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Special Double Sampling Plan with Fuzzy Parameter

KEYWORDS Triangular fuzzy number, Special double sampling plan, Operating characteristic function and Average outgoing quality function

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ABSTRACT *The classification of judgement in attribute acceptance sampling plan is based on binary classification such as defective or non-defective. However, in many practical cases it is difficult to classify the item as strictly conforming or nonconforming when quality data related to various items are expressed as good, almost good, bad, not so bad and so on. This introduces vagueness in the value of proportion defective in the lot / process. Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling uncertain systems which has incomplete and imprecise information.*

Hence, in this paper special double sampling plan under fuzzy environment is studied. The performance measures operating characteristic (OC)function and average outgoing quality (AOQ)functions under fuzzy and non fuzzy environment are evaluated and presented.

INTRODUCTION

Acceptance sampling is used for quality assurance and in recent years, it has become typical to work with suppliers to improve the process performance. The acceptance sampling is useful where testing is destructive, the cost of 100% inspection is extremely high and the product liability risks are serious. Single sampling plan for attribute is one of the widely used acceptance sampling methods for acceptance or rejection of the lot based on a single sample which is characterized by the parameters, n-sample size and c-acceptance number.

The classification of judgement in attribute acceptance sampling plans is based on binary classification such as defective or non-defective. However, in many practical cases it is difficult to classify the item as strictly conforming or nonconforming when quality data related to various items are expressed as good, almost good, bad, not so bad and so on. This introduces vagueness in the value of proportion defective in the lot / process. Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling uncertain systems which has incomplete and imprecise information.

Conflicting interest arise between the producer and the consumer in the use of single sampling plans either with c=1 or c=0 for product characteristics involving costly and destructive testing. Since single sampling plan with c=0 favours the consumer and with c=1 favours the producer. To overcome such conflicting interests a special double sampling plan is proposed with the following operating procedure.

From a given lot select a random sample of size n1 and count the number of defectives d1. If d1 ≥ 1 reject the lot, if d1 = 0 select a second sample of size n2 and count the number of defectives d2. If d2 ≤ 1, accept the lot otherwise reject the lot. In special double sampling plan the decision of acceptance is made only after inspecting the second sample. This aspect differs from usual double sampling plan in which decision of acceptance can be made even before the inspection of second sample.

Single sampling by attributes using membership functions were discussed by Kanagawa and Ohta (1990), Tamaki, Kanagawa and Ohta (1991) and Grzegorzewski (2001). Sampling plan by attributes for vague data were considered by Hryniewisz (1994). Buckley (2003) provided the basic concepts of fuzzy probability. Govindaraju (1991) and Balamurali and Kalyanasundaram (1997) studied the special type of double sampling plan under the assumption that the process fraction defective as a constant.

No research work has been carried out in the application of special double sampling plan under the fuzzy environment. This motivated to persue special double sampling plan with fuzzy parameter.

Definition1:

The fuzzy subset $\tilde{N}$ of real number IR, with the membership function mN :IR ® [0, 1] is a fuzzy number if and only if (a) $\tilde{N}$ is normal (b) $\tilde{N}$ is fuzzy convex (c) mN is upper semi continuous and (d). $\tilde{N}$ has a bounded support.

Definition2:

A triangular fuzzy number $\tilde{N}$ is fuzzy number with membership function defined by three numbers a1 < a2 < a3 where the base of the triangle is the interval [a1, a3] and vertex is at x = a2.

Definition3:

The a-cut of a fuzzy number $\tilde{N}$ is a non-fuzzy set defined as N[a] = {x Î IR ; mN(x) ³ a}. Hence we have N[a] = $[N^L_\alpha, N^U_\alpha]$ where

N^L_α =inf {x Î IR ; mN(x) ³ a} , N^U_α =sup {x Î IR ; mN(x) ³ a}

Definition4:

Let X be a random variable denoting the number of defectives in the process / lot with P(X = x_i) = ki ; for all i, where 0 < ki < 1 and S ki = 1. Let $\tilde{k}_i$ be the fuzzy number corresponding to ki such that $\tilde{P}$(X = xi) = $\tilde{k}_i$, where ki Î $\tilde{k}_i$[a], 0 ≤ a ≤ 1.

Definition5:

$\tilde{P}$[a, b] [a] be the fuzzy probability of acceptance to have the number of defectives to lie in a ≤ x ≤ b, is defined by $\tilde{P}$[a, b] [a] = [PL[a, b] [a], PU[a, b] [a]]

where PL[a, b] [a] PU[a, b] [a] are respectively the minimum and the maximum of $\sum_{r=a}^{b} \tilde{P}_r[\alpha]$ .n the case of binomial model

pÎ $\tilde{p}$[a], q Î $\tilde{q}$[a], p+ q = 1

$$PL[a,b][a] =, \min_p \left\{\sum_{d=a}^{b} nC_d\, p^d q^{n-d}\right\} PU[a,b][a] = \max_p \left\{\sum_{d=a}^{b} nC_d\, p^d q^{n-d}\right\}$$

In the case of Poisson model, l Î $\tilde{\lambda}$[a], l > 0.

$$PL[a, b] [a] = \min_\lambda \left\{\sum_{d=a}^{b} \frac{e^{-\lambda}\lambda^d}{d!}\right\}, PU[a, b] [a] = \max_\lambda \left\{\sum_{d=a}^{b} \frac{e^{-\lambda}\lambda^d}{d!}\right\}$$

OC Curve

Operating characteristic curve is a graphical representation of probability of acceptance of a lot against the proportion of the non conforming items in the lot. It displays performance

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and discriminating power of the acceptance sampling plans. It aids in selection of plans that are effective in reducing risks .To derive the fuzzy operating characteristic function, we consider the structure of parameters as follows.

Bionomial Model

$\tilde{p}$ = (k, a2 + k, a3 + k)

where p Î $\tilde{p}$[a], q Î $\tilde{q}$[a] and p + q = 1, 0 ≤ a ≤ 1

$\tilde{p}$[a] = [p1[a], p2[a]] with p1[a] = k + a2 a, p2[a] = a3 + k – (a3 – a2) a

Probability of acceptance for special double sampling plan is

$$Pa = q^n\left(1+\frac{\phi n p}{q}\right) \text{ with } f = \frac{n_2}{n},\ n = n1 + n2$$

Fuzzy probability of acceptance is defined by $\tilde{P}$[a] = [PL[a], PU[a]]

$$\text{where } PL[a] = \min\left\{q^n\left(1+\frac{\phi n p}{q}\right)\right\},\ PU[a] = \max\left\{q^n\left(1+\frac{\phi n p}{q}\right)\right\}$$

Poisson Model

$\tilde{\lambda}$ = n$\tilde{p}$ = (nk, na2 + nk, na3 + nk) where l Î $\tilde{\lambda}$[a], 0 ≤ a ≤ 1

$\tilde{\lambda}$[a] = [l1 [a], l2 [a]], with l1[a] = nk + na2a ,l2[a] =na3 + nk – (na3 – na2) a Probability of acceptance for special double sampling g plan is

Pa = e-l (1 + f l) with f = $\frac{n_2}{n}$, n = n1 + n2 and l = np

Fuzzy probability of acceptance is $\tilde{p}$[a] = [PL[a], PU[a]],

PL[a] = $\min_{\lambda}$ {e-l (1 + f l)} PU[a] = $\max_{\lambda}$ {e-l (1 + f l)}

a Cut on Probability of Acceptance

The degree of uncertainty in fraction defective is one of the factors which determine the band width of the fuzzy probability of acceptance. Thus the effect of different degree of fuzziness on probability of acceptance for special double sampling plan when fraction of defective items in a lot is a fuzzy parameter is evaluated.

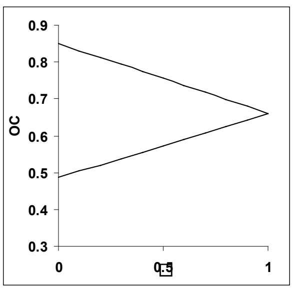
As an illustration consider a special double sampling plan with n1 = 10, n2 = 40 and the structure of the fuzzy parameter pas $\tilde{p}$=(0.01, 0.02, 0.03) Then $\tilde{p}$[a]=[p1[a],p2[a]] where p1[a] =0.01+0.01a and p2[a] = 0.03-0.01a where a is the uncertainty parameter representing the degree of fuzziness with 0£a£1. Then the fuzzy probability of acceptance under binomial model is

$$\tilde{P}[\alpha] = \left[\{1-p_2[\alpha]\}^n\left\{1+\frac{\phi n p_2[\alpha]}{1-p_2[\alpha]}\right\}, \{1-p_1[\alpha]\}^n\left\{1+\frac{\phi n p_1[\alpha]}{1-p_1[\alpha]}\right\}\right]$$

and under Poisson model is

$$\tilde{P}_a[\alpha] = [e^{-n p_2[\alpha]} (1 + f\, n\, p2[a]), e^{-n p_1[\alpha]} (1 + f\, n\, p1[a])]$$

The fuzzy probabilities of acceptance for various values of uncertainty parameter are presented in Figures 1 and 2.



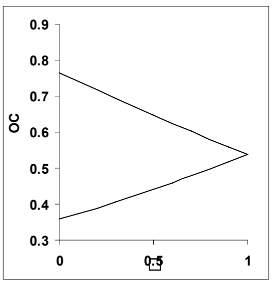


Fig.1 n1 = 10, n2 = 40 Fig.2 n1 = 20, n2 = 40
Effect of fuzziness on OC values :binomial model $\tilde{p}$ = [0.01, 0.02, 0.03]

From Fig.1and Fig.2 we observe that the increase in degree of fuzziness decreases the OC band width. The band width depends on the degree of uncertainty. The lower and upper bounds will coincide when there exists no fuzziness(i.e a = 1) which corresponds to the classical OC curve where the proportion of defective in the lot is a crisp value.

OC Band with Fuzzy Parameter

The fuzzy probability of acceptance under complete fuzziness is evaluated interms of fuzzy fraction defective .The structure of fuzzy parameter $\tilde{p}$ = (k, a2+k, a3+k) is considered as $\tilde{p}$ = (0, 0.0025, 0.005) for p.

$\tilde{p}[\alpha]$ = [k + a2 a, a3 + k – (a3 – a2) a] = [k + 0.0025 a, 0.005 + k - .0025 a]

With variation in k and under complete fuzziness,

$\tilde{p}[0]$ = [k, 0.005 + k]

The fuzzy probability of acceptance under binomial model is given by

$$\tilde{P}_a[0] = \left[(0.995-k)^n\left(1+\frac{\phi n(0.005+k)}{(995-k)}\right), (1-k)^n\left(1+\frac{\phi k}{1-k}\right)\right]$$

and under Poisson model

$\tilde{P}_a[0]$ = [e-n(0.005 + k) (1 + f n (0.005 + k)), e-nk (1 + f n k]

Fig.3 shows the OC band of special double sampling plan for n1 = 20, n2 = 40.

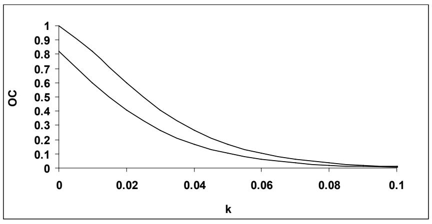


Figure 3. OC band of special double sampling plan n1 = 10, n2 =40

The fuzzy probability of acceptance of a lot in terms of fuzzy fraction defective items is a band with upper and lower bounds. The band width depends on the degree of uncertainty. The OC band under complete fuzziness indicate that when the lot quality is

(i) in a good state the OC band width is wider and
(ii) in an undesirable state the band width is narrow.

Average Outgoing Quality

The Average Outgoing Quality (AOQ), the expected proportion of non conforming items in all outgoing lots, measures the effectiveness of the acceptance sampling plan in use. Average Outgoing Quality Limit (AOQL) is the worst average quality the consumer will receive in the long run, no matter whatever the incoming quality is.

If the size of lot is N and the proportion of defective items in the lot is p and if N is very large compared to n, then AOQ ≃ p Pa.

The fuzzy average outgoing quality is FAOQ [a]=[FAOQL [a], FAOQU[a]]

FAOQL [a] = min {p Pa}, FAOQU [a] = max {p Pa}

Effect of a-Cut on AOQ

The a-cut fuzzy average outgoing quality FAOQ under binomial is

$$FAOQ[a] = \left[p_2^{[\alpha]}\{1-p_2[\alpha]\}^n\left\{1+\frac{\phi n p_2[\alpha]}{(1-p_2[\alpha])}\right\} p_1[\alpha]\{1-p_2[\alpha]\}^n\left\{1+\frac{\phi n p_1[\alpha]}{(1-p_1[\alpha])}\right\}\right]$$

and under the Poisson model is

FAOQ[a] = [p2[a]. $e^{-n p_2[\alpha]}$(1 + f n p2[a]), p1[a]. $e^{-n p_1[\alpha]}$(1 + f n p1[a]]

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where p1[a] = k + a2 a
p2[a] = a3 + k – (a3 – a2) a when the structure of $\tilde{p}$ = (k, a2 + k, a3 + k)

As an example consider the structure of p as $\tilde{p}$ = (0.01, 0.02, 0.03) then $\tilde{p}$[a] = [p1(a), p2(a)] where p1[a] = 0.01 + 0.01 a , p2[a] = 0.03 – 0.01a

For various values of the uncertainty parameter a the a-cut FAOQ values are obtained and presented in Figures 4 and 5.

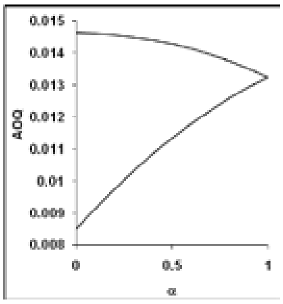

Fig.4 n1 = 10, n2 = 40

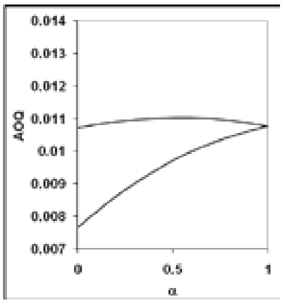

Fig.5 n1 = 20, n2 = 40

Effect of fuzziness on AOQ values Binomial model $\tilde{p}$ = [0.01, 0.02, 0.03]

Figures 4 and 5 indicate that increase in the degree of fuzziness decreases the width of FAOQ for binomial model. Fuzzy average outgoing quality values under complete fuzziness (fora = 0) for the special double sampling plan n1=10, n2=40 is given in Fig 6 and 7 signify the following facts

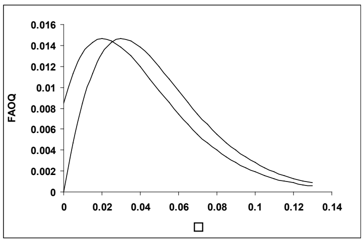

Fig. 6 FAOQ band of sampling plan : Binomial model - n1 = 10, n2 = 40

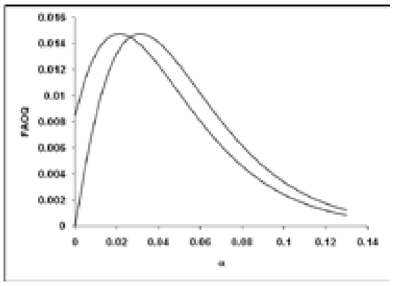

Fig. 7 FAOQ band of sampling plan : Poisson model - n1 = 10, n2 = 40

Under binomial model

FAOQ [a] = [FAOQ*, FAOQ**], 0 < k < 0.030535

= [FAOQ**, 0.014638216], 0.03053551 £ 0.30545

= [FAOQ**, FAOQ*], k > 0.30545

and the maximum of FAOQ occurs at $\tilde{p}$* [0.030535, 0.040535] with

FAOQL = [0.013772830, 0.014638216]

Under Poisson model

AOQ[a] = [FAOQ*, FAOQ**], 0 < k < 0.031076
= [FAOQ**, 0.01473857], 0.031076 £ k £ 0.031094
= [FAOQ**, FAOQ*], k > 0.031094

and the maximum occurs at
$\tilde{p}$*= [.031076, .041076] with FAOQL=[0.013923162, 0.01473857]

These values reveal that the FAOQ values are lower for lots of very good or very bad input quality.

The effect of fuzziness of the quality characteristic on OC and AOQ functions are discussed and also OC and AOQ functions for special double sampling plan with fuzzy quality characteristic is presented. If the fraction defective in the lot is a crisp value these functions reduce to that of classical plans.

K	$n_1 = 10, n_2 = 20$		$n_1 = 10, n_2 = 30$		$N_1 = 10, n_2 = 40$	
	Binomial	Poisson	Binomial	Poisson	Binomial	Poisson
0.000	[0.9460, 1.0000]	[0.9459, 1.0000]	[0.9417, 1.0000]	[0.9415, 1.0000]	[0.9348, 1.0000]	[0.9346, 1.0000]
0.005	[0.8876, 0.9460]	[0.8875, 0.9459]	[0.8717, 0.9417]	[0.8714, 0.9415]	[0.8495, 0.9348]	[0.8491, 0.9346]
0.010	[0.8271, 0.8876]	[0.8270, 0.8875]	[0.7959, 0.8717]	[0.7958, 0.8714]	[0.7558, 0.8495]	[0.7558, 0.8491]
0.015	[0.7659, 0.8271]	[0.7661, 0.8270]	[0.7186, 0.7959]	[0.7189, 0.7958]	[0.6615, 0.7558]	[0.6622, 0.7558]
0.020	[0.7054, 0.7659]	[0.7062, 0.7661]	[0.6426, 0.7186]	[0.6438, 0.7189]	[0.5712, 0.6615]	[0.5730, 0.6622]
0.025	[0.6466, 0.7054]	[0.6481, 0.7062]	[0.5701, 0.6426]	[0.5723, 0.6438]	[0.4878, 0.5712]	[0.4909, 0.5730]
0.030	[0.5900, 0.6466]	[0.5924, 0.6481]	[0.5022, 0.5701]	[0.5055, 0.5723]	[0.4127, 0.4878]	[0.4171, 0.4909]
0.035	[0.5363, 0.5900]	[0.5397, 0.5924]	[0.4396, 0.5022]	[0.4442, 0.5055]	[0.3464, 0.4127]	[0.3519, 0.4171]
0.040	[0.4857, 0.5363]	[0.4902, 0.5397]	[0.3827, 0.4396]	[0.3885, 0.4442]	[0.2886, 0.3464]	[0.2951, 0.3519]
0.045	[0.4383, 0.4857]	[0.4440, 0.4902]	[0.3314, 0.3827]	[0.3383, 0.3885]	[0.2389, 0.2886]	[0.2463, 0.2951]
0.050	[0.3943, 0.4383]	[0.4012, 0.4440]	[0.2857, 0.3314]	[0.2936, 0.3383]	[0.1967, 0.2389]	[0.2046, 0.2463]
0.055	[0.3537, 0.3943]	[0.3617, 0.4012]	[0.2453, 0.2857]	[0.2540, 0.2936]	[0.1611, 0.1967]	[0.1693, 0.2046]
0.060	[0.3164, 0.3537]	[0.3254, 0.3617]	[0.2098, 0.2453]	[0.2191, 0.2540]	[0.1313, 0.1611]	[0.1396, 0.1693]
0.065	[0.2823, 0.3164]	[0.2922, 0.3254]	[0.1788, 0.2098]	[0.1885, 0.2191]	[0.1065, 0.1313]	[0.1148, 0.1396]
0.070	[0.2513, 0.2823]	[0.2619, 0.2922]	[0.1518, 0.1788]	[0.1618, 0.1885]	[0.0861, 0.1065]	[0.0941, 0.1148]
0.075	[0.2231, 0.2513]	[0.2344, 0.2619]	[0.1285, 0.1518]	[0.1386, 0.1618]	[0.0693, 0.0861]	[0.0769, 0.0941]
0.080	[0.1976, 0.2231]	[0.2095, 0.2344]	[0.1084, 0.1285]	[0.1185, 0.1386]	[0.0555, 0.0693]	[0.0628, 0.0769]
0.085	[0.1747, 0.1976]	[0.1870, 0.2095]	[0.0912, 0.1084]	[0.1011, 0.1185]	[0.0444, 0.0555]	[0.0511, 0.0628]
0.090	[0.1541, 0.1747]	[0.1666, 0.1870]	[0.0765, 0.0912]	[0.0861, 0.1011]	[0.0353, 0.0444]	[0.0415, 0.0511]
0.095	[0.1357, 0.1541]	[0.1484, 0.1666]	[0.0641, 0.0765]	[0.0733, 0.0861]	[0.0281, 0.0353]	[0.0337, 0.0415]
0.100	[0.1192, 0.1357]	[0.1319, 0.1484]	[0.0535, 0.0641]	[0.0622, 0.0733]	[0.0222, 0.0281]	[0.0273, 0.0337]

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