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NUMERICAL SOLUTION OF KORTEWEG-DE VRIES -BURGERS' (KDVB) EQUATION BY SEMI-ANALYTICAL METHOD

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ABSTRACT Korteweg-De Vries -Burgers' (KdVB) equation is a nonlinear Partial Differential Equation(PDE) which has a wide range of applications in various fields of Mathematics, Science and Engineering. In this article KdVB equation with an inhomogeneous term and specified initial condition is solved by implementing a semi-analytic method called Laplace Decomposition Method (LDM).The essential features of implementation of LDM on nonlinear Partial differential equations (PDEs) are presented in the paper. A variety of examples are illustrated graphically and numerically by using MATLAB 7.0. The influence of initial condition and flow parameters on the wave nature and propagation is analyzed. The convergence and the effectiveness of the technique are presented through error analysis. The results obtained show that the final solution is dependent on the nature of initial condition and source term. The approach and technique can be implemented to solve a wide range of inhomogeneous non-linear dispersive-diffusive equations with different initial conditions and source terms.

KEYWORDS : Approximation, KdVB Equation, LDM, Nonlinear PDE, Source Term.

INTRODUCTION

Solving nonlinear Partial Differential Equations (PDEs) and the analysis of nonlinear wave phenomena has always been and remains, the topic of considerable interest in the fields of engineering and applied sciences. This is due to fact that the presence of nonlinear term in governing equations can manifest a wide range of wave behaviour. Korteweg-de Vries-Burgers' (KdVB) equation is an important basic model which forms an important class of PDE that occurs in many physical phenomena viz. nonlinear advection, diffusive-dispersive phenomena.

Korteweg-de Vries-Burgers' (KdVB) equation (Su & Gardner,1969),(Grad & Hu,1967), (Wijngaarden, 1972) is given by

$$u_t + uu_x + \nu u_{xx} + \delta u_{xxx} = 0, u = u(x,t), t > 0, x \in R \quad (1.1)$$

Here,$\nu > 0$,δ are real constants with the diffusive parameter ν and δ the dispersive parameter.The solution of (1.1) has important applications in weak plasma shock propagation, waves incoming from deep water, to analyze change in shapes long waves, fluid flow in elastic tubes, liquid in small bubbles and turbulence, etc. (Grad & Hu, 1967), (Wijngaarden, 1972). (1.1) is an alternate form of Burgers equation when $\delta=0$. For $\nu=0$, it takes the form of KdV equation and exhibits typical type of solution called soliton solutions (Wazwaz A. M., 2009)as the result of balance between nonlinearity, dispersion and diffusion. For negligible dispersive parameter, it may also exhibit shock wave solutions which have very important applications in biomedical and agricultural research(Jagadeesha & Natarajab, 2009).

Nature of the solution, wave profile and wave behaviour can be understood and hence physically realistic solution can be obtained when the nature of the source term is known and the initial condition is provided.

Let the KdVB equation with specified initial condition is as below;

$$u_t + uu_x + u = \nu u_{xx} + \delta u_{xxx}, u(x,0) = h(x), t \geq 0 \quad (1.2)$$

In the present work, a semi- analytical method called Laplace Decomposition Method (LDM) has been applied to solve (1.2) analytically. It is based on the concept of Laplace transform and a technique called Adomain Decomposition method (ADM) which uses Adomian polynomials for decomposition of nonlinear terms and the iterative technique to find infinite series solution.The convergence is decided by the initial approximation which is obtained by the source term and initial condition. The procedure, computation and convergence of the method are as given in the following papers and the references there in.(Khuri, 2001), (Khuri, 2004),(Adomian & Rach, 1986), (Wazwaz A. M., 1997), (Ismail, Raslan, & Abd Rabboh, 2004), (Biazar & Shafiof, 2007), (Djeriba & Belghaba, 2021), (Rao & Warke, 2022), (Beghami, Maayah, Bushnaq, & Arqub, 2022).

The Adomain polynomials $A_n(u)$, the recurrence relation and the solution $u(x,t)$ of (1.2) are given by (Wazwaz A. M., 2000), (Biazar & Shafiof, 2007) and (Rach, 2008),

$$A_n(u) = uu_x \quad (1.3)$$

$$u_{n+1}(x,t) = L^{-1}\left[\frac{1}{s+1}\left(L\left(\nu\frac{\partial^2}{\partial x^2} + \delta\frac{\partial^3}{\partial x^3}\right)(u_n(x,t))\right)\right] - L^{-1}\left[\frac{1}{s+1}L(A_n(u)\right], n \geq 0 \quad (1.4)$$

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \quad (1.5)$$

This paper will deal with the comparison of LDM approximations of (1.2) for hyperbolic and exponential initial conditions, continuous work of(Rao, 2021). The analysis of the results and presentation of graphical results are given in Section 2. Section 3 contains the concluding remarks and the references are given in Section4.

RESULTS AND DISCUSSION

We consider the initial value problem KdVB equation with an inhomogeneous term in the standard form as below,

$$u_t + uu_x + \alpha u = \nu u_{xx} + \delta u_{xxx}, u(x,0) = h(x) \quad (2.1)$$

The exponential and hyperbolic initial conditions are considered to solve (2.1) and to understand the impact of initial condition and flow parameters. By calculating initial approximations, Adomian polynomials and successive approximations for both the cases, we arrive at a solution of desired accuracy.

For $h(x) = e^{-2x}$ and $\alpha = \nu = \delta = 1$, the initial approximation u_0 is given by,

$$u_0 = e^{-t-2x} \quad (2.2)$$

and five-term approximate solution is given by,

$$u = e^{-2t}(16e^{-4x} - 12e^{-6x}) - e^{-t}(16e^{-4x} - 6e^{-6x}) + e^{-3t}(204e^{-6x} + 64e^{-8x}) + e^{-t}(3072e^{-4x} + 1620e^{-6x}) + \ldots \quad (2.3)$$

Table 2.1 depicts the results of various approximations and relative errors of (2.1) for different flow parameters and specified initial conditions. There is marginal decrease in u for $u(x,0)=e^{-2x}$ and increase in u for $(x,½0=sech^2(½x))$ as the number of terms of the approximattions increases. Higher rate of convergence is achieved for $u(x,0)=½\ sech^2(½x)$ the convergence and accuracy is achieved by using only five terms with negligible relative errors. The MATLAB graph depicting the wave profile of five terms approximations of (2.1) for inital conditions is shown in fig.2.1.

Table 2.1: Results of (2.1) for $t=0.001$, δ=1, ν=1

x	Approximations				Relative Errors	
	$u(x,0)=e^{-2x}$		$u(x,0)=½sech^2$		$u(x,0)=e^{-2x}$	$u(x,0)=½sech^2$
	u_0	5-term	u_0	5-term		
0.1	0.8179	0.8160	0.4983	0.4981	4.8731e-09	2.9210e-10
0.3	0.5483	0.5467	0.4884	0.4884	5.2652e-10	2.7495e-10
0.5	0.3675	0.3663	0.4695	0.4696	1.0208e-09	8.1214e-11
0.7	0.2464	0.2455	0.4430	0.4431	1.3370e-09	1.1117e-10
0.9	0.1651	0.1645	0.4106	0.4108	1.1842e-09	1.7658e-10

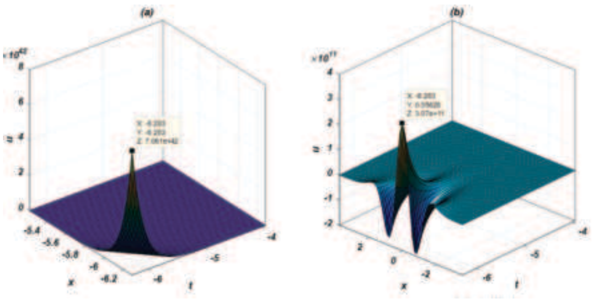

Fig. 2.1: Graph of the 5-term Approximation of Solution of (2.1) (a)$u(x,0)=e^{-2x}$ b)$u(x,0)=½sech^2(½x)$

Table 2.2 depicts the results of relative errors of (2.1) with specified initial conditions and other values of flow parameters. There is marginal increase in u and decrease in errors when there is an increase in t. The MATLAB graphs of relative errors for different values of t and the comparison of error graphs of (2.1) for specified initial conditions are shown in Fig.2.2 and Fig.2.3. Better accuracy is obtained when t values are smaller in both the cases.

Table 2.2: Results of relative errors of (2.1) for ν=1, δ=1

x	Relative errors for $u(x,0)=e^{-2x}$		Relative errors for $u(x,0)=½sech^2(½x)$	
	$t=0.002$	$t=0.004$	$t=0.002$	$t=0.004$
0.1	7.8376e-08	1.2574e-06	4.6767e-09	7.4871e-08
0.3	7.7988e-09	1.2400e-07	4.4030e-09	7.0454e-08
0.5	1.6572e-08	2.6756e-07	1.2973e-09	2.0745e-08
0.7	2.1412e-08	3.4504e-07	1.7782e-09	2.8423e-08
0.9	1.9095e-08	3.0784e-07	2.8268e-09	4.5171e-08

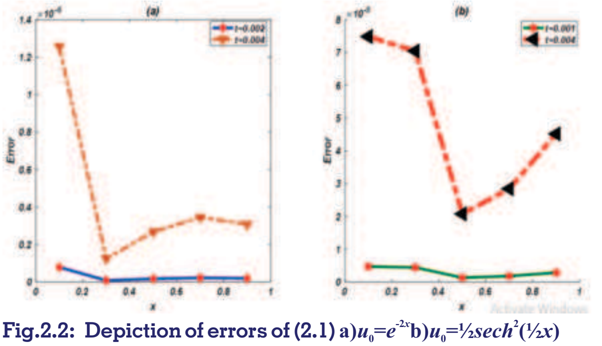

Fig.2.2: Depiction of errors of (2.1) a)$u_0=e^{-2x}$ b)$u_0=½sech^2(½x)$

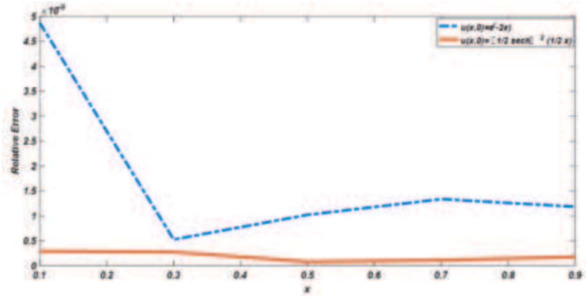

Fig.2.3: Comparison of relative errors of (2.1)

CONCLUSION

In research work, LDM is used to obtain solution of a model diffusive-dispersive equation; KdVB equation with hyperbolic and exponential initial condition. Various approximations with relative errors and MATLAB graphs of the wave profiles of approximations for different flow parameters are presented. The results obtained are analysed to understand the influence of initial condition on wave profile of the final solution. The results obtained show that, LDM has faster convergence rate and the desired level of accuracy is achieved by using very few terms. The LDM can be tested for solving various nonlinear PDE describing diffusive-dispersive phenomena with different initial conditions and source terms. As the solutions of KdVB equation is very important for understanding many physical phenomena, the results can be used in studying waves observed in fluids, plasma, astrophysics, water waves, etc. The approach can also be extended to study the influence of nature of initial condition on the wave nature by comparing the solutions under different types of initial conditions.

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