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Cost Deviation Method for Solving Transportation Problems

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ABSTRACT *A new method namely, cost deviation method for solving transportation problems is proposed in which an initial feasible solution and MODI method are used. The optimality of the solution obtained by the cost deviation method of a transportation problem is analytically proved. The proposed method is illustrated with a numerical example. The proposed method can help decision makers in the logistics related issues by aiding them in the decision making process and providing an optimal solution in a simple and effective manner.*

INTRODUCTION

The transportation problem is a special class of linear programming problem which has applications in Management Sciences, Engineering and Technology. The transportation problem deals with shipping commodities from different sources to various destinations. In the transportation problem, the objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. In literature, much effort has been concentrated on transportation problems with equality constraints [1,4,5,6,11]. Among them, the transportation algorithm for solving transportation problem s with equality constraints introduced by Dantzig [1] is the simplex method specialized to the format of a table called transportation table which involves two parts: first, finding an initial basic feasible solution to the transportation problem and then, checking the optimality of the basic feasible solution to the transportation problem. Shimshak et.al. [10] proposed a modified Vogel's approximation method (SVAM). Goyal [3] suggested another modified Vogel's approximation method (GVAM). Ramakrishnan [9] proposed a modified GVAM for solving unbalanced transportation problem. Nagaraj Balakrishnan [7] suggested further modification in SVAM. Pandian and Natarajan [8] introduced the zero point method (ZPM) for finding an optimal solution to the transportation problem. Edward Samuel [2] has suggested a modification in ZPM .

In this paper, we propose a new algorithm namely, cost deviation method for solving of transportation problems. Then, we prove analytically the solution to the transportation problem obtained by the proposed method is optimal. The proposed method is illustrated by a numerical example. In the proposed method, methods of finding a basic feasible solution to the transportation problem and methods of checking optimality to the transportation problem. Further, we extend the proposed method to fuzzy transportation problems. Decision makers may use the cost deviation method for taking decision to the logistics related real life problems in simple and effective manner.

TRANSPORTATION PROBLEMS

Consider the following transportation problem.

(P) Minimize Z = $\sum_{i=1}^{m} \sum_{j=1}^{n} c_j \, x_j$

subject to

$\sum_{j=1}^{n} x_j = a_i$, i = 1, 2, …, m (1)

$\sum_{i=1}^{m} x_j = b_j$,

$x_j \geq 0$

j = 1,2,…,n (2) , for all i and j and are integers (3)

where m is the number of supply points ; n is the number of demand points; x_j is the number of units shipped from supply point i to demand point j; c_j is the cost of shipping one unit from supply point i to the demand point j ; a_i is the supply at supply point i and b_j is the demand at supply point j.

Now, the above problem can put in the following table namely, transportation table.

	D_1	…	D_n	Supply
O_1	c_{11}	…	c_{1n}	a_1
⋮	⋮	⋮	⋮	⋮
O_m	c_{m1}	…	c_{mn}	a_m
Demand	b_1	…	b_n	

If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$,

the transportation problem (P) is said to be balanced. Otherwise, it is called unbalanced.

	D_1	…	D_n	Supply
O_1	(p_{11}, t_{11})	…	(p_{1n}, t_{1n})	a_1
⋮	⋮	⋮	⋮	⋮
O_m	$_{m1}, t_{m1})$	…	$_{mn}, t_{mn})$	a_m
Demand	b_1	…	b_n	

Any set of non-negative allocations of a transportation problem which satisfies (1) , (2) and (3), is called a feasible solution to the transportation problem. Optimal solution to a transportation problem is a feasible solution to the transportation problem which minimizes the total shipping cost of the problem.

Volume : 3 | Issue : 12 | Dec 2013 | ISSN - 2249-555X

COST DEVIATION METHOD
We, now define the following terms which are going to use in the cost deviation method.

The **row deviation of a cell** in the transportation table is the value which is equal to the transportation cost of the cell minus the minimum of the corresponding row transportation costs.

The **column deviation of a cell** in the transportation table is the value which is equal to the transportation cost of the cell minus the minimum of the corresponding column transportation costs.

The ordered pair (a,b) is said to be the **cost deviation vector** of a cell if *a* is the row cost deviation and *b* is the column cost deviation of the cell.

Let r_i be the minimum cost of the i^{th} row and s_j be the minimum cost of the j^{th} column. The row cost deviation of the $(i,j)^{th}$ cell, p_{ij} is given by $p_{ij}=c_{ij}-r_i$. The column cost deviation of the $(i,j)^{th}$ cell, t_{ij} is given by $t_{ij}=c_{ij}-s_j$. The cost deviation vector of the $(i,j)^{th}$ cell is (p_{ij}, t_{ij}).

The cost deviation table for the given transportation problem is as follows.

Leveling the cost deviation table means that the entry of the each cell (i,j) is reformed the cost deviation table by using the following formulae:

(i) New $p_{ij} = p_{ij} -$ min.{ p_{ij} : for all i} and (ii) New $t_{ij} = t_{ij} -$ min{ t_{ij} : for all j}.

Resetting the cost deviation table means that the entry of the each unallocated cell (i,j) is modified by using the following formulae:

(i) New $p_{ij} = p_{ij} -$ min.{ p_{ij} : for all j} and (ii) New $t_{ij} = t_{ij} -$ min{ t_{ij} : for all i}.

The new proposed method namely, cost deviation method is proposed to solve the transportation . The algorithm of the proposed method proceeds as follows.

Step 1. Construct the cost deviation table for the given transportation problem.

Step 2. Level the cost deviation table.

Step 3. Identify the cells having (0,0) in the leveled cost deviation obtained from the Step 2..

Step 4. Select the cell having (0,0) one by one from the largest original cost to least original cost and allot the maximum possible item to the selected cell.

Step 5. Reform the cost deviation table after deleting fully used supply points and fully received demand points and modify not fully used supply points and not fully received demand points.

Step 6. If only one column or row is available in the reduced cost deviation table obtained from the Step 5. to allocate, move to the Step 7.. If not, move to the Step 8..

Step 7. Allot the rest of the items such that row to row / column to column as it is and stop the computation. Then, go to the Step 9..

Step 8. Reset the cost deviation table obtained from the Step 5. and then, go to the Step 3..

Step 9. The combined allotments obtained from the Step 7.. yields a solution to the given transportation problem.

Now, we prove that the solution obtained by the cost deviation method to the transportation problem is an optimal solution to the transportation problem.

Theorem 1: The solution obtained by the cost deviation method to the transportation problem (P) is an optimal solution to the problem (P).

Proof. Now, the problem (P) is given below:

(P) Minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

subject to (1) , (2) and (3) are satisfied .

Now, using the Step 1. to the Step 3. of the proposed method, we find that a set of cells, say A who cost deviation vectors are (0,0). As per the selection of the allotment cell by the Step 4. , we can observe that the cell transportation cost at each selected cell is the minimum cost related to its row or / and its column such that its supply point is fully used or/ and its demand point is fully received.

Therefore, the objective of the problem (P) can be written as follows:

$$z = \sum_{(i,j)\in A}\sum c_{ij} x_{ij} + \sum_{(i,j)\in K}\sum c_{ij} x_{ij}$$

where $K = \{(i,j) : (i,j) \notin A\}$.

This implies that $\text{Minimize } z = c(A) + \text{Minimize } z_1$.

where $c(A)$ is the total transportation costs obtained by the allotment of elements of A and z_1 is the objective function of the reduced transportation problem obtained from the given problem after applying the Step 7..

Now, consider the following reduced transportation problem

(P_1) Minimize $z_1 = \sum_{i\in M_1} \sum_{j\in N_1} c_{ij} x_{ij}$

subject to

$\sum_{j\in N_1} x_{ij} = a^1_i \quad i \in M_1$,

$\sum_{i\in M_1} x_{ij} = b^1_j, \quad j \in N_1$

$x_{ij} \geq 0$, for all $i \in M_1$ and $j \in N_1$

and are integers

where $M_1 \subseteq \{1,2,...,m\}$, $N_1 \subseteq \{1,2,...,n\}$ and $M_1 \times N_1 \subseteq K$.

Now, using the Step 8. and the Step 3. to the Step 5. of the proposed method, we find that a set of cells, say B having zero cost deviation vector.

Therefore, the objective of the problem (P_1) can be written as follows:

$$z_1 = \sum_{(i,j)\in B}\sum c_{ij} x_{ij} + \sum_{(i,j)\in J}\sum c_{ij} x_{ij}$$

where $J = \{(i,j) : (i,j) \notin B\}$.

This implies that $Minimize\ z_1 = c(B) + Minimize\ z_2$.

where $c(B)$ is the total transportation costs obtained by the allotment of elements of B and z_2 is the objective function of the reduced transportation problem obtained from the reduced problem (P_1) after applying the Step 7..

Now, consider the following reduced transportation problem

(P_2) Minimize $z_2 = \sum_{i\in M_2} \sum_{j\in N_2} c_{ij} x_{ij}$

Volume : 3 | Issue : 12 | Dec 2013 | ISSN - 2249-555X

subject to

$\sum_{j\in N_2} x_{ij} = a^2_i$, $i \in M_2$

$\sum_{i\in M_2} x_{ij} = b^2_j$, $j \in N_2$

$x_{ij} \geq 0$, for all $i \in M_2$ and $j \in N_2$ and are integers where $M_2 \subseteq M_1 \cap \{1,2,...,m\}$, $N_2 \subseteq N_1 \cap \{1,2,...,n\}$ and $M_2 \times N_2 \subseteq J$..

Now, the above said process is repeated till the reduced cost deviation has one only unallocated column or one only unallocated.

Finally, we obtain a solution set $\{x^{\circ}_{rs}, x^{\circ}_{pq}, \ldots, x^{\circ}_{ab}, x^{\circ}_{cd}\}$ to the problem (P) by the cost deviation method such that

$$z = c_{rs}x^{\circ}_{rs} + c_{pq}x^{\circ}_{pq} + \ldots + c_{ab}x^{\circ}_{ab} + c_{cd}x^{\circ}_{cd}$$

is minimum.

Thus, the solution $\{x^{\circ}_{rs}, x^{\circ}_{pq}, \ldots, x^{\circ}_{ab}, x^{\circ}_{cd}\}$ obtained by the cost deviation method is an optimal solution to the given transportation problem (P).

Hence the theorem.

Remark 1: By the cost deviation method, allotments to the cells are made by step by step based on the cost of the cells. So, an unbalanced transportation can be solved directly without making balanced by the proposed method.

Now, with the following numerical example, the cost deviation method is illustrated.

Example 1: Consider the following transportation problem

	D_1	D_2	D_3	D_4	Supply
O_1	4	6	14	11	12
O_2	4	1	8	3	3
O_3	8	12	19	12	17
Demand	11	11	6	4	

The cost deviation table for the given problem is as follows.

	D_1	D_2	D_3	D_4	Supply
O_1	(0,0)	(2,5)	(10,6)	(7,8)	12
O_2	(3,0)	(0,0)	(7,0)	(2,0)	3
O_3	(0,4)	(4,11)	(11,11)	(4,9)	17
Demand	11	11	6	4	

Now, after leveling the cost deviation table, we have the following table

	D_1	D_2	D_3	D_4	Supply
O_1	(0,0)	(2,5)	(3, 6)	(5,8)	12
O_2	(3,0)	(0,0)	(0,0)	(0,0)	3
O_3	(0,0)	(4,7)	(4,7)	(2,5)	17
Demand	11	11	6	4	

Now, (0,0) cells are (1,1) with cost 4 , (2,2) with cost 1, (2,3) with cost 8, (2,4) with cost 3 and (3,1) with cost 8. After allotting the cell (2,3) to 3 and the cell (3,1) to 11, we have the following reduced cost deviation table

	D_1	D_2	D_3	D_4	Supply
O_1		(2,5)	(3, 6)	(5,8)	12
O_2			3		
O_3	11	(4,7)	(8,7)	(6,5)	6
Demand		11	3	4	

Now, after resetting the reduced cost deviation table, the following table is obtained.

	D_1	D_2	D_3	D_4	Supply
O_1		(0,0)	(1, 0)	(3,3)	12
O_2			3		
O_3	11	(0,2)	(4,1)	(2,0)	6
Demand		11	3	4	

Now, (0,0) cell is only one cell (1,2). After allotting the cell (1,2) to 11, we obtain the following table.

	D_1	D_2	D_3	D_4	Supply
O_1		11	(1, 0)	(3,3)	1
O_2			3		
O_3	11		(4,1)	(2,0)	6
Demand			3	4	

Now, after resetting the reduced cost deviation table, the following table is obtained.

	D_1	D_2	D_3	D_4	Supply
O_1		11	(0, 0)	(1,3)	1
O_2			3		
O_3	11		(2,1)	(0,0)	6
Demand			3	4	

Now, (0,0) cells are (1,3) with cost 14 and (3,4) with cost 12 . After allotting the cell (1,3) to 1 and the cell (3,4) to 4, we have

	D_1	D_2	D_3	D_4	Supply
O_1		11	1		
O_2			3		
O_3	11		(2,1)	4	2
Demand			2		

Now, since only one column is available, allot the cell (3,3) to 2 and then, we have the following optimal allotment

	D_1	D_2	D_3	D_4	Supply
O_1		11	1		
O_2			3		
O_3	11		2	4	
Demand					

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Thus, the optimal solution to the given transportation problem is $x_{12} = 11$; $x_{13} = 1$; $x_{23} = 3$; $x_{31} = 11$; $x_{33} = 2$ and $x_{34} = 4$ with minimum transportation cost 278.

CONCLUSIONS

In this paper, we propose the cost deviation method for finding an optimal solution to transportation problems. In the proposed method, we find an allotment one by one to the transportation problem. Initial basic feasible solution methods, optimality methods and the allotment condition of the zero point method are not used in the proposed method. An unbalanced transportation can be solved directly without making balanced by the cost deviation method. In near future, we extend the proposed method to fuzzy transportation problems. The cost deviation method can help decision makers in the logistics related issues by aiding them in the decision making process and providing an optimal solution in a simple and effective manner.

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