₹ 200 ISSN - 2249-555X
Volume : 1 Issue : 5 February 2012

Indian Journal Of Applied Research

Journal for All Subjects

www.ijar.in

Listed in International ISSN Directory, Paris.

ISSN - 2249-555X

Indian Journal of Applied Research

Journal for All Subjects

Editor,
Indian Journal Of Applied Research
8-A, Banans, Opp. SLU Girls College, New Congres Bhavan, Paldi,
Ahmedabad-380006**,** Gujarat, INDIA
Contact.: +91-9824097643 E-mail : editor@ijar.in

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Volume : 1 | Issue : 5 | February 2012 ISSN - 2249-555X

Research Paper Mathematics

Intuitionistic Fuzzy Primary And Semiprimary Ideal

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ABSTRACT

In this paper, we made an attempt to study an algebraic nature of an intuitionistic fuzzy primary and semiprimary ideal and its properties.

2000 AMS Subject Classification : 03F55, 20N25, 08A72.

Keywords : Intuitionistic fuzzy set, intuitionistic fuzzy primary ideal, intuitionistic fuzzy semiprimary ideal.

Introduction

Ever since an introduction of fuzzy sets by Zadeh[9], the fuzzy concept has invaded almost all branches of mathematics. The concept of intuitionistic fuzzy set was introduced by Atanassov [1], as a generalization of the notion of fuzzy set. KOG and BALKANAY [3] defined a θ-Euclidean L-fuzzy ideals of rings. In this paper, we introduce the concept of intuitionistic fuzzy primary and semiprimary ideal and prove some results .

Preliminaries

Definition

Let S be any nonempty set. A mapping $\mu : S \to [0, 1]$ is called a fuzzy subset of S.

Definition

A fuzzy ideal μ of a ring R is called fuzzy primary ideal, if for all a ,b$\in$ R, either $\mu(ab) = \mu(a)$ or else $\mu(ab) \leq \mu(b^m)$ for some $m \in z_+$

Definition

A fuzzy ideal μ of a ring R is called fuzzy semiprimary ideal , if for all a,b $\in$ R , either $\mu(ab) \leq \mu(a^n)$, for some $n \in z_+$ or else $\mu(ab) \leq \mu(b^m)$ for some $m \in z_+$.

Definition

Let μ be any fuzzy ideal of a ring R , $t \in [0, 1]$ and $t \leq \mu(o)$. The ideal μ_t is called a level ideal of μ .

Definition

An intuitionistic fuzzy set (IFS) A in X is defined as an object of the form $A = \{< x, \mu_A(x), \gamma_A(x) > / x \in X\}$, where μ_A:X [0, 1] and γ_A:X [0, 1] define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

Definition

A fuzzy ideal A of a ring R is called Intuitionistic fuzzy primary ideal if for all a, b $\in$ R either $\mu_A(ab) = \mu_A(a)$ and $\gamma_A(ab) = \gamma_A(a)$ or $\mu_A(ab) \leq \mu_A(b^m)$ and $\mu_A(ab) \geq \mu_A(b^m)$, for some $m \in Z_+$.

Example:

Consider $\mu_A(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0.8 & \text{if } x \in <4> \sim <0> \\ 0.6 & \text{if } x \in z \sim <4> \end{cases}$

$\gamma_A(x) = \begin{cases} 0 & \text{if } x = 0 \\ 0.1 & \text{if } x \in <4> \sim <0> \\ 0.3 & \text{if } x \in z \sim <4> \end{cases}$

Definition

A fuzzy ideal A of a ring R is called Intuitionistic fuzzy Semiprimary ideal if for all a, b $\in$R either $\mu_A(ab) \leq \mu_A(a^n)$ and $\gamma_A(ab) \geq \gamma_A(a^n)$, for some $n \in Z_+$ or else $\mu_A(ab) \leq \mu_A(b^m)$ and $\gamma_A(ab) \geq \gamma_A(b^m)$, for some $m \in Z_+$.

Some Properties Of Intuitionistic Fuzzy Primary And Semiprimary Ideal

Theorem:

If A is any semiprimary ideal of a ring R , A R than the fuzzy subset B of R defined by

$\mu_B(x) = \alpha$ if x A

β if x R~A, where $\alpha, \beta \in [0, 1], \alpha > \beta$

$\gamma_B(x) = \alpha_1$ if $x \in A$

β_1 if $x \in R \sim A$, where $\alpha_1, \beta_1 \in [0, 1], \alpha_1 < \beta_1$

is an intuitionistic fuzzy semiprimary ideal of R.

Proof:

Let a,b$\in$ R and let $\mu_B(ab) > \mu_B(a^n)$, for all $n \in Z_+$ then $\mu_B(a^n) \alpha$ so that $\mu_B(a^n) = \beta$ and $\mu_B(ab) = \alpha$, hence $a^n \notin A$ for all $n \in Z_+$ since $ab \in A$ and A is Semiprimary , it follows that $b^m \in A$, for some $n \in z_+$. Consequently $\mu_B(b^m) = \alpha = \mu_B(ab)$.

Let a,b $\in$ R and let $\gamma_B(ab) < \gamma_B(a^n)$, for all $n \in Z_+$,then $\gamma_B(a^n) \alpha_1$ so that $\gamma_B(a^n) = \beta_1$ and $\gamma_B(ab) = \alpha_1$, hence $a^n A$, for some $n \in Z_+$. Since $ab \in A$ and A is semiprimary it follows that $b^m \in A$, for some $m \in Z_+$. Consequently $\gamma_B(b^m) = \alpha_1 = \gamma_B(ab)$. Hence B is an intuitionistic fuzzy semiprimary ideal.

Theorem:

A fuzzy ideal A of a ring R is intuitionistic fuzzy semiprimary ideal iff each level ideal A_t, $t \in$ Im A is semiprimary ideal.

Proof:

To prove that the level ideal A_t is semiprimary.

Suppose that A is an intuitionistic fuzzy semiprimary ideal. Let a, b $\in$ R and let $ab \in \mu_{At}$. If $a^n \notin \mu_{At}$ for all $n \in Z_+$ then $_A(a^n) < t \leq \mu_A(ab)$, so that $_A(ab) \leq _A(b^m)$ for some $m \in z_+$. Hence $b^m \in \mu_{At}$

Let a,b$\in$ R and let a, b $\in \gamma_{At}$. If $b^m \in \gamma_{At}$ for all $m \in Z_+$, then $\gamma_A(b^m) < t < \gamma_A(ab)$. therefore $\gamma_A(ab) \geq \gamma_A(b^m)$ for some $m \in Z_+$. Hence the level ideal A_t is semiprimary.

To prove A is an intuitionistic fuzzy semiprimary ideal.

Suppose that the level ideal A_t , $t \in ImA$ is a semiprimary ideal of R.

Let $a,b \in R$, $_A(ab) = t$ and $_A(ab) > _A(a^n)$ for every $n \in z_+$,then $_A(a^n) < t$ so that $a^n \notin A_t$ for all $n \in z_+$. Since A_t is semiprimary and $ab \in A_t$ it follows that $b^m \in A_t$ for some $m \in z_+$. Hence $_A(b^m) \geq t \geq _A(ab)$ implies $_A(ab) \leq _A(b^m)$.

Let $a,b \in R, \gamma_A(ab) = t$ and $\gamma_A(b^m) \leq \gamma_A(ab)$ for some $m \in Z_+$, then $\gamma_A(b^m) \leq t$ so that $b^m \in A_t$ for all $m \in Z_+$. Since A_t is semiprimary and $a, b \in A_t$ it follows that $b^m \in A_t$ for some $m \in Z_+$ then $\gamma_A(b^m) \leq \gamma_A(ab)$ implies $\gamma_A(ab) \geq \gamma_A(b^m)$, which result that A is an Intuitionistic fuzzy semiprimary ideal.

Theorem:

Intersection of any two intuitionistic fuzzy semiprimary ideal is again an intuitionistic fuzzy semi primary ideal

Proof:

Let A and B be an intuitionistic fuzzy semiprimary ideal of a ring R

Let $x,y \in A \cap B$ implies $x,y \in A$ and $x,y \in B$

Since A is an intuitionistic fuzzy semiprimary ideal of ring R , $\mu_A(xy) \leq \mu_A(x^n)$ and $\gamma_A(xy) \geq \gamma_A(x^n)$

Since B is an intuitionistic fuzzy semiprimary ideal of ring R, $\mu_B(xy) \leq \mu_B(x^n)$ and $\gamma_B(xy) \geq \gamma_B(x^n)$

consider $x,y \in A \cap B$

consider $\mu_{A \cap B}(xy) = \min(\mu_A(xy), \mu_B(xy))$
$\leq \min(\mu_A(x^n), \mu_B(x^n))$
$= \mu_{A \cap B}(x^n)$

Therefore $\mu_{A \cap B}(xy) \leq \mu_{A \cap B}(x^n)$, for some $n \in z_+$

Consider $\gamma_{A \cap B}(xy) = \max(\gamma_A(xy), \gamma_B(xy))$
$\geq \max(\gamma_A(x^n), \gamma_B(x^n))$
$= \gamma_{A \cap B}(x^n)$

Therefore $\gamma_{A \cap B}(xy) \geq \gamma_{A \cap B}(x^n)$

Hence $A \cap B$ is an intuitionistic fuzzy semiprimary ideal

Theorem:

Union of any two intuitionistic fuzzy semiprimary ideal is again on intuitionistic fuzzy semiprimary ideal only when they are equal .

Proof:

Let A and B be on intuitionistic fuzzy semiprimary ideals of R and $A = B$

To prove $A \cup B$ is an intuitionistic fuzzy semiprimary idael of R.

Let $x,y \in A \cup B$ implies $x,y \in A$ or $x,y \in B$

Consider $\mu_{A \cup B}(xy) = \max(\mu_A(xy), \mu_B(xy))$
$= \max(\mu_A(xy), \mu_A(xy))$
$\leq \max(\mu_A(x^n), \mu_A(x^n))$
$= \max(\mu_A(x^n), \mu_B(x^n))$
$= \mu_{A \cup B}(x^n)$

Therefore $\mu_{A \cup B}(xy) \leq \mu_{A \cup B}(x^n)$

Consider $\gamma_{A \cup B}(xy) = \min(\gamma_A(xy), \gamma_B(xy))$
$= \min(\gamma_A(xy), \gamma_A(xy))$
$\geq \min(\gamma_A(x^n), \gamma_A(x^n))$
$= \min(\gamma_A(x^n), \gamma_B(x^n))$
$= \gamma_{A \cup B}(x^n)$

Therefore $\gamma_{A \cup B}(xy) \geq \gamma_{A \cup B}(x^n)$. This completes the result.

2.5 Theorem:

If A is any primary ideal of a ring R, A R then the fuzzy subset B of R defined by

$_B(x) =$ α if $x \in A$
β if $x \in RA$ where $\alpha, \beta \in [0,1]$, $\alpha > \beta$

$\gamma_B(x) =$ α_1 if $x \in A$
β_1 if $x \in R \sim A$ where $\alpha_1, \beta_1 \in [0,1]$, $\alpha_1 < \beta_1$

is an intuitionistic fuzzy primary ideal of R.

Theorem

A fuzzy ideal A of a ring R is intuitionistic fuzzy primary ideal if and only if each level ideal A_t, $t \in ImA$ is primary.

Theorem

Intersection of any two intuitionistic fuzzy primary ideal is an intuitionistic fuzzy primary ideal.

Theorem

Union of any two intuitionistic fuzzy semiprimary ideal is again an intuitionistic fuzzy semiprimary ideal only when they are equal.

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