VOLUME - 12, ISSUE - 04, APRIL - 2023 • PRINT ISSN No. 2277 - 8160 • DOI : 10.36106/gjra

Original Research Paper

Mathematics

RESPONSE OF HEAT SOURCES IN POROUS THERMOELASTIC MATERIAL WITH ONE RELAXATION TIME UNDER THERMOMECHANICAL LOADING.

Dr. Savita Devi
Department of Mathematics, Govt.P.G.College,Hisar(Haryana)-125001, India

ABSTRACT The present investigation is concerned with the response of heat sources in porous thermoelastic material with one relaxation time. The problem is solved subjected to thermomechanical boundary conditions by the use of Laplace and Fourier transforms. A concentrated or continuous source at the boundary surface has been taken to illustrate the utility of the approach. The transformed components of displacement, stress, change in volume fraction field and temperature distribution are inverted by using a numerical inversion technique. The numerical results of normal displacement, normal stress, change in volume fraction field and temperature distribution are illustrated graphically for various heat sources namely (i) distributed heat source(ii) continuous heat source (iii) heat source varying with depth. Some particular cases are also deduced from the present formulation.

KEYWORDS : Thermoelasticity; generalized thermoelasticity; concentrated or continuous source;heat sources.

1. INTRODUCTION

Biot [1] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for uncoupled and coupled theories of thermoelasticity, however, are of the diffusion type, predicting infinite speeds of propagation for heat waves contrary to physical observations.At present, there are various theories of generalized thermoelasticity. The first important generalization to the coupled theory is due to Lord and Shulman[2], who obtained a wave type heat equation by postulating a new law of heat conduction (the Maxwell-Cattaneo equation)to replace the classical Fourier law. Because the heat equation of this theory is of wave type, it automatically ensure finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and constitutive relations, remain the same as those for the coupled and uncoupled theories.Joseph and Preziosi[3,4]state the Maxwell-Cattaneo equation is the most obvious and simple generalization of the Fourier law that give rise to a finite propagation speed. The comprehensive work has been done in coupled theory(CT) and generalized theories of thermoelasticity with heat sources.

We consider a theory for the behavior of porous solids such that the matrix material is elastic and the interstices are void of material; it is a generalization of the classical theory of elasticity. The theory of porous elastic material has been established by Cowin and Nunziato [5,6,7]. In this theory the bulk density is the product of two scalar fields, the matrix material density and the volume fraction field; it is studied in the book of Ciarletta and Iesan [8,9]. This theory has practical use for investigating various types of geological and biological materials for which elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the voids volume isincluded among the kinematics variables and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity. The first investigation in the theory of thermoelastic materials with voids are due to Nunziato and Cowin [4] and Iesan [9]. The linear theory of thermoelastic maerials with voids was presented in [9](see also [10]). Different authors has been discussed different types of problem in linear thermoelastic materials with voids [11,12,13,14,15,16,17,18].

In the present investigation the response of heat sources in porous thermoelastic material with one relaxation time is studied by the use of application of integral transforms. At the half-space surface the tractions and temperature are prescribed. The transformed components of displacement, stress, change in volume fraction field and temperature distribution are obtained due to various heat sources subjected to thermomechanical loads. The results of papers may be applied to a wide class of geophysical problems involving temperature change. Application of the paper may be found in mechanics viz. in designing highways and airport runaways. The problem has practical applications in the field of geomechanics , engineering, fibre-wound composites and laminated composite materials.

2. Basic Equations

Following Cowin and Nunziato [6]and Lord and Shulman [2], the field equations and constitutive relations in porous thermoelastic material with one relaxation time, without body forces and extrinsic equilibrated body force can be written as:

$$(\lambda+2\mu)\nabla(\nabla\cdot\vec{u})-\mu(\nabla\times\nabla\times\vec{u})+b\nabla\phi-\beta\nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2}, \quad (1)$$

$$\alpha\nabla^2\phi-b\nabla\cdot\vec{u}-\xi_1\phi-\omega_0\frac{\partial\phi}{\partial t}+mT=\rho\psi\frac{\partial^2\phi}{\partial t^2}, \quad (2)$$

$$K\nabla^2T-\beta T_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla\cdot\vec{u}-mT_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\phi-\rho C_e\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T=-\rho\left(1+\tau_0\frac{\partial}{\partial t}\right)Q, \quad (3)$$

$$t_{ij}=\lambda u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+b\phi\delta_{ij}-\beta T\delta_{ij} \quad (4)$$

where λ,μ - Lame's constants, $\alpha, b, \xi_1, \omega_0, m, \psi$ - material constants due to presence of voids, T - the temperature distribution $\vec{u}$ - displacement vector, $\beta=(3\lambda+2\mu)\alpha_t$, α_t - coefficient of linear thermal expansion, ρ, C_e - density and specific heat respectively, K-thermal conductivity, ϕ - change in volume fraction field, T_0 - reference temperature, t_{ij} - components of stress tensor, τ_0 – the relaxation time, δ_{ij} – Kronecker delta,

$$\nabla=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}, \qquad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}.$$

3. Formulation And Solution of The Problem

We consider a homogeneous, isotropic, porous thermoelastic half space with one relaxation time in theundeformed temperature T_0. The rectangular Cartesian coordinate system (x, y, z) having origin on the surface $z = 0$ with z - axis pointing normally in to the medium is introduced. For two dimensional problem, we assume

$$\vec{u}=(u, 0, w) \quad (5)$$

To facilitate the solution, following dimensionless quantities are introduced:

$$x'=\frac{\omega_1^*}{c_1}x,\quad z'=\frac{\omega_1^*}{c_1}z,\quad u'=\frac{\omega_1^*}{c_1}u,\quad w'=\frac{\omega_1^*}{c_1}w,\quad t'_{33}=\frac{t_{33}}{\mu},$$

$$t'_{31}=\frac{t_{31}}{\mu},\quad \phi'=\frac{\omega_1^{*2}\psi}{c_1^2}\phi,\quad t'=\omega_1^*t,\quad \tau'_o=\omega_1^*\tau_o$$

$$\tau'_1=\omega_1^*\tau_1,\quad Q'=\frac{\beta Q}{K\omega_1^{*2}},\quad T'=\frac{T}{T_0},\quad P'_1=\frac{P_1}{\mu},\quad P'_2=\frac{P_2}{T_0}, \quad (6)$$

where

$$c_1=\left(\frac{\lambda+2\mu}{\rho}\right)^{\frac{1}{2}} \text{ and } \omega_1^*=\frac{\rho C_e c_1^2}{K}.$$

The expressions relating displacement components u(x, z, t) and w(x, z,t) to the scalar potential functionsy1(x, z, t) and y2 (x, z,t) in dimensionless form are given by

$$u=\frac{\partial\psi_1}{\partial x}-\frac{\partial\psi_2}{\partial z},\quad w=\frac{\partial\psi_1}{\partial z}+\frac{\partial\psi_2}{\partial x}. \quad (7)$$

Using (5)-(7), in equations (1)-(4) (after suppressing the primes)and applying the Laplace and Fourier transforms defined by

$$\hat{f}(x,z,s)=\int_0^\infty e^{-st}f(x,z,t)dt, \quad (8)$$

and

$$\tilde{f}(\xi,z,s)=\int_{-\infty}^\infty e^{i\xi x}\hat{f}(x,z,s)dx, \quad (9)$$

on resulting equations, we obtain

$$[D^2-(\xi^2+b_1s^2)]\tilde{\psi}_1+b_2\tilde{\phi}-b_3\tilde{T}=0 \quad (10)$$

$$a_1(D^2-\xi^2)\tilde{\psi}_1-[(D^2-(\xi^2+b_4)]\tilde{\phi}-a_9\tilde{T}=0, \quad (11)$$

$$\epsilon_1 b_5(D^2-\xi^2)\tilde{\psi}_1+\epsilon_2 b_5\tilde{\phi}-[D^2-(\xi^2+b_5)]\tilde{T}=a_{10}b_6\tilde{Q}. \quad (12)$$

$$(D^2-\lambda_4^2)\tilde{\psi}_2=0. \quad (13)$$

where

$$D=\frac{\partial}{\partial z},\ b_1=\frac{a_4}{1+a_1},\quad b_2=\frac{a_2}{1+a_1},\quad b_3=\frac{a_3}{1+a_1},\quad b_4=a_6+a_7s+a_9s^2,\ b_5=s+\tau_0s^2,$$

$$b_6=1+\tau_0s,\quad \lambda_4^2=\xi^2+a_4s^2.$$

with

$$a_1=\frac{\lambda+\mu}{\mu},\ a_2=\frac{bc_1^2}{\omega_1^{*2}\mu\psi},\ a_3=\frac{\beta T_0}{\mu},\ a_4=\frac{\rho c_1^2}{\mu},\ a_6=\frac{\xi_1 c_1^2}{\omega_1^{*2}\alpha},$$

$a_7=\frac{\omega_0 c_1^2}{\omega_1^*\alpha},\ a_9=\frac{\rho\kappa_1^2\psi}{\alpha},\ a_{10}=\frac{\rho c_1^2}{\beta T_0},\ \epsilon_1=\frac{\beta c_1^2}{K\omega_1^*},\ \epsilon_2=\frac{mc_1^4}{K\psi\omega_1^{*3}}$ and ϵ_1,ϵ_2 are the coupling coupling constants.

Eliminating $\tilde{\psi}_1$ and $\tilde{\phi}$ from equations (10)- (12), we get

$$(D^6+AD^4+BD^2+C)\tilde{T}=-a_{10}b_6(D^4+ED^2+F)\tilde{Q}, \quad (14)$$

where

$$A=-3\xi^2+f_1,$$
$$B=3\xi^4-2f_1\xi^2+f_2,$$
$$C=-\xi^6+f_1\xi^4-f_2\xi^2+f_3,$$
$$E=-2\xi^2+f_4,$$
$$F=\xi^4-f_4\xi^2+f_5,$$
$$f_1=(a_5b_2-\epsilon_1 b_2b_3b_5-(b_4+b_1s^2)-b_5),$$
$$f_2=(\epsilon_1 b_1b_4b_5-\epsilon_1 b_2b_5a_8+\epsilon_2 b_5(a_8-b_3a_5)-a_5b_2b_5+b_1b_2b_4s^2+b_5(b_4+b_1s^2)),$$
$$f_3=(-\epsilon_2 b_1b_5a_8s^2-b_1b_4b_5s^2),$$
$$f_4=(b_2a_5-b_4-b_1s^2),$$
$$f_5=b_1b_4s^2$$

with

$$a_5=\frac{b\psi}{\alpha},\quad a_8=\frac{mT_0\psi}{\alpha}.$$

Also from equations (10), we have

$$\tilde{\phi}=\frac{1}{b_2}\{b_3\tilde{T}-(D^2-(\xi^2+b_1s^2))\tilde{\psi}_1\}. \quad (15)$$

Using equation (15) in equation (11), we obtain

$$(D^4+ED^2+F)\tilde{\psi}_1=b_3(D^2-J)\tilde{T}, \quad (16)$$

where

$$J=\xi^2+\frac{b_1b_4-b_2a_8}{b_3}.$$

Equation (16) can be written as

$$(D^2-\lambda_1^2)(D^2-\lambda_2^2)(D^2-\lambda_3^2)\tilde{T}=-a_{10}b_6(D^4+ED^2+F)\tilde{Q} \quad (17)$$

where $\lambda_\ell^2(\ell=1,2,3.)$ are the roots of the following characteristic equation

$$\lambda^6+A\lambda^4+B\lambda^2+C=0. \quad (18)$$

The roots $\pm\lambda_1$, $\pm\lambda_2$ and $\pm\lambda_3$ of equation (20) satisfy the relations

$$\lambda_1^2+\lambda_2^2+\lambda_3^2=-A,$$
$$\lambda_1^2\lambda_2^2+\lambda_2^2\lambda_3^2+\lambda_3^2\lambda_1^2=B,$$
$$\lambda_1^2\lambda_2^2\lambda_3^2=-C.$$

The solution of equation (17) can be written in the form

$$\tilde{T}=\tilde{T}_1+\tilde{T}_2+\tilde{T}_3+\tilde{T}_p, \quad (19)$$

where $\tilde{T}_\ell$ is the solution of the homogeneous equation

$$(D^2-\lambda_\ell^2)\tilde{T}=0 \quad ;\ \ell=1,2,3. \quad (20)$$

and $\tilde{T}_p$ is a particular solution of equation (17).

The solutions of equations (20) and (13), which satisfies the radiations conditions, can be written as

$$\tilde{T}_\ell=A_\ell e^{-\lambda_\ell z} \quad ;\ \ell=1,2,3. \quad (21)$$

$$\tilde{\psi}_2=A_4e^{-\lambda_4 z}, \quad (22)$$

with

$$\tilde{\psi}_p=\frac{-a_{10}b_3b_6(D^2-J)\tilde{Q}}{(D^2-\lambda_1^2)(D^2-\lambda_2^2)(D^2-\lambda_3^2)} \quad (27)$$

Also using equations (23) and (26) in equation (16), we obtain the solution of $\tilde{\psi}_1$ as

$$\tilde{\psi}_1=\sum_{\ell=1}^{3}(m_\ell A_\ell e^{-\lambda_\ell z})+\tilde{\psi}_p, \quad (28)$$

and with the help of equations (23), (28) and (15), we get the value of $\tilde{\phi}$ as

$$\tilde{\phi}=\sum_{\ell=1}^{3}(d_\ell A_\ell e^{-\lambda_\ell z})+d_4, \quad (29)$$

where

$$m_\ell=\frac{b_3(\lambda_\ell^2-J)}{\lambda_\ell^4+E\lambda_\ell^2+F},\quad d_\ell=\frac{b_3-(\lambda_\ell^2-(\xi^2+b_1s^2))m_\ell}{b_2},$$

$$d_4=\frac{b_3\tilde{T}_p-(D^2-(\xi^2+b_1s^2))\tilde{\psi}_p}{b_2}.$$

Applying the Laplace transform and Fourier transform on equation (4) with the help of equations(5)-(7), then using equations(22) and (28)-(29), we get

$$\tilde{u}=\sum_{\ell=1}^{3}(-\iota\xi)m_\ell A_\ell e^{-\lambda_\ell z}+\lambda_4A_4e^{-\lambda_4 z}+(-\iota\xi)\tilde{\psi}_p, \quad (30)$$

$$\tilde{w}=\sum_{\ell=1}^{3}(-\lambda_\ell m_\ell A_\ell e^{-\lambda_\ell z})-(-\iota\xi)A_4e^{-\lambda_4 z}+D\tilde{\psi}_p \quad (31)$$

$$\tilde{t}_{33}=\sum_{\ell=1}^{4}(R_\ell A_\ell e^{-\lambda_\ell z})-B_1, \quad (32)$$

$$\tilde{t}_{31}=\sum^{4}(e_\ell A_\ell e^{-\lambda_\ell z})-B_2, \quad (33)$$

where

$$R_\ell=((a_{11}+2)m_\ell\lambda_\ell^2-a_{11}\xi^2m_\ell+a_2d_\ell-a_3),$$

$$e_\ell=2\iota\xi m_\ell\lambda_\ell,\quad B_1=\frac{(b_2(a_{11}+2)D^2-a_{11}\xi^2)-a_2^2(D^2-(\xi^2+b_1s^2)))\tilde{\psi}_p}{b_2},\quad B_2=2\iota\xi D\tilde{\psi}_p,$$

$$R_4=2\iota\xi\lambda_4,\quad e_4=-(\xi^2+\lambda_4^2);\quad (\ell=1,2,3)$$

with

$$a_{11}=\frac{\lambda}{\mu}.$$

4. Boundary Conditions

The appropriate boundary conditions are

(i) $t_{33}=-P_1f_1(x,t)$, (34)

(ii) $t_{31}=0$, (35)

(iii) $\frac{\partial\phi}{\partial z}=0$, (36)

(iv) $T=P_2f_2(x,t)$, (37)

Where P_1 is the magnitude of force, P_2 is the constant temperature applied on the boundary. $f_1(x,t)$ and $f_2(x,t)$ are known functions. Applying the Laplace and Fourier transform defined by (8) and (9) on boundary conditions (34)- (37), with the help of equation (6), we obtain

(i) $\tilde{t}_{33}=-P_1\tilde{f}_1(\xi,s)$, (38)

(ii) $\tilde{t}_{31}=0$, (39)

(iii) $\frac{d\tilde{\phi}}{dz}=0$, (40)

(iv) $\tilde{T}=P_2\tilde{f}_2(\xi,s)$ (41)

Substituting the value of $\tilde{\psi}_2,\tilde{T},\tilde{\psi}_1,\tilde{\phi},\tilde{t}_{33}$ and $\tilde{t}_{31}$ from equations (22)-(23), (28)-(29) and (32)-(33) in the boundary conditions (38)-(41), we obtain the parameters A_1, A_2, A_3 and A_4 from the resulting equations as

$$A_1=\{\Delta_1P_1\tilde{f}_1(\xi,s)+\Delta_2P_2\tilde{f}_2(\xi,s)+\Delta_3+\Delta_4+\Delta_5+\Delta_6\}/\Delta \quad (42)$$

$$A_2=\{\Delta_7P_1\tilde{f}_1(\xi,s)+\Delta_8P_2\tilde{f}_2(\xi,s)+\Delta_9+\Delta_{10}+\Delta_{11}+\Delta_{12}\}/\Delta, \quad (43)$$

$$A_3=\{\Delta_{13}P_1\tilde{f}_1(\xi,s)+\Delta_{14}P_2\tilde{f}_2(\xi,s)+\Delta_{15}+\Delta_{16}+\Delta_{17}+\Delta_{18}\}/\Delta, \quad (44)$$

$$A_4=\{\Delta_{19}P_1\tilde{f}_1(\xi,s)+\Delta_{20}P_2\tilde{f}_2(\xi,s)+\Delta_{21}+\Delta_{22}+\Delta_{23}+\Delta_{24}\}/\Delta, \quad (45)$$

where

$$\Delta=(R_1e_4-R_4e_1)(g_2-g_3)+(R_2e_4-R_4e_2)(g_3-g_1)+(R_3e_4-R_4e_3)(g_1-g_2),$$
$$\Delta_1=e_4(g_3-g_2),$$
$$\Delta_2=g_3(R_2e_4-R_4e_2)-g_2(R_3e_4-R_4e_3),$$
$$\Delta_3=-B_1\Delta_1,$$
$$\Delta_4=R_4(g_3-g_2)B_2,$$
$$\Delta_5=((R_4e_2-R_2e_4)+(R_3e_4-R_4e_3))B_3,$$
$$\Delta_6=B_4\Delta_2,$$
$$\Delta_7=e_4(g_3-g_1),$$
$$\Delta_8=g_3(R_4e_1-R_1e_4)+g_1(R_3e_4-R_4e_3),$$
$$\Delta_9=-B_1\Delta_7$$
$$\Delta_{10}=R_4(g_1-g_3)B_2,$$
$$\Delta_{11}=((R_1e_4-R_4e_1)+(R_4e_3-R_3e_4))B_3,$$

VOLUME - 12, ISSUE - 04, APRIL - 2023 • PRINT ISSN No. 2277 - 8160 • DOI : 10.36106/gjra

$\Delta_{12} = B_4 \Delta_8,$

$\Delta_{13} = e_4(g_2 - g_1),$

$\Delta_{14} = (R_1 e_4 - R_4 e_1) g_2 - (R_2 e_4 - R_4 e_2) g_1,$

$\Delta_{15} = -B_1 \Delta_{13},$

$\Delta_{16} = R_4(g_2 - g_1) B_2,$

$\Delta_{17} = ((R_4 e_1 - R_1 e_4) + (R_2 e_4 - R_4 e_2)) B_3,$

$\Delta_{18} = B_4 \Delta_{14}, \qquad \Delta_{19} = e_1(g_3 - g_2) + e_2(g_1 - g_3) + e_3(g_2 - g_1),$

$\Delta_{20} = (R_1 e_2 - R_2 e_1) g_3 + (R_2 e_3 - R_3 e_2) g_1 + (R_3 e_1 - R_1 e_3) g_2,$

$\Delta_{21} = -B_1 \Delta_{19}, \qquad \Delta_{22} = \{R_1(g_2 - g_3) + R_2(g_3 - g_1) + R_3(g_1 - g_2)\} B_2,$

$\Delta_{23} = \{(R_2 e_1 - R_1 e_2) + (R_3 e_2 - R_2 e_3) + (R_1 e_3 - R_3 e_1)\} B_3,$

$\Delta_{24} = B_4 \Delta_{20},$

with

$B_3 = D d_4, \quad B_4 = -\tilde{T}_p,$

$$g_\ell = -\lambda_\ell d_\ell; \quad (\ell = 1,2,3). \tag{46}$$

Substituting the values of A1, A2, A3, and A4 from (42)-(45) in equations (30)-(33), (29) and (23) with the help of equation(46), we obtain the values of displacement components, stresses, change in volume fraction field and temperature distribution. We shall take the heat sources of the form

(i) $Q(x,t) = Q_0 \delta(x) \sin(\frac{\pi t}{\tau})$

distributed heat source with

$\tilde{Q} = \frac{Q_0 \pi \tau (1 + e^{-s\tau})}{\pi^2 + s^2 \tau^2}$

(ii) $Q(x,t) = \delta(x) H(t)$

continuous heat source with

$\tilde{Q} = 1/s,$

(iii) $Q(x,t) = \frac{\delta(t) e^{-dz}}{x^2 + a^2}$

with heat source varying with depth

$$\tilde{Q} = \frac{\pi e^{-a|\xi|} e^{-dz}}{a}, \tag{47}$$

where Q_0, a, d are constants and $\delta()$ and $H()$ are the Dirac Delta function and Heaviside unit step function, respectively.

By replacing the values of $\tilde{Q}$ from equation (47) in equations (24) and (27), we obtain the components of displacement, stress, change in volume fraction field and temperature distribution, respectively.

5. Applications

we take $f_1(x,t)$ and $f_2(x,t)$ as $\quad (f_1(x,t), f_2(x,t)) = \delta(x) f(t), \qquad (48)$

where

$f(t) = \delta(t) / H(t),$

for concentrated/continuous load.

Applying the Laplace and Fourier transforms defined by (8) and (9) on equation (48), we obtain

$$(\tilde{f}_1(\xi, s) / \tilde{f}_2(\xi, s)) = \tilde{f}(s), \tag{49}$$

with

$\tilde{f}(s) = 1 / \frac{1}{s},$

for concentrated/continuous load.

The components of displacement, stress, change in volume fraction field and temperature distribution are obtain by replacing the values of $\tilde{f}_1(\xi, s)$ and $\tilde{f}_2(\xi, s)$ from equation (49) in equations (30)-(33), (29) and (23) with the help of equations(42)-(47).

6. Particular Cases

(i) The corresponding expressions are obtained for mechanical and thermal source by taking $P_2 = 0$ and $P_1 = 0$; in equations (42)-(45), respectively.

(ii) If $\tau_0 = 0$ in equations (30)-(33),(29) and (23) along with equations (42)-(47) and (49), we obtained the corresponding expressions of the porous thermoelastic half-space with heat sources.

(iii) If we neglect the voids effect $(\alpha = b = \xi_1 = m = \psi = \omega_0 = 0)$, in equations (30)-(33) and (23), and with the help of equations (42) -(47) and (49), we obtained the components of displacement, stress and temperature distribution in thermoelastic half - space with heat sources by replacing the values of Δ with Δ^*, Δ_ℓ with Δ^*_ℓ $(\ell = 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 24)$, $R_\ell, e_\ell, m_\ell, \tilde{T}_p, \tilde{\varphi}_p, \lambda_\ell, A, B, E, f_\ell, f_4$ with $R^*_\ell, e^*_\ell, m^*_\ell, \tilde{T}^*_p, \tilde{\varphi}^*_p, \lambda^*_\ell, A^*, B^*, E^*$, $f^*_\ell, f^*_4; (\ell = 1, 2.)$ and $a_2 = b_2 = b_4 = d_\ell = g_\ell = e_3 = R_3 = \lambda_3 = m_3 = f_3 = f_5 = C = F = J = \Delta_5 = \Delta_{11} = \Delta_{13} = \Delta_{14} = \Delta_{15} = \Delta_{16} = \Delta_{17} = \Delta_{18} = \Delta_{23} = A_3 = 0 \ (\ell = 1,2,3.)$ respectively,

where

$\Delta^* = e_4(R^*_2 - R^*_1) + R_4(e^*_1 - e^*_2),$

$\Delta^*_1 = -\Delta^*_3 = -\Delta^*_7 = \Delta^*_9 = e_4,$

$\Delta^*_4 = -\Delta^*_{10} = R_4,$

$\Delta^*_2 = \Delta^*_6 = R^*_2 e_4 - R_4 e^*_2,$

$\Delta^*_8 = \Delta^*_{12} = R_4 e^*_1 - R^*_1 e_4,$

$\Delta^*_{19} = -\Delta^*_{21} = e^*_2 - e^*_1,$

$\Delta^*_{22} = R^*_2 - R^*_1,$

$\Delta^*_{24} = \Delta^*_{20} = R^*_1 e^*_2 - R^*_2 e^*_1, \qquad R^*_\ell = \{(a_{11} + 2)\lambda^{*2}_\ell m^*_\ell - a_{11} m^*_\ell \xi^2 - a_3\},$

$e^*_\ell = 2(\iota\xi)\lambda^*_\ell m^*_\ell,$

$m^*_\ell = \frac{b_3}{\lambda^{*2}_\ell + E^*},$

$\tilde{T}^*_p = \frac{-a_{10} b_6 (D^2 + E^*)\tilde{Q}}{(D^4 + A^* D^2 + B^*)},$

$\tilde{\varphi}^*_p = \frac{-a_{10} b_3 b_6 \tilde{Q}}{(D^4 + A^* D^2 + B^*)}, \qquad \lambda^{*2}_\ell = \frac{-A^* + (-1)^{\ell+1}\sqrt{A^{*2} - 4B^*}}{2}; (\ell = 1,2)$

with $\quad A^* = -2\xi^2 + f^*_1, \ B^* = \xi^4 - f^*_1 \xi^2 + f^*_2, \ E^* = -\xi^2 + f^*_4, \qquad f^*_1 = -e_1 b_3 b_5 - b_5 - b_1 s^2,$

$f^*_2 = b_1 b_5 s^2, \quad f^*_4 = -b_1 s^2 \quad .$

7. Inversion of the transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms in equations (30)-(33), (29) and (23) by using equation (49) and with the help of equations (42) -(47) for coupled theory (CT), Lord and Shulman theory(L-S) with one relaxation time. These expressions are functions of z ,the parameters of Laplace and Fourier transforms s and ξ, respectively, and hence are of the form $\tilde{f}(\xi, z, s)$. To get the function $f(x, z, s)$. in the physical domain. First we invert the Fourier transform using

$$\hat{f}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\iota\xi x} \tilde{f}(\xi,z,s)\, d\xi \qquad = \frac{1}{\pi}\int_0^{\infty} (\cos(\xi x) f_e - \iota \sin(\xi x) f_0)\, d\xi, \tag{50}$$

Where f_e and f_0 are, respectively, even and odd parts of the function $\tilde{f}(\xi, z, s)$. Thus expression (50) gives us the Laplace transform $\hat{f}(x, z, s)$. of the function $f(x, z, t)$.

Then, for the fixed values of ξ, x and z the $\hat{f}(x, z, s)$. in the expression (50) can be considered as the Laplace transform $\hat{g}(s)$ of $g(t)$. Following Honig and Hirdes [19], the Laplace transformed function $\hat{g}(s)$ can be inverted as given below. The function g(t) can be obtained by using.

$$g(t) = \frac{1}{2\pi\iota}\int_{X - \iota\infty}^{X + \iota\infty} e^{st} \hat{g}(s)\, ds, \tag{51}$$

where X is an arbitrary real number greater than all the real parts of the singularities of $\hat{g}(s)$. Taking $s = X + \iota y$, we get

$$g(t) = \frac{e^{Xt}}{2\pi}\int_{-\infty}^{\infty} e^{\iota t y} \hat{g}(X + \iota y)\, dy. \tag{52}$$

Now, taking $e^{-Xt} g(t)$ as $h(t)$ and

expanding it as Fourier series in [0,2L], we obtain approximately the formula

$g(t) = g_\infty(t) + E_D,$

where $\quad g_\infty(t) = \frac{X_0}{2} + \sum_{k=1}^{\infty} X_k, \qquad 0 \le t \le 2L,$

and

$$X_k = \left(\frac{e^{Xt}}{L}\right) \mathrm{Re}\left[e^{\frac{ik\pi t}{L}} \hat{g}\left(X + \frac{ik\pi}{L}\right)\right] \tag{53}$$

E_D is the discretization error and can be made arbitrarily small by choosing X large enough; Honig and Hirdes [19]

As the infinite series in equation (50) can be summed up only to a finite number of N terms, so the approximate value of g(t) becomes

$$g_N(t) = \frac{X_0}{2} + \sum_{k=1}^{N} X_k, \quad 0 \le t \le 2L, \tag{54}$$

VOLUME - 12, ISSUE - 04, APRIL - 2023 • PRINT ISSN No. 2277 - 8160 • DOI : 10.36106/gjra

Now, we introduced a truncation error E_T, that must be added to the discretization error to produce the total approximate error in evaluating g(t) using the above formula. To accelerate the convergence, the discretization error and then the truncation erroris reduced by using the 'Korrektur- method' and the '∈-algorithm' , respectively as given by Honig and Hirdes [19].

The The Korrektur method formula, to evaluate the function g(t) is

$$g(t) = g_\infty(t) - e^{-2XL} g_\infty(2L + t) + E'_D,$$

where the descretization error $|E'_D| << |E_D|$

Thus, the approximate value of g(t) becomes

$$g_{N_K}(t) = g_N(t) - e^{-2XL} g_{N'}(2L + t),$$

where N' is an integer such that $N' < N$.

We shall now describe the ∈- algorithm, which is used to accelerate the convergence of the series in equation (54). Let N be an odd natural number and $S_m = \Sigma_{k=1}^{m} X_k$ be the sequence of partial sums of the equation (54). We define the' ∈- sequence' by

$$\epsilon_{0,m} = 0, \quad \epsilon_{1,m} = S_m, \quad \epsilon_{n+1,m} = \epsilon_{n-1,m+1} + \frac{1}{\epsilon_{n,m+1} - \epsilon_{n,m}}, \quad n, m = 1, 2, 3 \ldots$$

The sequence $\epsilon_{1,1}, \epsilon_{3,1}, \ldots, \epsilon_{N,1}$ converges to $g(t) + E_D - \frac{X_0}{2}$ faster than the sequence of partial sums S_m, m = 1, 2, 3,... The actual procedure to invert the Laplace transform consists of equation (52) together with the '∈-algorithm'.

The last step is to calculate the integral in equation (50). The method for evaluating this integral is described Press et al [20], which involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8. Numerical Results and Discussion

With the view of illustrating and compare the theoretical results obtained above in the context of the Lord and Shulman(L-S) theory of thermoelasticity,we now present somenumerical results. The physical data for thermoelastic material is taken from Sherief and Helmy [21]

$\lambda = 7.76\times10^{10} N/m^2$, $\mu = 3.86\times10^{10} N/m^2$, $K = 3.86\times10^2 W/m\,\text{deg}$, $\rho = 9.854\times10^3 Kg/m^3$, $T_0 = 293 K$, $\alpha_t = 1.78\times10^{-5}/K$, $C_e = 381\times10^3 J/Kg\,\text{deg}$,

and the voids parameters are

$\psi = 1.753\times10^{-15} m^2$, $\alpha = 3.688\times10^{-5} N$, $\xi_1 = 1.475\times10^{10} N/m^2$, $b = 1.13849\times10^{10} N/m^2$, $m = 2\times10^6 N/m^2\,\text{deg}$, $\omega_0 = 0787\times10^{-3} N/m^2 s$. The comparison were carried out for

$P_1 = P_2 = 1$, $\tau_0 = 0.02$, $Q_0 = a = d = \tau = 1$.

The comparison of normal displacement w, normal stress t_{33},change in volume fraction field ϕ, and temperature distribution T, due to various heat sources namely (i)distributed

heat source(QI) (ii)continuous heat source(QII) (iii)heat source varying with depth(QIII) with and without voids have been studied for concentrated or continuous thermomechanical sources, respectively. The variations of normal displacement w, normal stress t_{33}, and temperature distribution T, these components with distance x, have been shown by (i) solid line with voids (QIV) and solid line with centered symbol circle without voids (QIW) for (QI)(ii) long dashed line with voids (QIIV) and long dashed line with centered symbol square without voids (QIIW) for (QII) (ii) small dashed line with voids (QIIIV)and small dashed line with centered symbol triangle without voids (QIIIW) for (QIII) and the variations of change in volume fraction field ϕ, for (QIV,QIIV,QIIIV) have been shown by(i)sparse for QIV)(ii)dense for (QIIV) (iii)medium for (QIIIV) ; at non-dimensional time $t = 0.5$, in figures 1-6 and 7-8, respectively. The computations are carried in the range $0 \le x \le 10$.

8.1 Thermomechanical source (concentrated/continuous)

Fig.1. shows the variations of normal displacement w with distance x . The value of w for (QIV) increases in the range 0 ≤ x ≤ 2.2 whereas for (QIW) decreases in the same range and oscillatory in the remaining range of x . The value of w for (QIIV) increases slowly in the range 0 ≤ x ≤ 2.2 whereas for (QIIW) increases sharply in the same range and oscillatory in the remaining range of x . Also the behavior of variations of w for (QIIIV,QIIIW)is oscillatory in the whole range of x . Fig.2. shows the variations oft33 with distance x. The trend of variations of t33 for (QIV,QIW) is same whereas the correspondingvalues are different in magnitude. Also the behavior of variations of t33 , for (QIIV,QIIW) is oscillatory in the whole range of x , but the magnitude of oscillation is large for (QIIV) in comparison to (QIIW). Nearthe point of application of source, the value of t33 for(QIIIV) increases sharply in the range 0 ≤ x ≤ 2.3whereas for (QIIIW) decreases in the same range and in the remaining range of x the behavior of variations of normal stress is opposite oscillatory for (QIIIV,QIIIW),respectively. The magnitude of oscillations decreases as x increases further. Fig.3. shows the variations of temperature distribution T. with distance x . The values of T for all type of heat sources is same i.e. near the point of application of boundary source the values of Tdecrease in the range 0 ≤ x ≤ 2 and oscillatory as away from the boundary source. Fig.4. shows thevariations of normal displacement w with distance x . The behavior of variations w for (QIV,QIW) is opposite oscillatory in the whole range of x . Also the behavior of variations of w for (QIIV,QIIW) is same but magnitude of oscillation of w for (QIIV) is more in comparison to (QIIW). The trend of variations of w for (QIIIV,QIIIW) is same i.e. the values of w first decrease in the range 0 ≤ x ≤ 2.8 andas x increases further it oscillate around zero. Fig. 5. shows the variations of normal stress t_{33} withdistance x. The trend of variations of t^{33} for(except QIV) is same, i.e. the values of t33 for(except QIV)first increase in the range0 £ x £ 2 whereas for (QIV) decreases in the same range and then oppositeoscillatory in the remaining range of x .

Fig 6. shows the variations of temperature distribution T withdistance x .The values of T for (except QIV) decreses in the range 0 ≤x ≤ 2.2 and converge to zero valueas x .increases further.Also the value of T for (QIV) decreases in the range 0 ≤ x ≤ 2.5 whereas for (QIW) decrease in the same range and oscillate with small magnitude value in the remaining range of x . Fig.7 and 8. shows the variations of change in volume fraction field Φ due to concentrated/ continuous source with distance x. Near the point of application of source, the values of f for (QIV) is more ascompare for(QIIV,QIIIV). The value of f for (QIV) first decreases sharply in the range 0 ≤ x ≤2.4whereas for (QIIV,QIIIV) decrease gradually in the same range and as x . increases further the behavior ofvariations of Φ is oscillatory. To compare the variations the values of demagnified by multiplying 10^{-1}.

9 CONCLUSION

The comparison of theory of porous thermoelasticity i.e. Lord and Shulman theory(L-S) with one relaxation time due to thermomechanical(concentrated or continuous) source with various heat sources is carried out. It is observed that the behavior of variations of normal displacement, change in volume fraction field and normal strees for (QI,QII,QIII) due to concentrated source is similar to those for the continuous source, with only difference in their magnitude value; with and without voids, respectively.

Also it is noticed that the temperature distribution for (QIIV,QIIIV), with voids are more in comparison to without voids due to concentrated or continuous source, respectively. The behavior of normal stress for (QIV) due to concentrated source is opposite to that of the continuous source. It is observed that the magnitude of normal displacement, normal stress, change in volume fraction field and temperature distribution follow an oscillatory pattern as x diverges from the point of application of source.

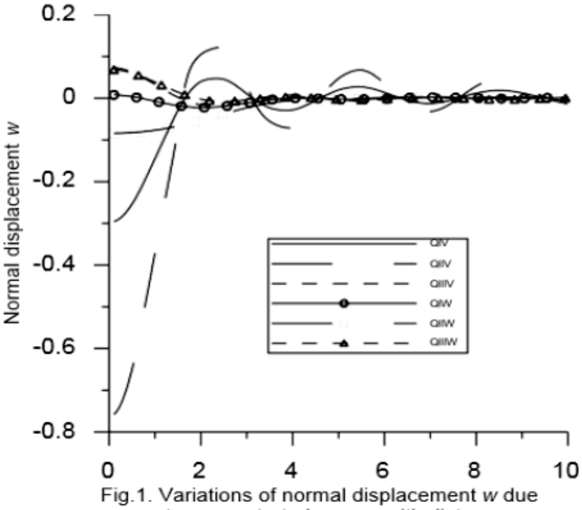

Fig.1. Variations of normal displacement w due to concentrated source with distance x.

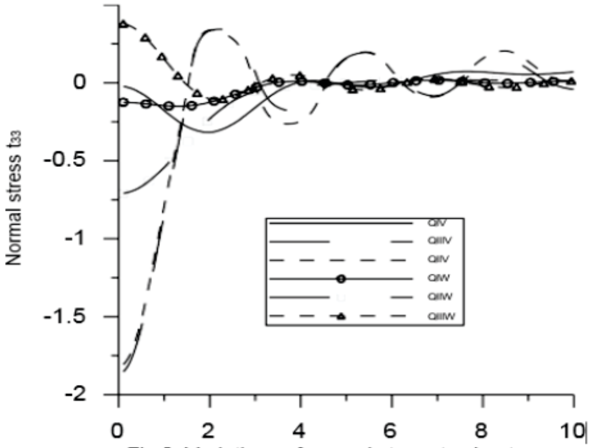

Fig.2. Variations of normal stress t_{33} due to concentrated source with distance *x*.

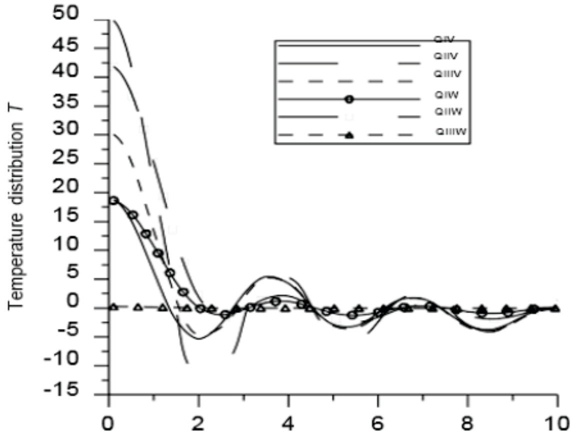

Fig.3. Variations of temperature distribution *T* due to concentrated source with distance *x*.

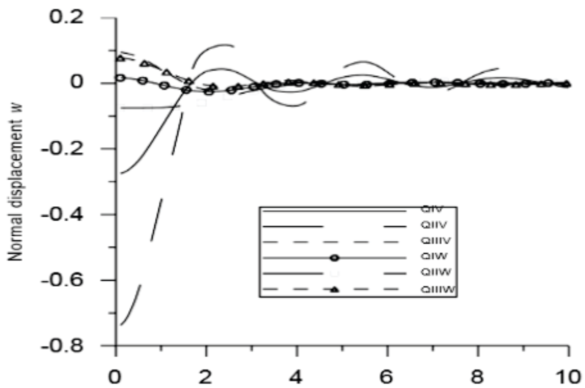

Fig.4. Variations of normal displacement *w* due to continuous source with distance *x*.

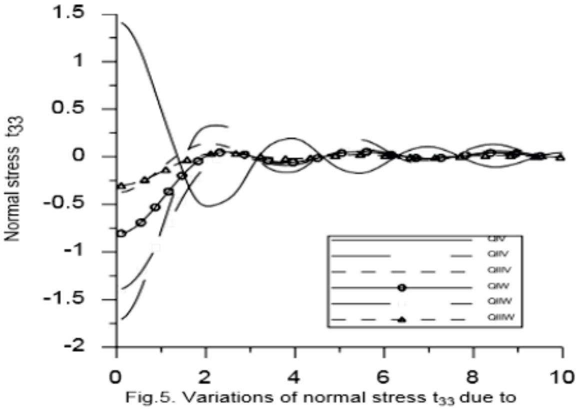

Fig.5. Variations of normal stress t_{33} due to continuous source with distance *x*.

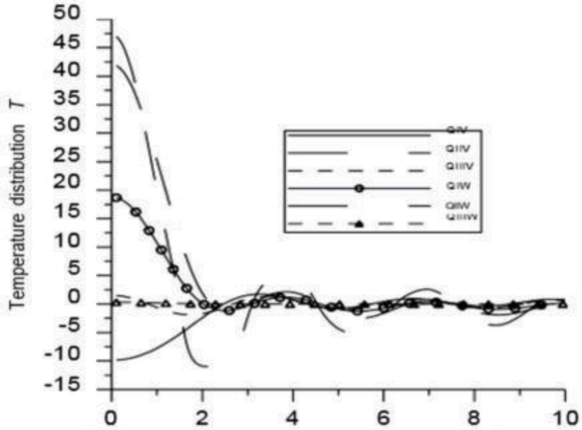

Fig.6. Variations of temperature distribution *T* due to continuous source with distance *x*.

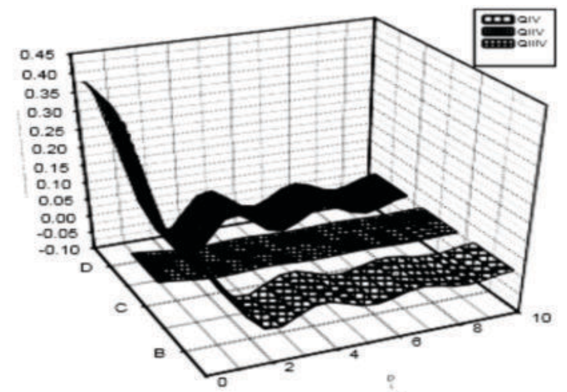

Fig.7. Variations of change in volume fraction field? due to concentrated source with distance x.

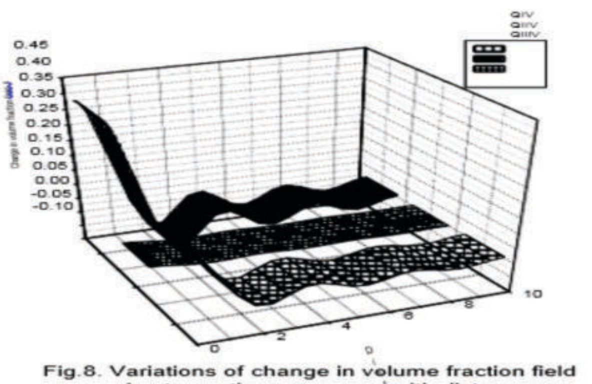

Fig.8. Variations of change in volume fraction field due to continuous source with distance x.

REFERENCES

1. Biot, M., "Thermoelasticity and irreversible thermodynamics", J.Appl.Phys., 1956,27:240 - 253.
2. Lord, H. and Shulman, Y. A., "Generalized dynamical theory of thermoelasticity", J. Mech. Phys. Solid,1967,15: 299-309.
3. Joseph,D.D. and Preziosi,L.,"Heat waves", Rev. Mod.Phys.,1989,61:41-73.
4. Joseph,D.D. and Preziosi,L.,"Addendum to the Paper:Heat waves", Rev. Mod.Phys.,1990,62:375-391.
5. Nunziato, J. W. and Cowin, S. C.,"A non-linear theory of elastic materials with voids", Arch. Rational Mech. Analy., 1979,72:175-201.
6. Cowin, S. C. and Nunziato, J. W.,"Linear elastic materials with voids" J. of Elasticity, 1983,13:125-147.
7. Cowin, S. C. "The viscoelastic behavior of linear elastic materials with voids" J.Elasticity,1985,15:185- 191.
8. Ciarletta, M. and Iesan, D.,"Non-classical", Pitman Research Notes in mathematics Series, 293. Longman Scientific and Technical,Essex(1993).
9. Iesan, D. "A theory of thermoelastic materials with voids" Acta Mechanica, 1986,60: 67-89.
10. Iesan, D. "Thermoelastic models of continua", Kluwer Academic Publishers, Boston/Dordrecht/London, 2004.
11. Chirita, S. and Scalia, A.,"On the spatial and temporal behavior in linear thermoelasticity of materials with voids" J. Thermal Stresses,2001,24: 433-455.
12. Iesan, D. and Nappa, L.,"Thermal stresses in plain strain of porous elastic bodies" Meccanica, 2004,39: 125-138.
13. Kumar, R. and Rani, L.,"Response of generalized thermoelastic half-space with voids to mechanical and thermal source" Meccanica, 2004,39: 563-584.

14. Kumar, R. and Rani, L.,"Deformation due to mechanical and thermal sources in thermoelastic half- space with voids" J. of Vibration and control, 2004,11: 499-517.
15. Kumar, R. and Rani, L.,"Elastodynamic response of mechanical and thermal sources in generalized thermoelastic half-space with voids" Mechanics and Mechanical Engineering, 2005,9: 29-45.
16. Kumar, R. and Rani, L.,"Deformation due to mechanical and thermal sources ingeneralized orthorhombic thermoelastic material" Sadhana, 2005,28: 123-145.
17. Kumar, R. and Rani, L.,"Deformation due to mechanical and thermal sources in generalized thermoelastic half-space with voids" J. of Thermal Stresses, 2005,28: 123- 145.
18. Kumar, R. and Rani, L., "Mechanical and thermal sources in generalized thermoelastic half-space with voids" Indian J. Pure and Appl. Math., 2005,36: 113-134.
19. Honig, G. and Hirdes, U., ''A method for the numerical inversion of Laplace transforms'' J. Comput. And Appl. Math., 1984, 10:113-132.
20. W. H. Press, S. A. Teukolshy, , W. T. Vellerling and B.P.Flannery, Numerical Recipes in FORTRAN (2ndedn.) Cambridge University Press, Cambridge, (1986).
21. Sherief, H. H., Helmy, A.K.,"A two dimensional problem for a half-space in magneto-thermoelasticity with thermal relaxation" Int. J. of Eng. Sc.,2002,40:587- 604.