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Some Properties Of Anti L-fuzzy Subnearrings Of A Nearring

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of anti L-fuzzy subnearring of a nearring.

Keywords : L-fuzzy set, L-fuzzy subnearring, anti L-fuzzy subnearring

Introduction

There are many concepts of universal algebras generalizing an associative ring ($R, +, \bullet$). Some of them in particular, near rings and several kinds of semi rings, have been proven very useful. An algebra ($R, +, \bullet$) is said to be a nearring if (R, +) is a group and (R, $\bullet$) is a semi group satisfying $a \bullet (b + c) = a \bullet b + a \bullet c$, for all a, b and c in R. After the introduction of fuzzy sets by L.A.Zadeh, several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subnearring and ideals were introduced by S.Abou Zaid. In this paper, we introduce some theorems in anti L-fuzzy subnearring of a nearring.

Preliminaries

Definition

Let X be a non-empty set. A L-fuzzy subset A of X is a function A: X -> L.

Definition

Let R be a nearring. A L-fuzzy subset A of R is said to be a L-fuzzy subnearring **(LFSNR)** of R if it satisfies the following conditions:

(i) $\mu_A(x\ y) \geq \mu_A(x) \wedge \mu_A(y)$,

(ii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$, for all x and y in R.

Definition

Let R be a nearring. A L-fuzzy subset A of R is said to be a anti L-fuzzy subnearring **(ALFSNR)** of R if it satisfies the following conditions:

(i) $\mu_A(x\ y) \leq \mu_A(x) \vee \mu_A(y)$,

(ii) $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in R.

Definition

Let R be a nearring. An anti L-fuzzy subnearring A of R is said to be an anti L-fuzzy normal subnearring **(ALFNSNR)** of R if it satisfies the following conditions:

(i) $\mu_A(x+y) = \mu_A(y+x)$,

(ii) $\mu_A(xy) = \mu_A(yx)$, for all x and y in R.

Definition

Let A and B be L-fuzzy subsets of sets G and H, respectively. The anti product of A and B, denoted by AxB, is defined as

$AxB = \{ (x, y), \mu_{AxB}(x,y)\ /$ for all x in G and y in H $\}$ where $\mu_{AxB}(x, y) = \mu_A(x) \vee \mu_B(y)$.

Definition

Let A be a L-fuzzy subset in a set S, the strongest anti L-fuzzy relation on S, that is an anti L-fuzzy relation on A is V given by $\mu_V(x, y) = \mu_A(x) \vee \mu_A(y)$, for all x and y in S.

Properties Of Anti L-Fuzzy Subnearring Of A Nearring

Theorem

Union of any two anti L-fuzzy subnearring of a nearring R is an anti L-fuzzy subnearring of R.

Proof

Let A and B be any two anti L-fuzzy subnearrings of a nearring R and x and y in R.

Let $A = \{(x, \mu_A(x)) / x \in R \}$ and $B = \{(x, \mu_B(x)) / x \in R \}$ and also

let $C = A \cup B = \{(x, \mu_C(x)) / x \in R \}$, where $\mu_A(x)\ \mu_B(x) = \mu_C(x)$.

Now, $\mu_C(xy) = \mu_A(xy) \vee \mu_B(xy) \leq \{ \mu_A(x) \vee \mu_A(y)\} \vee \{ \mu_B(x) \vee \mu_B(y)\}$

$= \{ \mu_A(x) \vee \mu_B(x)\} \vee \{ \mu_A(y) \vee \mu_B(y) \}$

$= \mu_C(x) \vee \mu_C(y)$.

Therefore, $\mu_C(x - y) \leq \mu_C(x) \vee \mu_C(y)$, for all x and y in R.

$\mu_C(xy) = \mu_A(xy) \vee \mu_B(xy) \leq \{ \mu_A(x) \vee \mu_A(y)\} \vee \{ \mu_B(x) \vee \mu_B(y)\}$

$= \{\mu_A(x) \vee \mu_B(x)\} \vee \{\mu_A(y) \vee \mu_B(y)\}$

$= \mu_C(x) \vee \mu_C(y)$.

Therefore, $\mu_C(xy) \leq \mu_C(x) \vee \mu_C(y)$, for all x and y in R.

Therefore C is an anti L-fuzzy subnearring of a nearring R.

Hence the union of any two anti L-fuzzy subnearrings of a nearring R is an anti L-fuzzy subnearring of R.

Theorem

The union of a family of anti L-fuzzy subnearrings of a nearring R is an anti L-fuzzy subnearring of R.

Proof

Let $\{ V_i : i \in I\}$ be a family of anti L-fuzzy subnearrings of a nearring R and

let $A = \bigcup_{i \in I} V_i$. Let x and y in R.

Then, $\mu_A(x-y) = \bigvee_{i \in I} \mu_{Vi}(x-y) = \bigvee_{i \in I} \{ \mu_{Vi}(x) \vee \mu_{Vi}(y)\}$

$= \{ \bigvee_{i \in I} \mu_{Vi}(x) \} \vee \{ \bigvee_{i \in I} \mu_{Vi}(y)\}$

$= \mu_A(x) \vee \mu_A(y)$.

Therefore, $\mu_A(x-y) = \mu_A(x) \vee \mu_A(y)$, for all x and y in R.

$\mu_A(xy) = \bigvee_{i \in I} \mu_{Vi}(xy) = \bigvee_{i \in I} \{ \mu_{Vi}(x) \vee \mu_{Vi}(y) \}$

$= \{ \bigvee_{i \in I} \mu_{Vi}(x) \} \vee \{ \bigvee_{i \in I} \mu_{Vi}(y) \}$

$= \mu_A(x) \vee \mu_A(y)$.

Therefore, $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in R.

That is, A is an anti L-fuzzy subnearring of a nearring R.

Hence, the union of a family of anti L-fuzzy subnearrings of R is an anti L-fuzzy subnearring of R.

3.3 Theorem

If A and B are any two anti L-fuzzy subnearrings of the nearrings R_1 and R_2 respectively, then anti-product AxB is an anti L-fuzzy subnearring of $R_1 x R_2$.

Proof

Let A and B be two anti L-fuzzy subnearrings of the nearrings R_1 and R_2 respectively.

Let x_1 and x_2 be in R_1 and y_1 and y_2 be in R_2.

Then (x_1, y_1) and (x_2, y_2) are in $R_1 x R_2$.

Now, $\mu_{AxB}[(x_1, y_1)-(x_2, y_2)] = \mu_{AxB}(x_1-x_2, y_1-y_2) = \mu_A(x_1-x_2) \vee \mu_B(y_1-y_2)$

$\leq\{\mu_A(x_1) \vee \mu_A(x_2)\}\vee \{\mu_B(y_1) \vee \mu_B(y_2)\}$

$=\{\mu_A(x_1) \vee \mu_B(y_1)\} \vee \{\mu_A(x_2) \vee \mu_B(y_2)\}$

$= \mu_{AxB}(x_1, y_1) \vee \mu_{AxB}(x_2, y_2)$.

Therefore, $\mu_{AxB}[(x_1, y_1)-(x_2, y_2)] \leq \mu_{AxB}(x_1, y_1) \vee \mu_{AxB}(x_2, y_2)$.

Also, $\mu_{AxB}[(x_1, y_1)(x_2, y_2)] = \mu_{AxB}(x_1x_2, y_1y_2) = \mu_A(x_1x_2)\vee \mu_B(y_1y_2)$

$\leq\{\mu_A(x_1)\vee\mu_A(x_2)\}\vee\{\mu_B(y_1)\vee \mu_B(y_2)\} = \{\mu_A(x_1) \vee \mu_B(y_1)\} \vee \{\mu_A(x_2) \vee \mu_B(y_2)\}$

$= \mu_{AxB}(x_1,y_1) \vee \mu_{AxB}(x_2, y_2)$.

Therefore, $\mu_{AxB}[(x_1, y_1)(x_2, y_2)] \leq \mu_{AxB}(x_1, y_1) \vee \mu_{AxB}(x_2, y_2)$.

Hence AxB is an anti L-fuzzy subnearring of nearring of R_1xR_2.

Theorem

Let A and B be L-fuzzy subsets of the nearrings R_1 and R_2 respectively.

Suppose that e and e^1 are the identity elements of R_1 and R_2 respectively.

If AxB is an anti L-fuzzy subnearring of R_1xR_2, then at least one of the following two statements must hold: (i) $\mu_B(e^1) \leq \mu_A(x)$, for all x in R_1,

(ii) $\mu_A(e) \leq \mu_B(y)$, for all y in R_2.

Proof

Let AxB be an anti L-fuzzy subnearring of R_1xR_2.

By contraposition, suppose that none of the statements (i) and (ii) holds.

Then we can find a in R_1 and b in R_2 such that $\mu_A(a) < \mu_B(e^1)$ and $\mu_B(b) < \mu_A(e)$.

We have, $\mu_{AxB}(a, b) = \mu_A(a) \vee \mu_B(b) < \mu_B(e^1) \vee \mu_A(e) = \mu_A(e)\vee \mu_B(e^1) = \mu_{AxB}(e, e^1)$.

Thus AxB is not an anti L-fuzzy subnearring of R_1xR_2.

Hence either $\mu_B(e^1) \leq \mu_A(x)$, for all x in R_1 or $\mu_A(e) \leq \mu_B(y)$, for all y in R_2.

3.5 Theorem

Let A and B be two L-fuzzy subsets of the nearrings R_1 and R_2 respectively and AxB is an anti L-fuzzy subnearring of R_1xR_2. Then the following are true:

(i) if $\mu_A(x) \geq \mu_B(e^1)$, then A is an anti L-fuzzy subnearring of R_1.

(ii) if $\mu_B(x) \geq \mu_A(e)$, then B is an anti L-fuzzy subnearring of R_2.

(iii) either A is an anti L-fuzzy subnearring of R_1 or B is an anti L-fuzzy subnearring ofR_2.

Proof

Let AxB be an anti L-fuzzy subnearring of R_1xR_2 and x, y in $R_1$and e^1 in R_2.

Then (x, e^1) and (y, e^1) are in R_1xR_2. Now, using the property that $\mu_A(x) \geq \mu_B(e^1)$,

for all x in R_1, we get, $\mu_A(x-y) = \mu_A(x-y) \vee \mu_B(e^1-e^1) = \mu_{AxB}((x-y), (e^1-e^1))$

$= \mu_{AxB}[(x, e^1)-(y, e^1)]$

$\leq \mu_{AxB}(x, e^1)\vee \mu_{AxB}(y, e^1)$

$= \{\mu_A(x)\vee \mu_B(e^1)\}\vee\{\mu_A(y) \vee \mu_B(e^1)\} = \mu_A(x)\vee \mu_A(y)$

Therefore, $\mu_A(x-y) \leq \mu_A(x)\vee \mu_A(y)$, for all x and y in R_1.

Also, $\mu_A(xy) = \mu_A(xy)\vee \mu_B(e^1e^1) = \mu_{AxB}((xy), (e^1e^1)) = \mu_{AxB}[(x, e^1)(y, e^1)]$

$\leq \mu_{AxB}(x, e^1) \vee \mu_{AxB}(y, e^1)$

$= \{\mu_A(x)\vee \mu_B(e^1)\}\vee\{\mu_A(y) \vee \mu_B(e^1)\}$

$= \mu_A(x)\vee \mu_A(y)$.

Therefore, $\mu_A(xy) \leq \mu_A(x)\vee \mu_A(y)$, for all x and y in R_1. Hence A is an anti L-fuzzy subnearring of R_1. Thus (i) is proved.

Now, using the property that $\mu_B(x) \geq \mu_A(e)$, for all x in R_2, let x and y in R_2 and e in R_1.

Then (e, x) and (e, y) are in R_1xR_2.

we get, $\mu_B(x-y) = \mu_B(x-y) \vee \mu_A(e-e) = \mu_A(e-e) \vee \mu_B(x-y)$

$= \mu_{AxB}((e-e), (x-y)) = \mu_{AxB}[(e, x)-(e, y)]$

$\leq \mu_{AxB}(e,x)\vee\mu_{AxB}(e,y)$

$= \{\mu_A(e) \vee \mu_B(x)\}\vee\{\mu_A(e)\vee \mu_B(y)\}$

$= \mu_B(x)\vee \mu_B(y)$

Therefore, $\mu_B(x-y)\leq \mu_B(x) \vee \mu_B(y)$, for all x and y in R_2.

Also, $\mu_B(xy) = \mu_B(xy) \vee \mu_A(ee) = \mu_A(ee) \vee \mu_B(xy)$

$= \mu_{AxB}((ee), (xy)) = \mu_{AxB}[(e, x)(e, y)]$

$\leq \mu_{AxB}(e, x) \vee \mu_{AxB}(e, y)$

$=\{\mu_A(e)\vee \mu_B(x)\}\vee\{\mu_A(e)\vee \mu_B(y)\}$

$= \mu_B(x)\vee \mu_B(y)$.

Therefore, $\mu_B(xy) \leq \mu_B(x)\vee \mu_B(y)$, for all x and y in R_2.

Hence B is an anti L-fuzzy subnearring of a nearring R_2. Thus (ii) is proved. (iii) is clear.

Theorem

Let A be a L-fuzzy subset of a nearring R and V be the strongest anti L-fuzzy relation of R. Then A is an anti L-fuzzy subnearring of R if and only if V is an anti L-fuzzy subnearring of RxR.

Proof

Suppose that A is an anti L-fuzzy subnearring of a nearring R.

Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR.

We have, $\mu_V(x-y) = \mu_V[(x_1, x_2) - (y_1, y_2)]$

$= \mu_V(x_1-y_1, x_2-y_2)$

$= \mu_A(x_1-y_1)\vee \mu_A(x_2-y_2)$

$\leq \{\mu_A(x_1)\vee \mu_A(y_1)\}\vee\{\mu_A(x_2)\vee \mu_A(y_2)\}$

$=\{\mu_A(x_1)\vee \mu_A(x_2)\}\vee\{\mu_A(y_1)\vee \mu_A(y_2)\}$

$= \mu_V(x_1, x_2)\vee \mu_V(y_1, y_2)$

$= \mu_V(x)\vee \mu_V(y)$.

Therefore, $\mu_V(x-y) \leq \mu_V(x)\vee \mu_V(y)$, for all x and y in RxR.

$\mu_V(xy) = \mu_V[(x_1, x_2)(y_1, y_2)] = \mu_V(x_1y_1, x_2y_2) = \mu_A(x_1y_1)\vee \mu_A(x_2y_2)$

$\leq \{\mu_A(x_1)\vee \mu_A(y_1)\}\vee\{\mu_A(x_2)\vee \mu_A(y_2)\}$

$=\{\mu_A(x_1)\vee \mu_A(x_2)\}\vee\{\mu_A(y_1)\vee \mu_A(y_2)\}$

$= \mu_V(x_1, x_2) \vee \mu_V(y_1, y_2)$

$= \mu_V(x)\vee \mu_V(y)$.

Therefore, $\mu_V(xy) \leq \mu_V(x)\vee \mu_V(y)$, for all x and y in RxR.

This proves that V is an anti L-fuzzy subnearring of RxR.

Conversely assume that V is an anti L-fuzzy subnearring of RxR, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR,

we have $\mu_A(x_1-y_1)\vee \mu_A(x_2-y_2) = \mu_V(x_1-y_1, x_2-y_2)$

$= \mu_V[(x_1, x_2)-(y_1, y_2)]$

$= \mu_V(x-y)$

$\leq \mu_V(x)\vee \mu_V(y)$

$= \mu_V(x_1, x_2)\vee \mu_V(y_1, y_2)$

$=\{\mu_A(x_1)\vee \mu_A(x_2)\}\vee\{\mu_A(y_1)\vee \mu_A(y_2)\}$.

If we put $x_2 = y_2 = 0$, we get, $\mu_A(x_1-y_1) \leq \mu_A(x_1) \vee \mu_A(y_1)$, for all x_1 and y_1 in R.

$\mu_A(x_1y_1) \vee \mu_A(x_2y_2) = \mu_V(x_1y_1, x_2y_2)$

$= \mu_V[(x_1, x_2)(y_1, y_2)]$

$= \mu_V(xy)$

$\leq \mu_V(x)\vee \mu_V(y)$

$= \mu_V(x_1, x_2) \vee \mu_V(y_1, y_2)$

$= \{\mu_A(x_1)\vee \mu_A(x_2)\}\vee\{\mu_A(y_1)\vee \mu_A(y_2)\}$.

If we put $x_2 = y_2 = 0$, we get $\mu_A(x_1y_1) \leq \mu_A(x_1) \vee \mu_A(y_1)$, for all x_1 and y_1 in R.

Therefore A is an anti L-fuzzy subnearring of R.

Theorem

If A is an anti L-fuzzy subnearring of a nearring (R, +, •), then $\mu_A(-x) = \mu_A(x)$ and $\mu_A(x) \geq \mu_A(0)$, for x in R, 0 is the identity element of R.

Proof

For x in R and 0 is the identity in R.

Now, $\mu_A(x) = \mu_A(-(-x)) \leq \mu_A(-x) \leq \mu_A(x)$.

Therefore, $\mu_A(-x) = \mu_A(x)$, for all x in R.

Now, $\mu_A(0) = \mu_A(x-x) \leq \mu_A(x)\vee \mu_A(-x)=\mu_A(x)$.

Therefore, $\mu_A(0) \leq \mu_A(x)$ for all x in R.

Theorem

If A is an anti L-fuzzy subnearring of a nearring (R, +, •), then $\mu_A(x-y) = \mu_A(0)$ gives $\mu_A(x) = \mu_A(y)$, for x and y in R and 0 is the identity in R.

Proof

Let x and y in R and 0 is the identity element of R.

Now, $\mu_A(x) = \mu_A(x-y+y) \leq \mu_A(x-y) \vee \mu_A(y) = \mu_A(0) \vee \mu_A(y) = \mu_A(y) = \mu_A(x-(x-y))$

$\leq \mu_A(x-y)\vee \mu_A(x) = \mu_A(0) \vee \mu_A(x) = \mu_A(x)$.

Therefore, $\mu_A(x) = \mu_A(y)$, for all x and y in R.

Theorem

A is an anti L-fuzzy subnearring of a nearring (R, +, •) if and only if $\mu_A(x-y) \leq \mu_A(x)\vee \mu_A(y)$, $\mu_A(xy) \leq \mu_A(x)\vee \mu_A(y)$, for all x and y in R.

Proof

Let A be an anti L-fuzzy subnearring of a nearring (R, +, •) and x and y in R.

Then, $\mu_A(x-y) \leq \mu_A(x)\vee \mu_A(-y) \leq \mu_A(x)\vee \mu_A(y)$.

Therefore, $\mu_A(x-y) \leq \mu_A(x) \vee \mu_A(y)$. Clearly $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$.

Conversely, if $\mu_A(x-y) \leq \mu_A(x) \vee \mu_A(y)$, $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in R, replace y by x, then $\mu_A(x) \geq \mu_A(0)$, for all x in R.

Now, $\mu_A(-x) = \mu_A(0-x) \leq \mu_A(0) \vee \mu_A(x) = \mu_A(x)$.

Therefore, $\mu_A(-x) \leq \mu_A(x)$, for all x in R.

It follows that, $\mu_A(x+y) = \mu_A(x-(-y)) \leq \mu_A(x) \vee \mu_A(-y) \leq \mu_A(x) \vee \mu_A(y)$.

Therefore, $\mu_A(x+y) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in R.

Clearly, $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in R.

Hence A is an anti L-fuzzy subnearring of R.

Theorem

Let A be a L-fuzzy subset of a nearring (R, +, •) .

If $\mu_A(e) = 0$ and $\mu_A(x-y) \leq \mu_A(x)\vee \mu_A(y)$, $\mu_A(xy) \leq \mu_A(x)\vee \mu_A(y)\}$, then A is an anti L-fuzzy subnearring of R, for all x and y in R and e is the identity element of R.

Proof

Let x and y in R and e is the identity element of R.

Now $\mu_A(-x) = \mu_A(e-x) \leq \mu_A(e) \vee \mu_A(x) = 0 \vee \mu_A(x) = \mu_A(x)$.

Therefore, $\mu_A(-x) \leq \mu_A(x)$, for all x in R.

Now, $\mu_A(x+y) = \mu_A(x-(-y)) \leq \mu_A(x) \vee \mu_A(-y) \leq \mu_A(x) \vee \mu_A(y)$.

Therefore, $\mu_A(x+y) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in R.

Clearly, $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in R.

Hence A is an anti L-fuzzy subnearring of R.

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