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Research on Unsteady State Heat Conduction in Slab & Cylinder

KEYWORDS: Proposed Biot Number, Assumed Temperature Profile, Unsteady state heat conduction

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ABSTRACT *This paper mainly concerned with the analytical solution of unsteady state one dimensional heat conduction problem. In this report as a base, improved lumped parameter model is taken to find the variation in temperature field in a slab and cylinder type geometry. Polynomial Approximation Method is used to solve the un-steady state conduction equation for both the geometry. Along with this analysis on other models like heat generation in both slab & cylinder and models with boundary heat flux is performed. Based on the analysis, a proposed Biot number has been suggested that predicts the temperature variation irrespective the geometry of the problem. A closed form solution is obtained between Biot number, time, heat source parameter and temperature for all cases. The result of the present analysis work is compared with earlier numerical and analytical results. A good analogy of results is achieved between the present data and the already available results.*

1. Introduction

Heat transfer occurs by three basic mechanisms or modes: conduction, convection, and radiation. Heat transmission, in majority of real situations, occurs as a result of combinations of these modes of heat transfer. Conduction heat transfer is defined as heat transfer in solids and fluids without bulk motion. Heat conduction generally takes place in solids, through it may occur in fluids without bulk motion or with rigid body motion. In fluids, conduction is due to the Collusions of the molecules during their random motion. In solids, it is due to the combination of vibrations of molecules in a lattice and the energy transport by free electrons. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion.

A plane slab and cylinder are considered one dimensional heat conduction when one of the surfaces of these geometries in each direction is very large compared to the region of thickness. When the temperature variation in the region is described by two and three variables, it is said to be two-dimensional and three-dimensional respectively. Generally the heat flow through the heat transfer medium dominates with only one direction.

2. Heat Conduction Problem

The solution of the heat conduction problems involves the functional dependence of temperature on various parameters such as space and time. Obtaining a solution means determining a temperature distribution which is consistent with conditions on the boundaries.

One Dimensional Analysis: In general, the flow of heat takes place in different spatial coordinates. In some cases the analysis is done by considering the variation of temperature in one-dimension. In a slab one dimension is considered when face dimensions in each direction along the surface are very large compared to the region thickness, with uniform boundary condition is applied to each surface. Cylindrical geometries of one-dimension have axial length very large compared to the maximum conduction region radius. At a spherical geometry to have one-dimensional analysis a uniform condition is applied to each concentric surface which bounds the region.

Steady state Analysis: A system is said to attain steady state when variation of various parameters namely, temperature, pressure and density vary with time. We can use steady-state thermal analysis to determine temperatures, thermal gradients, heat flow rates, and heat fluxes in an object which do not vary over time. A steady-state thermal analysis may be either linear, by assuming constant material properties or can be nonlinear case, with material properties varying with temperature. The thermal properties of most material do vary with temperature, so the analysis becomes nonlinear. Furthermore, by considering radiation effects system also become nonlinear.

Unsteady state analysis: A thermal system is said to be Unsteady when there is a variation of temperature with time. Unsteady-heat-flow problem involves also periodic variations of temperature and heat flow. Periodic heat flow occurs in air-conditioning, internal-combustion engines, instrumentation, and process control.

3. Description of analytical method and numerical method

In general, we employ either an analytical method or numerical method to solve steady or transient conduction equation valid for various dimensions (1D/2D). Numerical technique generally used is finite difference, finite element, relaxation method etc. The most of the practical two dimensional heat problems involving irregular geometries is solved by numerical techniques. The main advantage of numerical methods is it can be applied to any two dimensional shape irrespective of its complexity or boundary condition. The numerical analysis, due to widespread use of digital computers these days, is the primary method of solving complex heat transfer problems. The heat conduction problems depending upon the various parameters can be obtained through analytical solution. An analytical method uses Laplace equation for solving the heat conduction problems. Heat balance integral method, Hermite-type approximation method, polynomial approximation method, Wiener–Hopf Technique is few examples of analytical method.

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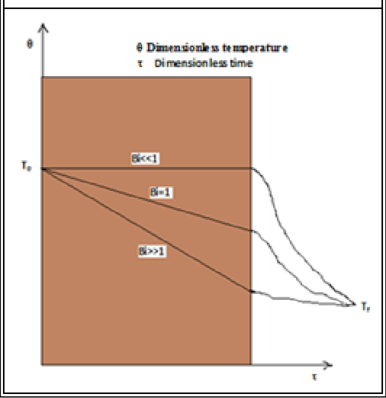
4. Low Biot number in 1-D heat conduction problems

The Biot number represents the ratio of the time scale for heat removed from the body by surface convection to the time scale for making the body temperature uniform by heat conduction. However, a simple lumped model is only valid for very low Biot numbers. In this preliminary model, solid resistance can be ignored in comparison with fluid resistance, and so the solid has a uniform temperature that is simply a function of time. The criterion for the Biot number is about 0.1, which is applicable just for either small solids or for solids with high thermal conductivity. In other words, the simple lumped model is valid for moderate to low temperature gradients. In many engineering applications, the Biot number is much higher than 0.1, and so the condition for a simple lumped model is not satisfied. Additionally, the moderate to low temperature gradient assumption is not reasonable in such applications, thus more accurate models should be adopted. Lots of investigations have been done to use or modify the lumped model. The purpose of modified lumped parameter models is to establish simple and more precise relations for higher values of Biot numbers and large temperature gradients. For example, if a model is able to predict average temperature for Biot numbers up to 10, such a model can be used for a much wider range of materials with lower thermal conductivity.

Figure 1: Schematic of Variation of Biot Number in a Slab



5. Solution of heat conduction problem

The objective of conduction analysis is to determine the temperature field in a body and how the temperature varies within the portion of the body. The temperature field usually depends on boundary conditions, initial condition, material properties and geometry of the body. Why one need to know temperature field. To compute the heat flux at any location, compute thermal stress, expansion, deflection, design insulation thickness, heat treatment method, these all analysis leads to know the temperature field. The solution of conduction problems involves the functional dependence of temperature on space and time coordinate.

Analytical Exact Solutions

Keshavarz and Taheri obtained the temperature distribution as:

Jian Su have analyzed unsteady cooling of a long slab by asymmetric heat convection within the framework of lumped parameter model.

The expression was written as:

Correa and Cotta have demonstrated heat conduction problems and examined against the Classical Lumped System Analysis (CLSA) and the exact solutions of the fully differential systems.

6. Solution procedure for current work

Polynomial Approximation Method (PAM) is one of the simplest, and in some cases, accurate methods used to solve unsteady conduction problems. The method involves two steps: first, selection of the proper assumed temperature profile, and second, to convert a partial differential equation into an equation. This can then be converted into an ordinary differential equation, where the dependent variable is average temperature and independent variable is time. The steps are applied on dimensionless governing equation .

Assuming constant physical properties, k and α, the generalized transient heat conduction valid for slab, cylinder and sphere can be expressed as:

where, m = 0 for slab, 1 and 2 for cylinder and sphere, respectively. We have covered different heat conduction problems for the analysis. The analytical method used is polynomial approximation method. Two problems are taken for heat flux, and two for heat generation. At the last a simple slab and cylinder is considered with different profiles. The result and discussion from the above analysis has been presented in the figures and tables, illustrated in following sections. Furthermore, the present prediction is compared with the analysis of Correa and Cotta (1998), Jian Su (2001) and Keshavarz and Taheri (2007).

7. Results and discussion:

We have tried to analyze the heat conduction behavior for both Cartesian and cylindrical geometry. Based on the previous analysis closed form solution for temperature, Biot number (B), heat source parameter (Q), and time for both slab and tube has been obtained.

Fig. 2 shows the variation of temperature with time for various Biot numbers, having heat source parameter as constant for a slab. With lower value of Biot numbers, the temperature inside the slab does not vary with time. However for higher value of Biot numbers, the temperature decreases with the increase of time.

Figure 2: Average Dimensionless Temperature vs. Dimensionless Time for Slab Q=1

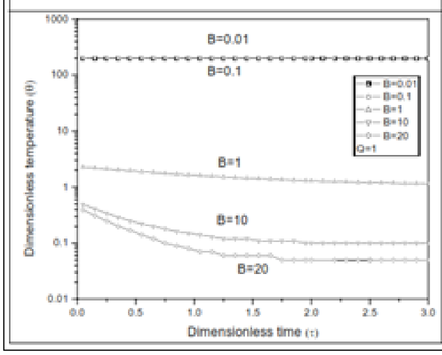


Fig.3 shows the variation of temperature with time for various Biot numbers, having heat source parameter as constant for a tube. With lower value of Biot numbers, the temperature inside the tube does not vary with time. For higher value of Biot numbers, the temperature decreases with the increase of time.

Figure 3: Average Dimensionless Temperatue vs. Dimensionless Time for Cylinder Q=1

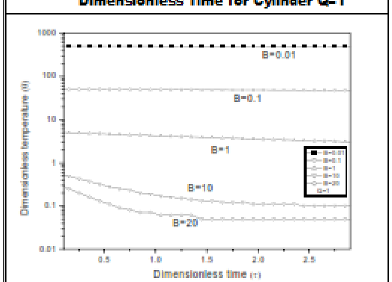


Fig .4 shows the variation of temperature with time for various

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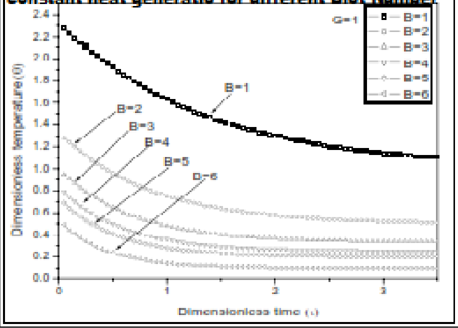
Biot numbers, having constant heat generation parameter for a slab. With lower value of Biot numbers, the temperature inside the tube does not vary with time. As the Biot number increases, the temperature varies more with increase of time.

Figure 4: Average Dimensionless Temperatue vs. Dimensionless Time in a slab with constant heat generatio for different Biot Number



We have considered a variety of temperature profiles to see their effect on the solution. Based on the analysis a proposed Biot number has been suggested, which is independent of geometry of the problem. It is seen that, for higher values of P(Proposed Biot Number) represent higher values of Biot number. Therefore the heat removed from the solid to surrounding is higher at higher Biot number. This leads to sudden change in temperature for higher value of P. By Polynomial Approximation method, we get the value of P as for a slab and for a cylinder in the general equation =exp(-Pτ) for a assumed Temperature profile of

Figure 5: Variation of average temperature with dimensionless time, for P=1 to 40 for a slab

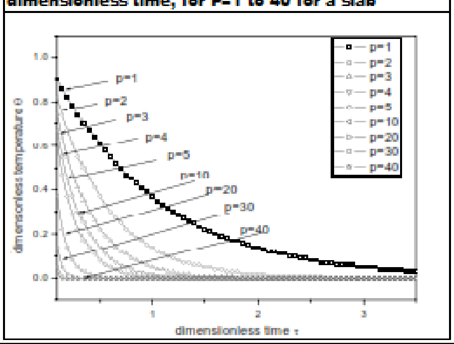


Fig.6 shows the comparison of present analysis with the other available results. These include classified lumped system analysis and exact solution by E.J. Correa and R.M. Cotta [4] of a slab. It is observed that the present prediction shows a better result compared to CLSA. The present prediction agrees well with the exact solution of E.J. Correa and R.M. Cotta [4] at higher time. However at shorter time, the present analysis under predicts the temperature in solid compared to the exact solution. This may be due to the consideration of lumped model for the analysis.

Figure 6: Comparision of solution of PAM, CLSA and Exact solution for a slab having internal heat generation

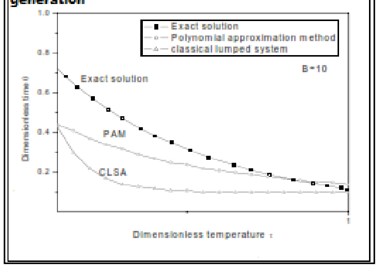


Table 1 is a Comparison of solutions of average temperature obtained from different heat conduction problems.

Table 1: Comparison of Solutions of Average Temperature Obtained from Different Heat Conduction Problems

Average Temperature	Slab with Heat Flux	Slab with Heat Generation	Tube with Heat Flux	Tube with Heat Generation
θ	$\theta=\left(\frac{e^{-U\tau}+V}{U}\right)$ where, $U=\frac{B}{1+B/3}$ $V=\frac{Q}{1+B/3}$	$\theta=\left(\frac{e^{-U\tau}+V}{U}\right)$ where, $U=\frac{B}{1+B_3}$ $V=\frac{G}{1+B_3}$	$\theta=\left(\frac{e^{-U\tau}+V}{U}\right)$ where, $U=\frac{B}{(4+B)/8}$ $V=\frac{Q}{(4+B)/8}$	$\theta=\left(\frac{e^{-U\tau}+V}{U}\right)$ where, $U=\frac{2B}{1+B_4}$ $V=\frac{G}{1+B_4}$

8. Conclusion:

An improved lumped parameter model is applied to the Unsteady heat conduction in a long slab and long cylinder. Polynomial approximation method is used to predict the transient distribution temperature of the slab and tube geometry. Four different cases namely, boundary heat flux for both slab and tube and, heat generation in both slab and tube has been analyzed. A unique number, known as proposed Biot number is, obtained from the analysis. It is seen that the proposed Biot number, which is a function of Biot number, plays important role in the transfer of heat in the solid.

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