The 3-Stage Optimization of K-Out-Of-N Redundant IRM With Multiple Constraints

Dr. A. Sridhar	S. Pavankumar
Department of Basic Sciences & Humanities, Vignan Institute of Engineering for Women, Visakhapatnam, A.P., India	Department of Basic Sciences & Humanities SBIT Engineering College, Khammam, A.P
V. Srinnivas Rao	**Dr. C. Umashankar**
Department of Basic Sciences & Humanities, Vignan Institute of Engineering for Women, Visakhapatnam, A.P., India	OR and SQC Department Rayalaseema University, Kurnool , A.P., India

ABSTRACT In the present scenario Reliability plays a key role for solving complex executive problems. This paper addresses a redundant Integrate Reliability Model (IRM) optimization for the k-out-of-n configuration system with multiple constraints. Generally the reliability of a system is treated as function of cost, but in many real life situations other considerations apart from conventional cost constraint like weight, volume, size, space etc., play vital role in optimizing the system reliability. Quite a few IRM's are reported with cost constraint only in optimizing the system reliability. As the literature informs that few authors mentioned IRM'S with Redundancy and this paper focuses a novel method of optimizing a Redundant IRM with multiple constraints to encounter the hidden impact of additional constraints apart from the cost constraint while the system is optimized by considering the k-out-of-n configuration system.

1. INTRODUCTION

The paper is focused to design and to optimize an Integrated Reliability Model for Redundant system with multiple constraints for the k-out-of-n configuration system as a beginning in the mentioned area of the research and initiated optimizing the system reliability. Integrated Reliability Model (IRM)refers to the determination of the number of components (x_j), component reliabilities (r_j), Stage reliabilities (R_j) and the system reliability (R_s) where in the problem considers both the unknowns that is the components reliabilities and the number of components in each stage for the given cost constraint to maximize the system reliability. So far in literature the integrated reliability models are optimized using cost constraint alone where there is an established truth between cost and reliability.

This prompted the author to present a piece of novel aspect of Reliability Optimization through modeling by considering an Integrated Reliability Model for a Redundant System by treating Weight and Volume as additional constraints apart from the conventional Cost constraint to optimize the System Reliability, to negotiate the hidden impact of the additional constraints like Weight and Volume for the k-out-of-n configuration reliability model.

3. MATHEMATICAL MODEL

The objective function and the constraints of the model

$$R_s = \prod_{j=1}^{n} R_j$$

maximize $R_j = \sum_{k=2}^{x_j} \binom{x_j}{k} r_j^{k} (1-r_j)^{x_j-k}$ (1)

subject to the constraints

$$\sum_{j=1}^{n} c_j . x_j \leq C_0 \quad (2)$$

$$\sum_{j=1}^{n} w_j . x_j \leq W_0 \quad (3)$$

$$\sum_{j=1}^{n} v_j . x_j \leq V_0 \quad (4)$$

non-negative restriction that x_j is an integer and r_j, $R_j > 0$

4. MATHEMATICAL FUNCTION

To establish the mathematical model, the most commonly used function is considered for the purpose of reliability design and analysis. The proposed mathematical function

$$c_j = e^{[(1-f_j)(r_j-r_{j,\min})/(r_{j,\max}-r_j)]}$$ (5)

Where aj, b_j are constants.

System reliability for the given function

$$R_s = \prod_{i=1}^{n} R_j$$ (6)

The number of components at each stage X_j is given through the relation

$$x_j = \frac{\ln(R_j)}{\ln(r_j)}$$ (7)

The problem under consideration is

maximize $R_s = \prod_{j=1}^{n} [1 \ (1 \ r_j)^{x_j}]$ (8)

subject to the constraints

$$\sum_{j-1}^{n} e^{[(1-f_j)(r_j-r_{j\min})/(r_{j\max}-r_j]} x_j - C_0 \leq 0$$ (9)

$$\sum_{j-1}^{n} e^{[(1-g_j)(r_j-r_{j\min})/(r_{j\max}-r_j]} x_j - W_0 \leq 0$$ (10)

$$\sum_{j-1}^{n} e^{[(1-h_j)(r_j-r_{j\min})/(r_{j\max}-r_j]} x_j - V_0 \leq 0$$ (11)

5. THE LAGRANGIAN METHOD :

Solving the proposed formulation using the Lagrangean method

A Lagrangian function is formulated as

$$F = R_S + \lambda_1\left[\left\{\sum_{j=1}^{n}\left\{e^{[1-f_j)(r_j-r_{j,min})(r_{j,max}-r_j)}.\frac{\ln(1-R_j)}{\ln(1-r_j)}\right\}\right\}-C_0\right]+$$

$$\lambda_2\left[\left\{\sum_{j=1}^{n}\left\{e^{[1-g_j)(r_j-r_{j,min})(r_{j,max}-r_j)}.\frac{\ln(1-R_j)}{\ln(1-r_j)}\right\}\right\}-W_0\right]+\lambda_3\left[\left\{\sum_{j=1}^{n}\left\{e^{[1-h_j)(r_j-r_{j,min})(r_{j,max}-r_j)}.\frac{\ln(1-R_j)}{\ln(1-r_j)}\right\}\right\}-V_0\right]$$

(12)

The stationary point can be obtained by differentiating the Lagrangean function with respect to $R_j, r_j, \lambda_1, \lambda_2$ and λ_3

$$\frac{\partial F}{\partial r_j}=\lambda_1\left[\sum_{j=1}^{n}e^{[1-f_j)(r_j-r_{j,min})(r_{j,max}-r_j)}.\ln(1-R_j)/\ln(1-r_j)\left\{(1-f_j)(r_{j,max}-f,min)(r_{j,max}-r_j)^2+1/(\ln(1-r_j)(1-r_j))\right\}\right]$$

$$+\lambda_2\left[\sum_{j=1}^{n}e^{[1-g_j)(r_j-r_{j,min})(r_{j,max}-r_j)}.\ln(1-R_j)/\ln(1-r_j)\left\{(1-g_j)(r_{j,max}-f,min)(r_{j,max}-r_j)^2+1/(\ln(1-r_j)(1-r_j))\right\}\right]$$

$$+\lambda_3\left[\sum_{j=1}^{n}e^{[1-h_j)(r_j-r_{j,min})(r_{j,max}-r_j)}.\ln(1-R_j)/\ln(1-r_j)\left\{(1-h_j)(r_{j,max}-f,min)(r_{j,max}-r_j)^2+1/(\ln(1-r_j)(1-r_j))\right\}\right]=0$$

(13)

$$\frac{\partial F}{\partial R_j}=1-\lambda_1\left[\left\{\sum_{j=1}^{n}e^{[1-f_j)(r_j-r_{j,min})(r_{j,max}-r_j)}\right\}[1/\ln(1-r_j)][1(1-R_j)]\right]$$

$$-\lambda_2\left[\left\{\sum_{j=1}^{n}e^{[1-g_j)(r_j-r_{j,min})(r_{j,max}-r_j)}\right\}\{1/\ln(1-r_j)][(1-R_j)]\right]-\lambda_3\left[\left\{\sum_{j=1}^{n}e^{[1-h_j)(r_j-r_{j,min})(r_{j,max}-r_j)}\right\}[1/\ln(1-r_j)][(1-R_j)]\right]=0.$$

(14)

$$\frac{\partial F}{\partial \ddot{e}_1}=\left[\left\{\sum_{j=1}^{n}\left\{e^{[(1-f_j)(r_j-r_{jmin})/(r_{jmax}-r_j]}.\frac{\ln(1-R_j)}{\ln(1-r_j)}\right\}\right\}-C_0\right]=0$$

(15)

$$\frac{\partial F}{\partial \ddot{e}_2}=\left[\left\{\sum_{j=1}^{n}\left\{e^{[(1-g_j)(r_j-r_{jmin})/(r_{jmax}-r_j]}.\frac{\ln(1-R_j)}{\ln(1-r_j)}\right\}\right\}-W_0\right]=0$$

(16)

$$\frac{\partial F}{\partial \ddot{e}_3}=\left[\left\{\sum_{j=1}^{n}\left\{e^{[(1-h_j)(r_j-r_{jmin})/(r_{jmax}-r_j]}.\frac{\ln(1-R_j)}{\ln(1-r_j)}\right\}\right\}-V_0\right]=0$$

(17)

Where $\lambda_1, \lambda_2, \lambda_3$ are Lagrangean multipliers.

CASE PROBLEM:

To derive the optimum component reliability(r), stage reliability(R), number of components in each stage(x) and the system reliability(R) not to exceed system cost Rs.1000, weight of the system 1500 kg and volume of the system 2000 cm³.

CONSTANTS:

Stage	f_j	g_j	h_j	$r_{j,min}$	$r_{j,max}$
1	0.9	0.5	0.2	0.5	0.99
2	0.9	0.5	0.2	0.5	0.99
3	0.9	0.5	0.2	0.5	0.99

Reliability design relating to Cost, weight and volume – Without X_j Rounding Off:

i) RELIABILITY DESIGN RELATING TO COST

Stage	r_j	R_j	x_j	C_{jX100}	$c_j. x_j$
01	0.701	0.9399	2.33	110.27	256.93
02	0.70	0.9518	2.52	110.00	277.2
03	0.80	0.9886	2.78	110.51	307.21
Total Cost					841.34

ii) RELIABILITY DESIGN RELATING TO WEIGHT

Stage	r_j	R_j	X_j	W_{jx100}	$W_j. X_j$
01	0.701	0.9399	2.33	163.08	379.97
02	0.70	0.9518	2.52	161.6	407.23
03	0.80	0.9886	2.78	164.8	458.14
Total Weight					1245.34

iii) RELIABILITY DESIGN RELATING TO VOLUME

Stage	r_j	R_j	X_j	V_{jx100}	$V_j .X_j$
01	0.701	0.9399	2.33	218.7	509.57
02	0.70	0.9518	2.52	215.5	543.06
03	0.80	0.9886	2.78	222.6	618.83
Total Volume					1671.46

SYSTEM RELIABILITY = R_s = 0.8844

RELIABILITY DESIGN RELATING TO COST, WEIGHT AND VOLUME – WITH X_j ROUNDING OFF:
RELIABILITY DESIGN RELATING TO COST

Stage	r_j	R_j	x_j	c_{ix100}	$c_j. x_j$
01	0.701	0.9732	3	110.27	330.31
02	0.70	0.9730	3	110.0	330.00
03	0.80	0.9920	3	110.51	331.53
Total Cost					991.84

VARIATION IN TOTAL COST = 18.82%

RELIABILITY DESIGN RELATING TO WEIGHT

Stage	r_j	R_j	X_j	W_{jx100}	$W_j. X_j$
01	0.701	0.9732	3	163.08	489.24
02	0.70	0.9730	3	161.6	484.8
03	0.80	0.9920	3	164.8	494.4
Total Weight					1468.44

VARIATION IN TOTAL WEIGHT = 18.83%

RELIABILITY DESIGN RELATING TO VOLUME

Stage	r_j	R_j	X_j	V_{jx100}	$V_j .X_j$
01	0.701	0.9732	3	218.7	656.1
02	0.70	0.9730	3	215.5	646.5
03	0.80	0.9920	3	222.6	667.8
Total Volume					1970.4

VARIATION IN TOTAL VOLUME = 18.79%
SYSTEM RELIABILITY = 0.9394
VARIATION IN SYSTEM RELIABILITY = 06.22%

6. HEURISTIC METHOD

The Lagrangean multipliers method gives a solution to arrive at an optimal design quickly rather than sophisticated algorithms. This is of course done at the cost of treating the number of components in each stage (x_j) as real. This disad-

vantage can be overcome, by the heuristic approach. Heuristic methods, in most cases employ experimentation and trial-and-error techniques. A heuristic method is particularly used to come rapidly to a solution that is reasonably close to the best possible answer, or 'optimal solution'.

i) RELIABILITY DESIGN RELATING TO COST

Stage	r_j	R_j	x_j	c_{jx100}	$c_j . x_j$
01	0.701	0.9732	3	110.27	330.31
02	0.70	0.9730	3	110.0	330.00
03	0.80	0.9600	2	110.51	221.02
Total Cost					881.33

VARIATION IN TOTAL COST   = 11.86%

ii) RELIABILITY DESIGN RELATING TO WEIGHT

Stage	r_j	R_j	X_j	W_{jx100}	$W_j . X_j$
01	0.701	0.9732	3	163.08	489.24
02	0.70	0.9730	3	161.6	484.8
03	0.80	0.9600	2	164.8	329.60
Total Weight					1303.64

VARIATION IN TOTAL WEIGHT   = 13.09%

iii) RELIABILITY DESIGN RELATING TO VOLUME

Stage	r_j	R_j	X_j	V_{jx100}	$V_j . X_j$
01	0.701	0.9732	3	218.7	656.1
02	0.70	0.9730	3	215.5	646.5
03	0.80	0.9600	2	22.6	445.2
Total Volume					1747.8

SYSTEM RELIABILITY (RS)	=	0.9091
VARIATION IN TOTAL VOLUME	=	12.61%
VARIATION IN SYSTEM RELIABILITY	=	02.79%

7.2 SENSITIVITY ANALYSIS

It is observed that when the input data of constraints is increased by 10% there is only a 4.09% increase in system reliability. When the input data is decreased 10%, there is only an 8.3% decrease in system reliability. When one factor is varied, keeping all the other factors unchanged, the variation in the system reliability is as shown in the following Table.

Variation in factors		System Reliability
Cost	10% decrease	No change
	10% Increase	No change
Weight	10% decrease	No change
	10% Increase	No change
Volume	10% decrease	8.37% decreases
	10% Increase	4.09% increase

The analysis confirms that the volume factor is more sensitive to input data than are cost and weight.

8. DYNAMIC PROGRAMMING

The heuristic approach commonly provides a workable solution which is approximate one. To validate the established redundant reliability system and to obtain the much needed integer solution the Dynamic Programming method is applied. The Lagrangean Method can be used as the input for the Dynamic Programming Approach, in order to determine the stage Reliabilities, System Reliabilities, Stage Cost and

the System Cost. The Dynamic Programming Approach provides flexibility in determining the number of components in each stage; Stage Reliabilities and the System Reliability for the given System Cost. As per the procedure the parameter values derived from the Lagrangean are given as inputs for the Dynamic Programming Approach to obtain the integer solution.

i) RELIABILITY DESIGN RELATING TO COST CONSTRAINT

Stage	r_j	R_j	X_j	C_j	$C_j . X_j$
01	0.9075	0.9794	1	1920	1920
02	0.9277	0.9811	1	1590	1590
03	0.9278	0.9416	1	878	878
Total Cost					4388

VARIATION IN TOTAL COST = 12.24%

ii) RELIABILITY DESIGN RELATING TO WEIGHT CONSTRAINT

Stage	r_j	R_j	X_j	W_j	$W_j . X_j$
01	0.9075	0.9794	1	2560	2560
02	0.9277	0.9811	1	2385	2385
03	0.9278	0.9416	1	1676	1676
Total weight					6621

VARIATION IN TOTAL WEIGHT = 11.72%

iii) RELIABILITY DESIGN RELATING TO VOLUME:

Stage	r_j	R_j	X_j	V_j	$V_j . X_j$
01	0.9075	0.9794	1	1920	1920
02	0.9277	0.9811	1	1590	1590
03	0.9278	0.9416	1	878	878
Total volume					4388

SYSTEM RELIABILITY (R_s)	=	0.9047
VARIATION IN TOTAL VOLUME	=	12.24%
VARIATION IN SYSTEM RELIABILITY	=	19.42%

9. CONCLUSIONS

The integrated reliability models for redundant systems with multiple constraints for the k-out-of-n configuration system is established for the commonly used mathematical function using Lagrangean method approach where component reliabilities (r_j) and the number of components (x_j) in each stage are treated as unknowns. The system reliability (R_s) is maximized for the given cost, weight and volume by determining the component reliabilities(r_j) and the number of components required for each stage(x_j). The Lagrangean Multiplier Method provide a real valued solution, the Heuristic approach is considered for analysis purpose which provided a near optimum solution wherein the values of component reliabilities (r_j) are taken as input to carry out heuristic analysis. The analysis of Heuristic approach results in gaining a solution which ought to be an approximate one even after its validation and to derive the much needed scientific integer solutions for the defined problem, the Dynamic Programming approach is applied. The advantage of Dynamic Programming is that the number of components required for each stage (xj) directly gives an integer value along with the other values of the parameters, which is very convenient for practical implementation for the real life problems.

REFERENCE [1] BALAGURUSWAMY.E. (2002). Reliability engineering –Tata McGraw Hill, New Delhi. | [2] AGGARWAL,K.K., GUPTA.J.S. (1975). On minimizing the cost of reliable systems, IEEE Transactions on Reliability, Vol.R-24, No.3. | [3] MISHRA.K.B. (1972). Reliability Optimization of series – parallel system, IEEE Transactions on Reliability, Vol R-21, No.4. | [4] MISRA.K.B. (1971). A method of solving Redundancy optimization Problems, IEEE Transactions on Reliability, Vol R-20, No.3. | [5] FAN.L.T & WANG, TILLMAN. (1997). Optimization of system reliability IEEE Transactions on Reliability. Vol R-16. | [6] AGGARWAL,K.K , MISHRA.K.B, GUPTA.T.S.(1975). Reliability evaluation: A comparative Study of different techniques", Microelectronics and Reliability, Vol.14, pp.49- 56. | [7] METTAS.A. (2000). Reliability Allocation and Optimization for complex systems, Proceedings Annual Reliability and Maintainability Symposium, Los Angeles, California, USA. | [8] USHAKOV-LEVITIN-LISNIANSKI: (2002). Multi State system reliability: from theory to Practice: proceedings of the 3rd international conference on mathematical methods in reliability, Trondheim, Norway, 635-638. | [9] WAY KUO AND RAJENDRA PRASAD.V.(2000). An Annotated overview of system reliability Optimization", IEEE Transactions on Reliability, Vol. 49, No. 2, PP. 176-187. |