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Bivariate Manpower Model for Permanent and Temporary Grades Under Equiulibrium

KEYWORDS: steady-state analysis, Manpower planning model, graded system, Poisson processes.

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ABSTRACT *Manpower planning is an important task of managing large organizations such as government, public , private sector and corporate. Since the personal behavior is random, the stochastic models provide the basic framework for efficient control and management of manpower systems. In this paper, the steady state analysis of a bivariate manpower planning model for permanent and temporary grades is studied. The recruitment, promotion and leaving processes of an employee in the organization are characterized by Poisson processes. The model characteristics such as the joint probability generating function, the average number of employees in each grade, the mean duration of stay of an employee in each grade, the variance of the number of employees in each grade are derived and analyzed. This model also includes some of the earlier model as particular cases.*

1. INTRODUCTION.

Manpower planning is a fundamental aspect of human research management in organizations. The objective of the manpower planning is developing plans to meet the future human resource recruitments. A shortage as well as a surplus of staff would be highly undesirable. It would lead to lower production, loss of orders and customers, higher costs and / or Less profit. Ashis Kumar etal (2007) especially for companies confronted with an ageing workforce or shortage on the labor market, manpower planning becomes a crucial instruments to create a sustainable competitive advantage. Statistical techniques have extensively been developed to support organization in their manpower planning challenges.

Starting from the pioneering work by Seal (1954) a large number of manpower models have been developed and analyzed with various assumptions in order to analyze the practical situations. Bartholomew has utilized the probability distribution for developing the manpower models using complete length of service of an employee. Butlers (1971) has studied the probability distribution of the number of leavers in a graded organization in which the grade sizes are fixed. S.I.Mc.clean and Abodende, T (1978) has studied the usage of entropy as a measure of stability in manpower models. Woodard, (1983) and Vivekananda Murthy, M (1996) has studied the manpower models through Markov chains. Glen J.J.(1977), Grinold, R.C, and Stanford R.E. (1974), Subramanyan,V (1996) and others studied the various graded manpower systems. Srinivasa Rao, K. etal (2006) developed a two graded manpower model using cumulant generating function. Anne-Guerry (2009) studied the manpower planning of wastage and the internal transition for homogeneous groups of employees in a manpower system.

But in many organizations having two grades the recruitment is done in both the grades. For example, in government organizations, the employees are recruited to both temporary and permanent grades with different recruitment rates. Once an employee is joined in the temporary grade he may be promoted to the permanent grade after a random period of time or he may be terminated (left) from the organization.

2. MANPOWER MODEL FOR PERMANENT AND TEMPORARY STAFF IN THE ORGANIZATION.

In this section, we consider that the growth of both permanent and temporary staff in the organization can be approximated by stochastic processes. We assume that recruitment process of temporary staff follows a Poisson process with parameter λ_1. The recruitment process of the permanent staff also follows a Poisson process with parameter λ_2. The promotion process from temporary to permanent also follows a Poisson process with parameter μ_1. The leaving processes of temporary and permanent staff are also Poisson with parameters α and μ_2 respectively. The schematic diagram representing manpower model in the organization is given in fig:1

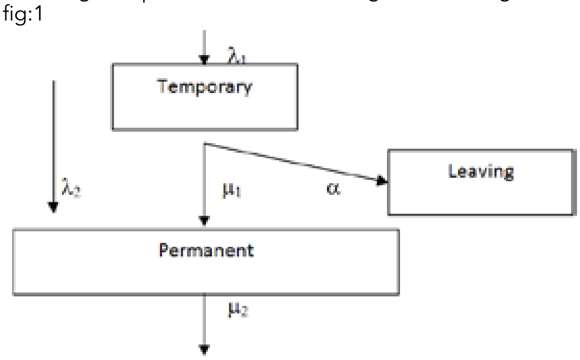

Fig. 1: The schematic diagram of the model.

Let $P_{n,m}(t)$ denote the probability that there are 'n' temporary staff members and 'm' permanent staff members at time 't' in the organization.

With these assumptions the postulates of the model are:

1. The occurrences of events in non-overlapping intervals of time are statistically independent.
2. The probability that a temporary employee is appointed during a small interval of time 'h' is $\lambda_1 h + o(h)$.
3. The probability that an employee is directly recruited as permanent employee during a small interval of time 'h' is $\lambda_2 h + o(h)$.
4. The probability that a temporary employee is promoted as permanent employee during a small interval of time 'h' when, there are 'n' temporary employees in the organization is $n\mu_1 h + o(h)$.
5. The probability that a temporary employee is terminated (or left) during a small interval of time 'h' when, there are 'n' temporary employees in the organization is $n\alpha h + o(h)$.
6. The probability that a permanent employee left the organization during a small interval of time 'h' when there are 'm' permanent employees in the organization is $m\mu_2 h$

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+ o(h).

7. The probability that there is no recruitment of either temporary or permanent employees and no employee left the organization during a small interval of time 'h' is [1 - ($\lambda_1+\lambda_2$ + $n\mu_1+n\alpha+m\mu_2$)h] +o(h) and
8. The probability that other than the above events during a small interval of time 'h' is o(h) Therefore the difference-differential equations of the model are

$$\frac{\partial P_{n,m}(t)}{\partial t}=[-(\lambda_1+\lambda_2+n\alpha+n\mu_1+m\mu_2)]P_{n,m}(t)+\lambda_1 P_{n-1,m}(t)+(n+1)\alpha P_{n+1,m}(t)$$

$+(m+1)\mu_2 P_{n,m+1}(t)+\lambda_2 P_{n,m-1}(t)$, for n, m >0 (1)

$$\frac{\partial P_{n,0}(t)}{\partial t}=[1-(\lambda_1+\lambda_2+n\alpha+n\mu_1)]P_{0,0}(t)+(n+1)\alpha P_{n+1,0}(t)+\mu_2 P_{n,1}(t)+\lambda_1 P_{n-1,0}(t)$$

for n >0 (2)

$$\frac{\partial P_{0,m}(t)}{\partial t}=[1-(\lambda_1+\lambda_2+m\mu_2)]P_{0,m}(t)+\alpha P_{1,m}(t)+\mu_1 P_{1,m-1}(t)+(m+1)\mu_2 P_{1,m+1}(t)+\lambda_2 P_{0,m-1}(t)$$

for m >0 (3)

$$\frac{\partial P_{0,0}(t)}{\partial t}=[1-(\lambda_1+\lambda_2)]P_{0,0}(t)+\alpha P_{1,0}(t)+\mu_2 P_{0,1}(t) \quad (4)$$

Let $P(Z_1,Z_2;t)$ be the joint probability generating function

Then $P(Z_1,Z_2;t)=\sum_n\sum_m Z_1^n Z_2^m P_{n,m}(t)$ (5)

Multiplying the equations (5) to (8) with corresponding Z_1^n, Z_2^m and summing over all n=0, 1, 2,…; m= 0,1,2,…; we get

$$\frac{\partial P}{\partial t}=-[(\lambda_1+\lambda_2)P]-(\alpha+\mu_1)Z_1\frac{\partial P}{\partial Z_1}-\mu_2 Z_2\frac{\partial P}{\partial Z_2}+\lambda_1 Z_1 P+\alpha\frac{\partial P}{\partial Z_1}+\mu_1\frac{\partial P}{\partial Z_2}+\lambda_2 Z_2 P \quad (6)$$

After simplification we have

$$\frac{\partial P}{\partial t}=[-(\alpha+\mu_1)Z_1+\mu_1 Z_2+\alpha]\frac{\partial P}{\partial Z_1}+[-\mu_2 Z_2+\mu_2 Z_3]\frac{\partial P}{\partial Z_2}+[\lambda_1 P(Z_1-1)+\lambda_2 P(Z_2-1)] \quad (7)$$

Solving the equation (7) by Lagrangian's method, the auxiliary equations are

$$\frac{\partial t}{1}=\frac{-\partial Z_1}{[-(\alpha+\mu_1)Z_1+\mu_1 Z_2+\alpha]}=\frac{-\partial Z_2}{[\mu_2(Z_2-1)]}=\frac{\partial P(Z_1,Z_2;t)}{[\lambda_1(Z_1-1)+\lambda_2(Z_2-1)]P(Z_1,Z_2;t)} \quad (8)$$

with the initial conditions that there are N_0 temporary and M_0 permanent employees in the organization at time t=0. i.e.,

$P_{N_0,M_0}(0)=1, \quad P_{N_0,M_0}(t)=0, \quad$ for $\quad t>0$.

Solving the equations, we get

$a=e^{-\mu_2 t}(Z_2-1)$ (9)

$$b=e^{-(\alpha+\mu_1)t}\left[(Z_1-1)-\frac{\mu_1(Z_2-1)}{(\mu_2-\mu_1)}\right]$$

$$c=p(Z_1,Z_2;t)\left\{\exp\left\{-\left[\frac{\lambda_1 b e^{(\alpha+\mu_1)t}}{\alpha+\mu_1}+\frac{a e^{\mu_2 t}}{\mu_2}\left(\lambda_2-\frac{\lambda_1\mu_1}{\mu_2-\alpha-\mu_1}\right)\right]\right\}\right\}$$

where, a, b and c are arbitrary constants.

The general solution of the equation (9) gives the joint probability generating function of the model as

$$P(Z_1,Z_2;t)=c.\exp\left\{\frac{\lambda_1 b e^{(\alpha+\mu_1)t}}{\alpha+\mu_1}+\frac{a e^{\mu_2 t}}{\mu_2}\left[\lambda_2-\frac{\lambda_1\mu_1}{\mu_2-\alpha-\mu_1}\right]\right\}X.Y. \qquad |Z_1|<1;\ |Z_2|<1$$

(10)

where,

$$X=\left[1-(1-Z_1)e^{-\mu_1 t}-\left(\frac{\mu_1}{\mu_1-\mu_2}\right)(1-Z_2)\left(e^{-\mu_1 t}-e^{-\mu_2 t}\right)\right]^{N_0}$$

; $Y=\left[1-(1-Z_2)e^{-\mu_2 t}\right]^{M_0}$.

Substituting the value of 'c' from equation (9) in the equation (10), the joint probability generating function of the number of employees in the temporary and permanent grades is obtained as

$$P(Z_1,Z_2;t)=\exp\left\{(Z_1-1)\left(\frac{\lambda_1}{\alpha+\mu_1}\right)\left(1-e^{-(\alpha+\mu_1)t}\right)\right.$$

$$+(Z_2-1)\left(\frac{\lambda_1\mu_1\left(1-e^{-(\alpha+\mu_1)t}\right)}{(\alpha+\mu_1)(\mu_2-\alpha-\mu_1)}+\left(\lambda_2-\frac{\lambda_1\mu_1}{\mu_2-\alpha-\mu_1}\right)\left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)\right)$$

$$\left.+\left(\left(\lambda_2-\frac{\lambda_1\mu_1}{\mu_2-\alpha-\mu_1}\right)\left(\frac{1-e^{-\mu_2 t}}{\mu_2}\right)\right)XY\right.$$

where, X and Y are given in equation (10) (11)

3. CHARACTERISTICS OF THE MODEL

In this section we study the steady state behavior of the manpower model. The steady state analysis of the model can be done by assuming that the organization is stable and under equilibrium conditions.

i.e. $\lim_{t\to\infty} P_{n,m}(t)=P_{n,m}$ and $\lim_{t\to\infty} P(Z_1,Z_2;t)=P(Z_1,Z_2)$

Using the equation (11) we get the joint probability generating function of the number of temporary and permanent employees in the organization when, the system is under equilibrium as

$$P(Z_1,Z_2)=e^{\left\{(Z_1-1)\left(\frac{\lambda_1}{\alpha+\mu_1}\right)+(Z_2-1)\left(\frac{\lambda_1\mu_1}{(\alpha+\mu_1)(\mu_2-\alpha-\mu_1)}+\left(\lambda_2-\frac{\lambda_1\mu_1}{\mu_2-\alpha-\mu_1}\right)\left(\frac{1}{\mu_2}\right)\right)\right.},$$

$|Z_1|<1$, $|Z_2|<1$ (12)

Expanding $P(Z_1,Z_2)$ and collecting constant terms, we get the probability that there is no employee in the organization when the system is under equilibrium as

$$P_{0,0}=e^{-\left(\frac{\lambda_1}{\alpha+\mu_1}\right)-\left\{\left(\frac{\lambda_1\mu_1}{(\alpha+\mu_1)(\mu_2-\alpha-\mu_1)}+\left(\lambda_2-\frac{\lambda_1\mu_1}{\mu_2-\alpha-\mu_1}\right)\left(\frac{1}{\mu_2}\right)\right)\right\}} \quad (13)$$

Taking Z_2 = 1, we get the probability generating function of the number of temporary employees in the organization as

$P(Z_1)=e^{\left(\frac{\lambda_1}{\alpha+\mu_1}\right)(Z_1-1)}$; $|Z_1|<1$

Expanding $P(Z_1)$ and collecting the constant terms, we get the probability that there is no temporary employee in the organization as

$$P_{0,}=e^{-\frac{\lambda_1}{\alpha+\mu_1}} \quad (14)$$

Similarly, taking Z_1 = 1, we get the probability generating function of the number of permanent employees in the organization as

$P(Z_2)=e^{(Z_2-1)\left\{\frac{\lambda_1\mu_1}{(\alpha+\mu_1)(\mu_2-\alpha-\mu_1)}+\left(\lambda_2-\frac{\lambda_1\mu_1}{\mu_2-\alpha-\mu_1}\right)\left(\frac{1}{\mu_2}\right)\right\}}$; $|Z_2|<1$

Expanding $P(Z_1)$ and collecting the constant terms, we get the probability that there is no permanent employee in the organization as

$$P_{,0}=e^{-\left\{\frac{\lambda_1\mu_1}{(\alpha+\mu_1)(\mu_2-\alpha-\mu_1)}+\left(\lambda_2-\frac{\lambda_1\mu_1}{\mu_2-\alpha-\mu_1}\right)\left(\frac{1}{\mu_2}\right)\right\}}$$

(15)

The mean number of temporary employees in the organization is

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$$L_1 = \frac{\lambda_1}{\alpha + \mu_1} \quad (16)$$

The probability that there is at least one temporary employee in the organization is

$$U_1 = 1 - e^{-\frac{\lambda_1}{\alpha + \mu_1}} \quad (17)$$

The mean number of permanent employees in the organization is

$$L_2 = \frac{\lambda_1 \mu_1}{(\alpha + \mu_1)(\mu_2 - \alpha - \mu_1)} + \left(\lambda_2 - \frac{\lambda_1 \mu_1}{\mu_2 - \alpha - \mu_1}\right)\left(\frac{1}{\mu_2}\right) \quad (18)$$

The probability that there is at least one permanent employee in the organization is

$$U_2 = 1 - e^{-\left\{\frac{\lambda_1 \mu_1}{(\alpha+\mu_1)(\mu_2-\alpha-\mu_1)} + \left(\lambda_2 - \frac{\lambda_1 \mu_1}{\mu_2 - \alpha - \mu_1}\right)\left(\frac{1}{\mu_2}\right)\right\}} \quad (19)$$

The mean number of employees in the organization is

$L = L_1 + L_2$

Therefore,

$$L = \frac{\lambda_1}{\alpha + \mu_1} + \frac{\lambda_1 \mu_1}{(\alpha + \mu_1)(\mu_2 - \alpha - \mu_1)} + \left(\lambda_2 - \frac{\lambda_1 \mu_1}{\mu_2 - \alpha - \mu_1}\right)\left(\frac{1}{\mu_2}\right)$$

The average duration of stay of a temporary employee in the organization is

$$W_1 = \frac{L_1}{(\alpha + \lambda_1)(1 - P_{0,})} = \frac{\frac{\lambda_1}{\alpha + \mu_1}}{(\alpha + \lambda_1)\left(1 - e^{-\frac{\lambda_1}{\alpha + \mu_1}}\right)}$$

(20)

The average duration of stay of permanent employees in the organization is

$$W_2 = \frac{L_2}{\mu_2 (1 - P_{.0})} = \frac{\frac{\lambda_1 \mu_1}{(\alpha + \mu_1)(\mu_2 - \alpha - \mu_1)} + \left(\lambda_2 - \frac{\lambda_1 \mu_1}{\mu_2 - \alpha - \mu_1}\right)\left(\frac{1}{\mu_2}\right)}{\mu_2 \left\{1 - \exp\left\{-\left[\frac{\lambda_1 \mu_1}{(\alpha + \mu_1)(\mu_2 - \alpha - \mu_1)} + \left(\lambda_2 - \frac{\lambda_1 \mu_1}{\mu_2 - \alpha - \mu_1}\right)\left(\frac{1}{\mu_2}\right)\right]\right\}\right\}}$$

The variance of the number of temporary employees in the organization is

$$V_1 = \frac{\lambda_1}{\alpha + \mu_1} \quad (22)$$

The variance of the number of permanent employees in the organization is

$$V_2 = \frac{\lambda_1 \mu_1}{(\alpha + \mu_1)(\mu_2 - \alpha - \mu_1)} + \left(\lambda_2 - \frac{\lambda_1 \mu_1}{\mu_2 - \alpha - \mu_1}\right)\left(\frac{1}{\mu_2}\right) \quad (23)$$

4. NUMERICAL ILLUSTRATIONS.

Using the equations the average number of employees in temporary grade, average number of employees in permanent grade , the average duration of stay of a temporary employee, the average duration of stay of a permanent employee, the variance of the number of employees in the temporary grade , the variance of the number of employees in the permanent grade and coefficient of variation of the number of employees in both temporary and permanent grades are computed and presented in table 2 for different values of the parameters as α = 2, 3, 4 λ_1 = 1, 3, 5 λ_2 = 1, 4, 5μ_1 = 6, 7, 8 μ_2 = 7, 9, 11 N_0 = 100, 300, 700 and M_0 = 300, 500, 700.

From table 1, it is observed that the leaving rate from temporary grade (α) increases from 2 to 5, the average number of employees in both the grades are decreasing. The decrease is more rapid in temporary grade than that of permanent grade. The average duration of stay of the employees in both the grades are also decreasing, when α increases for fixed values of the other parameters. As the recruitment rate of the temporary employees increases from 1 to 5, the average number of employees in temporary grade is increasing from 0.167 to 0.833. Similarly, the average number of employees in the permanent grade is increasing from 0.733 to 1.267. The average duration of stay of an employee in temporary grade and permanent grade are also increasing. The variances of the number of employees in both the grades are also increasing, when the other parameters are fixed.

When the direct recruitment rate of the permanent employees (λ_2) increases from 1 to 5, the values of L_2, W_2, and V_2 are increasing for fixed values of the other parameters. But there is no influence on L_1 W_1, and V_1. As the promotion rate from temporary to permanent grade increases from 6 to 8, the average number of employees in the temporary grade is decreasing. But there is a positive relationship between μ_1 and average number of employees in permanent grade. As (μ_1) increases, L_2 is also increasing, when other parameters remain fixed. As a result of it the average duration of stay of a permanent employee in the organization is also increasing, but this increase is very small.

When the leaving rate of a permanent employee (μ_2) is increasing then the average number of employees in the permanent grade , the variance of the number of employees in permanent grade , the average duration of stay of an employee in permanent grade are decreasing, when other parameters are remain fixed. But there is no influence on L_1, W_1, and V_1.

A comparative study of a steady grade results with the transient results reveals that the time't' has a significant influence on the characteristics of the model. Hence, for short term forecasts of the number of employees in the organization one can use transient results and for long term forecasts one can use steady grade solutions.

Table. 1

α	λ_1	λ_2	μ_1	μ_2	N_0	M_0	L_1	L_2	W_1	W_2	V_1	V_2
2	2	3	4	5	500	100	0.333	0.867	0.294	0.299	0.333	0.867
3	2	3	4	5	500	100	0.286	0.829	0.23	0.294	0.286	0.829
4	2	3	4	5	500	100	0.25	0.8	0.188	0.291	0.25	0.8
2	1	3	4	5	500	100	0.167	0.733	0.362	0.282	0.167	0.733
2	3	3	4	5	500	100	0.5	L1	0.254	0.316	0.5	1
2	5	3	4	5	500	100	0.833	1.267	0.211	0.353	0.833	1.267
2	2	1	4	5	500	100	0.333	0.467	0.294	0.25	0.333	0.467
2	2	4	4	5	500	100	0.333	1.067	0.294	0.325	0.333	1.067
2	2	5	4	5	500	100	0.333	1.267	0.294	0.353	0.333	1.267
2	2	3	6	5	500	100	0.25	0.9	0.283	0.303	0.25	0.9
2	2	3	7	5	500	100	0.222	0.911	0.279	0.305	0.222	0.911
2	2	3	8	5	500	100	0.2	0.92	0.276	0.306	0.2	0.92
2	2	3	4	7	500	100	0.333	0.619	0.294	0.192	0.333	0.619

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2	2	3	4	9	500	100	0.333	0.481	0.294	0.14	0.333	0.481
2	2	3	4	11	500	100	0.333	0.394	0.294	0.11	0.333	0.394
2	2	3	4	5	100	100	0.333	0.867	0.294	0.299	0.333	0.867
2	2	3	4	5	300	100	0.333	0.867	0.294	0.299	0.333	0.867
2	2	3	4	5	700	100	0.333	0.867	0.294	0.299	0.333	0.867
2	2	3	4	5	700	300	0.333	0.867	0.294	0.299	0.333	0.867
2	2	3	4	5	700	500	0.333	0.867	0.294	0.299	0.333	0.867
2	2	3	4	5	700	700	0.333	0.867	0.294	0.299	0.333	0.867

The values of L_1, L_2, W_1, W_2, V_1 and V_2 for different values of parameters

5. CONCLUSIONS.

This paper discusses a novel methodology for transient analysis of a bivariate manpower model for permanent and temporary grades. In the modern competitive business environment the manpower models provide the basic framework for developing the optimal recruitment and promotion policies of the organization. Assuming the recruitment in both temporary and permanent grades, promotion leaving processes follow Poisson processes. The joint probability generating function of the different grade size distributions is obtained. The steady-state analysis provide assistance for predicting the mean number of customers in both temporary and permanent grades and the mean duration of stay in each grade which are very much important for human resource managers to take optimal manpower decisions in scheduling the recruitments and promotions of their employees. The two grade system is sufficient for multi graded systems also. The direct recruitment to the permanent grade consider in this model is more appropriate in approximating manpower system characteristics close to the reality. This model is also includes some of the earlier models given by Sally Mc.Clean (2000) and Srinivasa Rao, K. etal (2006), when the direct recruitment rate λ_2 =0. It is possible to extent this model under non-Markovian recruitment and promotion processes which will be taken up elsewhere.

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