) \propto_S^2 (x)}, \beta_R(x)\beta_S(x)\rangle | x \epsilon X\}$
iv) $R \times S = \{ \langle x, \alpha_R(x)\alpha_S(x), \sqrt{\beta_R^2 (x) + \beta_S^2 (x) - \beta_R^2 (x) \beta_S^2 (x)} \rangle | x \epsilon X\}$
v) $R^c = \{ \langle x, \beta_R(x), \alpha_R(x)\rangle | x \epsilon X.$

Definition 2.5

Let R be the Pythagorean fuzzy set then the score and Accuracy functions of R is defined as follows:

$S_c (R) = \alpha_R^2(x) - \beta_R^2(x)$, Where $S_c (R) \in [-1, 1]$
$A_c (R) = \alpha_R^2(x) + \beta_R^2(x)$, Where $A_c (R) \in [0, 1]$.

For any 2 pythagorean fuzzy sets, R and T is compared as follows:

i) If $Sc (R) > Sc (T)$ then $R > T$
ii) If $Sc (R) = Sc (T)$ then
a) if $Ac (R) > Ac (T)$ then $R > T$
b) if $Ac (R) = Ac (T)$ then $R \sim T$.

Definition 2.6

Let R and T be 2 pythagorean fuzzy sets. Then the following similarity measures are defined as below:

i) Cosine Similarity:

$$C_s (R,T) = \frac{1}{n}\sum_{i=1}^{n} \frac{\alpha_R^2(x_i)\alpha_T^2(x_i) + \beta_R^2(x_i)\beta_T^2(x_i) + \gamma_R^2(x_i)\gamma_T^2(x_i)}{\sqrt{\alpha_R^4(x_i) + \beta_R^4(x_i) + \gamma_R^4(x_i)} \sqrt{\alpha_T^4(x_i) + \beta_T^4(x_i) + \gamma_T^4(x_i)}}$$

ii) Similarity Measure based on Hamming Distance:

$$S_H\,(R,T) = 1 - \frac{\sum_{i=1}^{n}\left(\,|\alpha_R^2(x_i) - \alpha_T^2(x_i)\,| + |\beta_R^2(x_i) - \beta_T^2(x_i)\,| + |\gamma_R^2(x_i) - \gamma_T^2(x_i)\,|\,\right)}{2n}$$

Definition 2.7

Let R, S and T be 3 pythagorean fuzzy sets on X. A similarity measure S (R, S) is a mapping S : P (X) × P (X)   [0,1] possessing the following properties.

I) $0 \le S(R,S) \le 1$

ii) $S(R,S) = S(S,R)$

iii) $S(R,S) = 1$ iff $R = S$

iv) $S(R,R^c) = 0$ iff R is a crisp set.

v) If $R \subseteq S \subseteq T$, then $S(R,T) \le S(R,S)$ and $S(R,T) \le S(S,T)$.

3. Algorithm

This algorithm yields the shortest path through shortest path length from source node to destination node in a directed acyclic network with pythagorean fuzzy set.

Step 1 : Initialization

Assign Pythagorean fuzzy number for each arc between all pair of nodes.

Step 2 : Analyzation

Analyze the path length for all possible paths from source node to destination node in the network using the formula

$$R + S = \{\, \langle x, \sqrt{\alpha_R^2(x) + \alpha_S^2(x) - \alpha_R^2(x)\,\alpha_S^2(x)},\ \beta_R(x)\beta_S(x)\rangle \mid x \epsilon X \}$$

Where R and S are two pythagorean fuzzy numbers.

Step 3 : Calculation

Calculate the minimum path length from all possible paths by the following formula

$$R \cap S = \{\, \langle x, \min(\alpha_R(x),\ \alpha_S(x)), \max(\beta_R(x),\ \beta_S(x))\ \rangle \mid x \epsilon X \}$$

Step 4 : Computation

Calculate the value of using cosine similarity for all possible paths with minimum path length.

$$C_s\,(R,T) = \frac{1}{n}\sum_{i=1}^{n} \frac{\alpha_R^2(x_i)\alpha_T^2(x_i) + \beta_R^2(x_i)\beta_T^2(x_i) + \gamma_R^2(x_i)\gamma_T^2(x_i)}{\sqrt{\alpha_R^4(x_i) + \beta_R^4(x_i) + \gamma_R^4(x_i)}\ \sqrt{\alpha_T^4(x_i) + \beta_T^4(x_i) + \gamma_T^4(x_i)}}$$

Step 5 : Identification of Shortest Path

Shortest path is fixed with the path which is having the highest similarity measure.

4. Numerical Example

In this section, a wide telecommunicated network is assigned to illustrate our proposed method.

Consider a zoom meeting, which is organized by one country (source) to another country (destination)through various countries (intermediate nodes), while we are organizing the zoom meeting through the network there may be various paths available to connect from source to destination. Pythagorean fuzzy number is assigned to the flow of frequency. The organizer wants to find the shortest path in which we can get good result in sound clarity (x_1), video clarity (x_2) and minimum number of network traffic (x_3) in the given finite universe X { x_1, x_2, x_3 }.

In these three, the organizer is focusing on the clarity (α_i) and the speed of the network (βi).

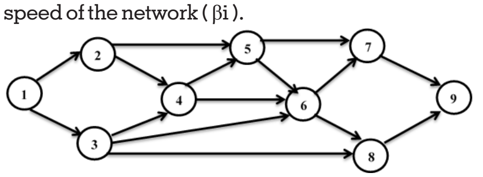

Fig. 4.1 An example of PFS network

Solution:

Step 1 :

Pythagorean fuzzy number is assigned to each arc.

1 – 2 ➔ < x_1 0.4, 0.5 > < x_2 0.3, 0.4 > < x_3 0.1, 0.2 >
1 – 3 ➔ < x_1 0.1, 0.3 > < x_2 0.5, 0.7 > < x_3 0.3, 0.6 >
2 – 4 ➔ < x_1 0.2, 0.4 > < x_2 0.4, 0.5 > < x_3 0.6, 0.7 >
2 – 5 ➔ < x_1 0.3, 0.4 > < x_2 0.1, 0.3 > < x_3 0.5, 0.8 >
3 – 4 ➔ < x_1 0.2, 0.3 > < x_2 0.4, 0.6 > < x_3 0.4, 0.5 >
3 – 6 ➔ < x_1 0.4, 0.6 > < x_2 0.2, 0.3 > < x_3 0.4, 0.7 >
3 – 8 ➔ < x_1 0.4, 0.7 > < x_2 0.5, 0.6 > < x_3 0.3, 0.6 >
4 – 5 ➔ < x_1 0.6, 0.7 > < x_2 0.1, 0.5 > < x_3 0.6, 0.7 >
4 – 6 ➔ < x_1 0.6, 0.7 > < x_2 0.3, 0.7 > < x_3 0.6, 0.8 >
5 – 6 ➔ < x_1 0.5, 0.6 > < x_2 0.2, 0.4 > < x_3 0.3, 0.6 >
5 – 7 ➔ < x_1 0.3, 0.4 > < x_2 0.5, 0.6 > < x_3 0.4, 0.6 >
6 – 7 ➔ < x_1 0.3, 0.6 > < x_2 0.4, 0.5 > < x_3 0.6, 0.7 >
6 – 8 ➔ < x_1 0.5, 0.8 > < x_2 0.3, 0.5 > < x_3 0.1, 0.5 >
7 – 9 ➔ < x_1 0.2, 0.5 > < x_2 0.5, 0.7 > < x_3 0.3, 0.6 >
8 – 9 ➔ < x_1 0.3, 0.5 > < x_2 0.2, 0.4 > < x_3 0.4, 0.5 >

Step 2 : Analyzation

There are 11 possible path lengths between source node to destination node.

The possible paths with path lengths are

P_1 = < x_1 0.49, 0.11 > < x_2 0.68, 0.17 > < x_3 0.55, 0.18 >
P_2 = < x_1 0.58, 0.04 > < x_2 0.7, 0.05 > < x_3 0.66, 0.05 >
P_3 = < x_1 0.71, 0.04 > < x_2 0.67, 0.02 > < x_3 0.78, 0.04 >
P_4 = < x_1 0.78, 0.05 > < x_2 0.49, 0.01 > < x_3 0.67, 0.03 >
P_5 = < x_1 0.52, 0.06 > < x_2 0.74, 0.08 > < x_3 0.75, 0.17 >
P_6 = < x_1 0.65, 0.07 > < x_2 0.61, 0.04 > < x_3 0.6, 0.11 >
P_7 = < x_1 0.78, 0.01 > < x_2 0.79, 0.03 > < x_3 0.86, 0.05 >
P_8 = < x_1 0.83, 0.02 > < x_2 0.69, 0.02 > < x_3 0.79, 0.04 >
P_9 = < x_1 0.69, 0.12 > < x_2 0.81, 0.09 > < x_3 0.79, 0.08 >
P_{10} = < x_1 0.69, 0.02 > < x_2 0.79, 0.11 > < x_3 0.85, 0.1 >
P_{11} = < x_1 0.77, 0.03 > < x_2 0.71, 0.06 > < x_3 0.77, 0.06 >

Step 3 : Calculation

Minimum path length is calculated by def. 2.4

P_{min} = < x_1 0.49, 0.11 > < x_2 0.49, 0.17 > < x_3 0.55, 0.18 >

Step 4 : Computation

Cosine similarity measure for all possible path length with minimum path length.

$$C_s\,(P_{min}, P_1) = \frac{1}{3}\left[\frac{0.6048}{(0.7777)(0.7777)} + \frac{0.4761}{(0.7619)(0.6847)} + \frac{0.5447}{(0.738)(0.738)}\right]$$

$C_s(P_{min}, P_1)$ = 0.9708

Similarly,

$C_s(P_{min}, P_2)$ = 0.9527
$C_s(P_{min}, P_3)$ = 0.8836
$C_s(P_{min}, P_4)$ = 0.9108
$C_s(P_{min}, P_5)$ = 0.9042
$C_s(P_{min}, P_6)$ = 0.9705
$C_s(P_{min}, P_7)$ = 0.7412
$C_s(P_{min}, P_8)$ = 0.8017
$C_s(P_{min}, P_9)$ = 0.812
$C_s(P_{min}, P_{10})$ = 0.7868
$C_s(P_{min}, P_{11})$ = 0.8457

Step 5 : Identification of Shortest Path

The highest similarity measure is $C_s(P_{min}, P_1)$ = 0.9708.

This similarity measure corresponds to the Shortest path 1-3-8-9 which is identified as shortest path. This path is having more clarity and high speed.

5. RESULT AND DISCUSSION

The clarity and network speed from source node to destination node are checked. In each arc sound clarity, video clarity and network traffic is maintained through out the path. Using cosine similarity shortest path between source node to destination node the highest speed and perfect clarity are calculated.

The results are tabulated and compared with Score, Accuracy and Similarity measure based on Hamming distance also. Any type of fuzzy number will not show the clear variation between the paths but using Pythagorean fuzzy number will show the clear variations between the paths.

Table 5.1 Comparison Table

S. No.	Possible Paths	$C_s(P_{min}, P_i)$	$S_H(P_{min}, P_i)$	$S_c(P_i)$	$A_c(P_i)$	Rank
1.	P_1 = 1-3-8-9	0.9708	0.9629	0.3105	0.3595	1
2.	P_2 = 1-2-5-7-9	0.9527	0.8404	0.4185	0.4229	3
3.	P_3 = 1-2-5-6-7-9	0.8836	0.7398	0.5193	0.5217	6
4.	P_4 = 1-2-5-6-8-9	0.9108	0.8168	0.4313	0.4336	4
5.	P_5 = 1-3-6-7-9	0.9042	0.7914	0.4472	0.4731	5
6.	P_6 = 1-3-6-8-9	0.9705	0.8774	0.3787	0.3911	2
7.	P_7 = 1-3-4-5-6-7-9	0.7412	0.6059	0.6562	0.6585	11
8.	P_8 = 1-3-4-5-6-8-9	0.8017	0.6654	0.5956	0.5972	9
9.	P_9 = 1-3-4-5-7-9	0.812	0.6759	0.5758	0.5951	8
10.	P_{10} = 1-3-4-6-7-9	0.7868	0.6539	0.6001	0.6151	10
11.	P_{11} = 1-3-4-6-8-9	0.8457	0.7004	0.5606	0.566	7

6. CONCLUSION

In this paper, cosine similarity for Pythagorean fuzzy set based on membership and non membership function is proposed. And an algorithm is also developed on the basis of the new proposed method which offers a suitable solution to address the recognized path in a network and the minimum path length to diverse the fastness and efficiency. It is much more closer to the actual situation and it reveals the intuitive judgments. In future, it can to the other applications also like medical diagnosis, machine learning system etc.

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