



Element Locations in System Identification

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ABSTRACT

Prediction of element location is still a challenging issue in the field of structural system identification. This paper presents a new approach for prediction of element locations using inverse technique. A finite element method (FEM) algorithm is presented here to detect the element location without having any prior assumptions. The distributed stiffness parameter model of the finite element model matrix is used instead of lumped mass/stiffness matrix formulation for the global stiffness matrix. The algorithm is able to identify the single as well as multiple element location case. Advantage of the current approach is to overcome the coupling terms problems, even for multiple element locations can be identified in the system parameter matrix. The example on pin-jointed truss is presented for the better understanding of the approach.

Keywords : System identification, Inverse problem, Structural health Monitoring, element location.

Introduction

Developments in the field of computational technique ultimately forced the structural behavior prediction using faster computation techniques. The finite element technique is universally accepted as a fast computation tool for the structural behavior prediction.

The existence of damage can be identified by comparing the undamaged structures response and the present response of structure, any adverse change in the structural response indicates the existence of damage. Once the existence of damage is identified, the next step is to find the damage location. Locations of damage are not known for any structure initially. In fact, the location of damage is predicted from the field response data of damaged structure therefore it is an inverse type problem. Many researchers are trying to find the location of damage using various approaches. A brief review is presented here:

Grafe (1998) mentioned that the location detection of damage and the noise are still the unresolved issues in the modal updating technique. Fripp and Atalla (2001) reviewed the modal sensing and actuation techniques using shape distributed modal transducer and discrete element modal transducer for practical data. Araki and Miyagi (2005) formulated damage detection as a mixed integer nonlinear least-square problem. A linearized error function was applied to the line search and for grouping of unknown parameters.

Another most common used technique for damage location prediction approached by sensitivity analysis or its derivatives. Hwu and Liang (2001) applied the strain energy sensitivity approaches in damage detection. Pothisiri and Hjelmstad (2003) proposed an optimization scheme to localize damaged sensitivity with measured noise. Bernal and Gunes (2004) used stochastic excitation technique with the change in flexibility matrix. Duan et al. (2005) used the changes of flexibilities matrix using presented a damage localization approach. A comparative study was presented by Catbas et al. (2007) they reviewed the practical aspects of monitoring of bridges on inverse problems.

Gladwell (1999) in an inverse finite element vibration showed that there is an infinite family of possible systems, of

the required form, corresponding to the given data. Even when the data consists of just one spectrum, then the analysis can be used to construct an infinite family of isospectral pairs of mass and stiffness matrices, all of the correct generic form, from one such pair. In finite element vibration theoretical problem that it is not possible

It is concluded from the above literature review that the location of damage element and type of sensors, number and location of measurement sensors, nonlinear structural response, energy loss etc. are among still the challenges in the field of damaged element location identification.

In this paper a new concept of element location is introduced for system element location in case of inverse problem. A finite element approach using the distributed stiffness matrix is used instead of lumped mass/stiffness matrix formulation of global stiffness matrix. The element location in the system matrix are identifying for single as well as multiple change in the system parameters. Numerical example is tested on a truss model.

Matrix Transformation from Local to Global

The most common, matrix transformation from the local to global in the finite element system matrix is usually carried out using following expression

$$[K] = [U]^T [k][U] \quad (1)$$

In the most of the papers the diagonal terms were used for the system identification by taking the lumped mass/stiffness matrix parameters. The diagonal terms in the system global matrix never represent the true element property. Rather it indicates the property of all the elements meeting at that joint. Hence the differentiations of the diagonal terms affect all the elements meeting at that particular joint. Generally, assumption regarding the damage element locations is inherent in some or other form for damage detection problem. Accordingly, in the optimization process the differential coefficients of elements are taken. For any unknown parameter location, the differential coefficients are not known. It may be any unknown constant, may be zero or may be one. Hence derivatives are not possible due to unknown parameter locations.

In other words, any optimization technique cannot be applied without knowing the prior element location. The above concept is applicable to the sensitivity analysis also. Some of the general observations on damage location prediction algorithms using inverse problem are:

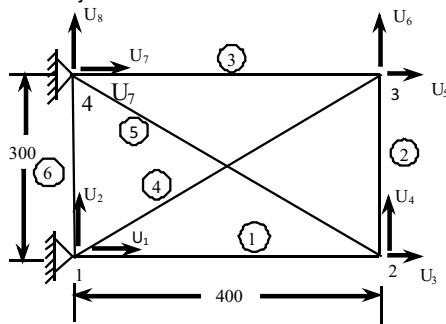
- Damaged element locations are unknown
- Matrix inversion should be avoided. (Dewangan, 2011)
- Damage prediction with noisy data set on diagonal member is a difficult task

In the damage parameter estimation the

Global stiffness/mass matrices use the diagonal terms for calculations. The diagonal element does not represent a single element property. They represent the mixed property of the members meeting at a joint in the corresponding row according to degree of freedom (DOF) of the joint in the row or column. The drawback in using the diagonal terms of a matrix is that, the element properties of the local elements are being added in terms of global matrix. Even minimizations of diagonal terms with noisy data are a difficult task, as noise locations are not known. Therefore it is concluded that the minimizations of diagonal terms with noisy output are difficult to handle.

In order to demonstrate the elements locations in the global stiffness matrix using the distributed stiffness parameters the global matrix is assembled for a pin jointed truss structure.

Figure 1: Pin jointed redundant truss



For the truss structure, Fig. 1, the modulus of elasticity of all elements are 206.8 GPa and initial undamaged cross sectional area of all members is 322.6 mm².

In order to demonstrate the elements locations in the global stiffness matrix, assuming a hypothetical local element stiffness values for the elements are 1, 2, 3, 4, 5 and 6 as k_1, k_2, k_3, k_4, k_5 and k_6 respectively. The global transformation of such all type of elements is performed using finite element transformation Eq.1. The assembled global stiffness matrix K is shown in Fig.2. From the global stiffness matrix, the element locations can be readout directly.

Considering the upper diagonal of global stiffness matrix the element locations are: Element 1 (r_1, r_2, c_3, c_4), Where r represents the row number and c is the column number. Similarly the other Elements are located at: Element 2 (r_3, r_4, c_5, c_6), Element 3 (r_1, r_2, c_3, c_4), Element 4 (r_3, r_4, c_5, c_6), Element 5 (r_3, r_4, c_7, c_8) and Element 6 at (r_1, r_2, c_3, c_4) locations.

It is to be noted that the off diagonal terms in global stiffness matrix relates to the element property directly, Fig. 2. Due to symmetry of matrix in the linear range, one can consider as lower diagonal portion also. These element locations reflect the individual element property directly in global stiffness matrix.

Figure 2 : Element locations in global stiffness matrix

$$k_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad k_2 = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

$$k_3 = \begin{bmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix} \quad k_4 = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix}$$

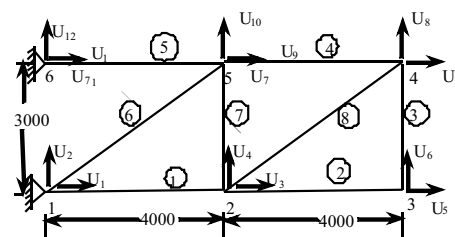
$$k_5 = \begin{bmatrix} 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 \end{bmatrix} \quad k_6 = \begin{bmatrix} 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 \end{bmatrix}$$

DOF	1	2	3	4	5	6	7	8
K= 1	10	10	1	1	3	3	6	6
2	10	10	1	1	3	3	6	6
3	1	1	8	8	2	2	5	5
4	1	1	8	8	2	2	5	5
5	3	3	2	2	9	9	4	4
6	3	3	2	2	9	9	4	4
7	6	6	5	5	4	4	15	15
8	6	6	5	5	4	4	15	15

If a member is having its end node numbers as I and J | (here J |) then it can be placed in the global matrix at a row location (2I-1, 2I) and column location (2J-1, 2J) in the upper diagonal. Here off diagonal terms, reflect the element property directly. The locations in the global coordinates are demonstrated for damage prediction. This element location plays a key role in separation of the damaged members individually. The above concept is explained with the help of numerical examples in the following paragraph.

Following paragraph demonstrate the developed algorithm in the context of a pin jointed truss model. The structural configuration is shown in Fig. 3. [2].

Figure 3: Pin jointed truss structure



The load displacement relationship is assumed as a linear one for small displacements. For a two dimensional linear finite element model, the element locations are shown encircled. The Joint numbers are shown without encircled along with their displacement degree of freedom. The modulus of elasticity $E = 29.5 \times 10^5 \text{ N/mm}^2$ and the original cross-sectional area of all members are $A=100 \text{ mm}^2$ for the undamaged condition are considered. Global stiffness matrix K for the original structure is generated using linear FEM model.

Change in the element stiffness parameter is introduced as the reduction in the cross sectional area of the member. A change value of stiffness parameters 10%, 20% and 25% are introduced in the elements 1, 3 and 4 respectively. The global stiffness matrix for changed structure is generated multiplying the reduced stiffness values constants to respective element stiffness at their corresponding locations. The assembled global stiffness matrix K_d for changed structure is arrived by assembling stiffness of various members $0.9k_1$, k_2 , $0.8k_3$ and $0.75k_4$. Where k_1 to k_4 are the element stiffness matrices of original unchanged structure. The computed matrices of K , K_d and $(K - K_d)$ are computed. The results are computed for various examples are tabulated in **Table 1** for this example.

Table 1. Element locations prediction for Pin Jointed truss, Fig. 3

Case	member location	Measured data	Member locations identified from its location
1	2,3	Displacements	2,3
2	2,3,8	Displacements	2,3,8
3	2,3,4	Displacements	2,3,4
4	2,3,4,7,8	Displacements	2,3,4,7,8
5	All	Displacements	All

With reference to the above Table, Case 3 is explained here. Changes are introduced at some of the unknown locations. The joint displacements are measured with the sets of forces as stated above. The row echelon form is used for computation of changed stiffness matrix. Once the changed stiffness matrix (K_d) is known, elements locations are read out from their respective locations as discussed above.

Conclusions

In this paper, a completely new approach for element location using finite element model was presented. The physical meaning of the matrix transformation $[K]=[U]^T[k][U]$ was established with special emphasizing element location. Element locations in the global stiffness matrix system for a two-dimensional pin jointed structure were predicted for system matrix changed parameters. The same conclusions can be extended to a three-dimension structure. The beauty of the element locations identification technique is that these locations are unaffected by other element locations or the coupling terms.

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