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Design and Implementation of Fuzzy Multi Objective Optimization Model for Production Planning

KEYWORDS: Production Management, Fuzzy multiple Objective Linear Programming, Fuzzy Set Theory.

C.Kavitha
Department of Mathematics, Sathyabama University, Chennai, India

C. Vijayalakshmi
Department of Mathematics, SAS, VIT University, Chennai, India

ABSTRACT *Manufacturing firm focuses on maximizing the profit by satisfying the customer demands. To achieve the goal they give importance to optimum utilization of available resources. Information available in real life system is of vague, imprecise and uncertain nature. The impreciseness and uncertainty aspects are handled using fuzzy sets to obtain optimal solution. The present paper demonstrates a novel solution approach for fuzzy goals and fuzzy constraints to satisfy the aspiration level of decision maker. Production management problem is modelled using fuzzy sets and applied to the real world data of a packaging industry. It is able to consider fuzziness in the parameters in the presence of multiple objective functions. Fuzzy linear programming approach exhibits greater computational efficiency by employing the linear membership functions to represent fuzzy numbers.*

1 Introduction

Manufacturing enterprise makes multi product to obtain the maximum profit by determining the product, raw material cost etc to meet the market demand in a long term. In production planning model, uncertainty occurs in the form of production wastage, demand in the market, variable cost, sales value etc. the objective function are frequently imprecise / fuzzy because some information is incomplete or unobtainable. Lai and Hwang [2] mention that traditional operation research approach is not acceptable for the practical decision making problems because of the nature of fuzziness. Fuzzy set theory is identified as an alternative approach to handle such vagueness of the planning objectives. In Fuzzy Linear Programming [FLP] the fuzziness of available resources is characterized by the membership function over the tolerance range. Zimmermann 1976[5] first introduced fuzzy set theory into an ordinary linear programming problem with fuzzy goal and constraints. An application of fuzzy optimization techniques to linear programming problems with multiple objectives has been presented by Zimmermann 1978, 1996 [6 & 7] advantages of the linear membership function are the simplicity and flexibility of the fuzzy arithmetic operations. Werners [4] presents the method of solving linear programming model with crisp or fuzzy constraints and crisp or fuzzy goals. Fung et al [1] developed a fuzzy multi product aggregate production planning model whose solutions were introduced to cater to different scenarios under various decision making preferences by using parametric programming. Wang and liang [3] developed a fuzzy multi objective linear programming model for solving the multi product aggregate production planning decision making problems in a fuzzy environment. In this paper an industrial case is used to demonstrate the feasibility of applying the FLP method to tackle the presence of multiple objective functions.

The remainder of the paper is organized as follows. Section 2 considers the fuzzy multi objective model. Section 3 describes the fuzzy additive model, augmented max-min model and the proposed fuzzy model. Section 4 denotes the mathematical model for production planning. Section 5 and 6 represents the conclusion, references.

2. Fuzzy multi objective linear programming

Fuzzy multi objective programming with fuzzy objectives and fuzzy constraints are

Find x

Such that

$$\begin{aligned} &C_k x \gtrsim Z_k, k \in I_1 \\ &C_k x \lesssim Z_k, k \in I_2 \\ &a_r x \cong b_r, r \in T \\ &x \in X \end{aligned} \qquad (1)$$

Where $I_1 \cup I_2 = \{1,2.....l\}$, $I_1 \cap I_2 = \phi$ and X is a set of deterministic linear constraints and sign restrictions.

$$C_k x = \sum_{i=1}^{n} C_{k} x_i, k = 1,2...............l$$

$$a_r x = \sum_{i=1}^{n} a_{r} x_i, r = 1,2...............s$$

For $k \in I_{1,2}, Z_k$ is the imprecise aspiration level for the kth objective function. $Z_k \in [Z_k^L, Z_k^U]$ denote the imprecise lower and upper bounds respectively for the kth objective function. $b_r \in [b_r^L, b_r^U]$ denote the imprecise lower and upper bounds respectively for the rth fuzzy constraints.

Membership function for maximization and minimization objectives and constraints are as follows

$$\mu_k(c_k x) = \begin{cases} 1 & if\ c_k x \geq z_k^U \\ \dfrac{(c_k x) - z_k^L}{z_k^U - z_k^L} & if\ z_k^L \leq c_k x \leq z_k^U, \\ & \forall k \in I_1 \\ 0 & if\ c_k x \leq z_k^L \end{cases} \qquad (2)$$

$$\mu_k(c_k x) = \begin{cases} 1 & if\ c_k x \leq z_k^L \\ \dfrac{z_k^U - (c_k x)}{z_k^U - z_k^L} & if\ z_k^L \leq c_k x \leq z_k^U, \\ & \forall k \in I_2 \\ 0 & if\ c_k x \geq z_k^U \end{cases} \qquad (3)$$

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$$\mu_r(a_r x) = \begin{cases} 1 & if \quad a_r x \le b_r^L \\ \dfrac{a_r x - b_r^L}{b_r - b_r^L} & if \quad b_r^L \le a_r x \le b_r \\ \dfrac{b_r^U - (a_r x)}{b_r^U - b_r} & if \quad b_r \le a_r x \le b_r^U, \\ & \forall r \in T \\ 0 & if \quad a_k x \ge b_r^U \end{cases} \tag{4}$$

Equation (2) represents linear monotone increasing membership function $\mu_k(c_k x)$ for maximization type objectives with fuzzy aspiration levels. Equation (3) represents linear monotone decreasing membership function $\mu_k(c_k x)$ for minimization type objectives with fuzzy aspiration levels and Equation (4) represents triangular membership function for $\mu_r(a_r x)$ for constraints.

3. Fuzzy additive model

Multi objective programming (1) based on Tiwari et al (1987) is

$$\max \frac{\left(\sum_{k=1}^{l} \lambda_k + \sum_{r=1}^{s} \lambda_r\right)}{l+s}$$

Such that $\lambda_k \le \mu_k(c_k x), k \in I_1 \cup I_2$

$\lambda_r \le \mu_r(a_r x), r \in T$

$g_p(x) \le b_p$, p = h+1,h+2……m [for deterministic constraints]

$\lambda_k, \lambda_r \in [0,1], \quad k = 1,2..........l, r = 1,2.....s$

$$x \in X \tag{5}$$

3.1 Augmented max-min model

Augmented max-min model for (1) based on Lai and Hwang's (1993, 1996) approach is

$$\max \lambda + \frac{\left(\sum_{k=1}^{l} \mu_k(c_k x) + \sum_{r=1}^{s} \mu_r(a_r x)\right)}{l+s}$$

Such that $\lambda_k \le \mu_k(c_k x), k = 1,2....................l$

$\lambda_r \le \mu_r(a_r x), r = 1,2.................s$

$g_p(x) \le b_p$, p = h+1,h+2……m [for deterministic constraints]

$x \in X$

$$\lambda \in [0,1] \tag{6}$$

λ is the minimum satisfaction degree and defined as follows

$$\min_{k,r} \{\mu_k(c_k x), \mu_r(a_r x)\} \; for \; k = 1,2.........l, \; r = 1,2.........s \tag{7}$$

3.2 Proposed Fuzzy Model

λ is the minimum satisfaction degree in (7)

$$\max \lambda + \frac{\left(\sum_{k=1}^{l} \mu_k(c_k x) + \sum_{r=1}^{s} \mu_r(a_r x)\right)}{l+s}$$

Such that $\lambda_k \le \mu_k(c_k x), k = 1,2....................l$

$\lambda_r \le \mu_r(a_r x), r = 1,2.................s$

$g_p(x) \le b_p$, p = h+1,h+2……m [for deterministic constraints]

$\sum_{k=1}^{l} c_k x \ge \alpha_k z_k^U, k = 1,2..........l$

$\sum_{k=1}^{l} c_k x \le \alpha_k z_k^U, k = 1,2..........l$

$\sum_{r=1}^{s} a_r x \le \alpha_r b_r^U, r = 1,2..........s$

$x \in X$

$$\alpha_k, \alpha_r, \lambda, \lambda_k, \lambda_r \in [0,1] \tag{8}$$

Parameters $\alpha_k and \ \alpha_r$ represent the minimum acceptable achievement levels for the kth objective and rth constraint determined by the decision makers.

4. MODEL FORMULATION

Index

i	Index of Products (i=1,2……………..m)
j	Index of Process (j=1,2………………..n)
k	Index of Resources (k=1,2……………..K)
t	Index of Period (t=1,2………………T)

Decision variables

Q_{ijt}	Quantity of each product (i) produced by process (j) in period (t)
Q_{it}^I	Quantity of product (i) held in inventory at the end of period (t)

Parameters

$\tilde{C}_{ijt}$	Cost of each product (i) produced by process (j) in period (t)
$\tilde{C}_{it}^M$	Machine running Cost of each product (i) produced in period (t)
$\tilde{C}_{ijt}^L$	Labour Cost of each product (i) produced by process (j) in period (t)
$\tilde{C}_{ijt}^W$	Wastage value of each product (i) produced by process (j) in period (t)
$\tilde{C}_{it}^I$	Cost to hold the product (i) in inventory for period (t)
$\tilde{S}_{ijt}$	Sales value of each product (i) produced by process (j) in period (t)
m_{ij}	Machine hours required for each product (i) produced by process (j) in each period.
$m_t^{\max}$	Maximum machine hours available in period (t)
$\tilde{D}_{it}$	Demand for product (i) in period (t)

Objective function

$$Min \sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{j=1}^{n} [\tilde{C}_{ijt} + \tilde{C}_{it}^M + \tilde{C}_{ijt}^L + \tilde{C}_{ijt}^W] Q_{ijt} + \tilde{C}_{it}^I Q_{it}^I$$

$$Max \sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{S}_{ijt} Q_{ijt}$$

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Constraints

Constraints on machine:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} m_{ij} Q_{ijt} \leq m_t^{\max}, t = 1,2 \ldots\ldots\ldots\ldots T$$

Constraints on carrying inventory:

$$Q_{it-1}^{I} + \sum_{j=1}^{n} Q_{ijt} - Q_{it}^{I} = \widetilde{D}_{it}, t = 1,2 \ldots\ldots T, i = 1,2 \ldots\ldots m$$

Non-negativity constraints on decision variables:

$$Q_{ijt}, Q_{it}^{I} \geq 0$$

4.1 Proposed Fuzzy Mathematical model

Multi objective model is constructed for aggregate production planning. The objectives are determined by fuzzy parameters to achieve the aspiration level of the decision maker. Upper bound, Lower bound of the objectives is calculated by solving the objectives separately for maximum and minimum condition. Fuzzy right hand side constraint is determined by triangular numbers to achieve the aspiration level. Fuzzy triangular numbers for demands are (950, 1000, 1100), (1000, 1050, 1150), (1050, 1100, 1200), (450, 500, 600), (550, 600, 700) and (500, 550, 600). Then determine the linear membership for fuzzy goals and constraints. The model is solved as (9) to satisfy the decision maker aspiration level. It is illustrated through a real world data.

Multi-flex lami-print Limited is a leading flexible packaging company in India. It manufacture flexible packaging materials by using the input raw materials like Packaging grade polyester film, Biaxially oriented poly propylene [BOPP], polyethylene film, Aluminum foil, Inks, Adhesives, Solvents etc. Flexible packaging materials are flexible in nature. These flexible can be used for packaging any products in liquid form, solid and powder form. Specifications are best suitable to the product to be packed. It manufactures five different products (Packaging materials in different composition for packaging various products) namely (i) 12 mic. Polyester/40mic. Polyethylene for shampoo, Tea, Salt, Oil etc (P1). (ii)12mic. Polyester/12mic.Metallised polyester/40mic. Polyethylene for shampoo, supari etc (P2) .Products are manufactured in two different types of machines for machine line of printing, Lamination and Slitting processes namely High speed advanced machine (M1) ,High speed machine (M2). Table 1 represent the solution for fuzzy additive model and augmented max-min model and Table 2 represent the solution for proposed fuzzy model.

Maximize

$$\lambda + (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8)/8$$

Such that

$$\lambda \leq \lambda_1$$

$$\lambda \leq \lambda_2$$

$$\lambda \leq \lambda_3$$

$$\lambda \leq \lambda_4$$

$$\lambda \leq \lambda_5$$

$$\lambda \leq \lambda_6$$

$$\lambda \leq \lambda_7$$

$$\lambda \leq \lambda_8$$

$$\lambda_1 \leq (481816.7 - (\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}[\widetilde{C}_{ijt} + \widetilde{C}_{it}^{M} + \widetilde{C}_{ijt}^{L} + \widetilde{C}_{ijt}^{W}]Q_{ijt} + \widetilde{C}_{it}^{I} Q_{it}^{I})) / 113060.5$$

$$\lambda_2 \leq ((\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{S}_{ijt} Q_{ijt}) - 268950) / 289200$$

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}[\widetilde{C}_{ijt} + \widetilde{C}_{it}^{M} + \widetilde{C}_{ijt}^{L} + \widetilde{C}_{ijt}^{W}]Q_{ijt} + \widetilde{C}_{it}^{I} Q_{it}^{I} \leq 1 * z^{U}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{S}_{ijt} Q_{ijt} \geq 0.8 * z^{U}$$

$$\lambda_3 \leq (1100 - (100 + p111 + p121 - i11))/100$$

$$\lambda_3 \leq ((100 + p111 + p121 - i11) - 950)/50$$

$$100 + p111 + p121 - i11 \geq 0.75 * z^{U}$$

$$\lambda_4 \leq (1150 - (i11 + p112 + p122 - i12))/100$$

$$\lambda_4 \leq ((i11 + p112 + p122 - i12) - 1000)/50$$

$$i11 + p112 + p122 - i12 \geq 0.75 * z^{U}$$

$$\lambda_5 \leq (1200 - (i12 + p113 + p123 - i13))/100$$

$$\lambda_5 \leq ((i12 + p113 + p123 - i13) - 1050)/50$$

$$i12 + p113 + p123 - i13 \geq 0.75 * z^{U}$$

$$\lambda_6 \leq (600 - (50 + p211 + p221 - i21))/100$$

$$\lambda_6 \leq ((50 + p211 + p221 - i21) - 450)/50$$

$$50 + p211 + p221 - i21 \geq 0.75 * z^{U}$$

$$\lambda_7 \leq (700 - (i21 + p212 + p222 - i22))/100$$

$$\lambda_7 \leq ((i21 + p212 + p222 - i22) - 550)/50$$

$$i21 + p212 + p222 - i22 \geq 0.75 * z^{U}$$

$$\lambda_8 \leq (650 - (i22 + p213 + p223 - i23))/100$$

$$\lambda_8 \leq ((i22 + p213 + p223 - i23) - 500)/50$$

$$i22 + p213 + p223 - i23 \geq 0.75 * z^{U}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} m_{ij} Q_{ijt} \leq m_t^{\max}, t = 1,2 \ldots\ldots\ldots\ldots T$$

$$Q_{ijt}, Q_{it}^{I} \geq 0$$

$$\lambda_i \in (0,1) \qquad (9)$$

Table 1: Solution for Fuzzy Additive model and augmented max-min model

	Fuzzy Additive model	Augmented Max-min model
$Min\ z_1$ $[481816.7, 368756.2]$	368760.6	369693
$Max\ z_2$ $[558150, 268950]$	444588.4	445993.8
$\mu_1(c_1 x)$	0.9999	0.9917
$\mu_2(c_2 x)$	0.6073	0.6122
$\mu_1(a_1 x)$	1	1

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$\mu_2(a_2x)$	1	1
$\mu_3(a_3x)$	1	0.3333
$\mu_4(a_4x)$	1	1
$\mu_5(a_5x)$	1	1
$\mu_6(a_6x)$	1	1
p111,p121,p211, p221 p112,p122,p212, p222 p113,p123,p213, p223	900, 0, 116.875, 527.5 1051, 0, 405.625, 0 3.999, 1095, 550, 0	900, 0, 509.51, 3.98 0.90, 1050.86, 536.50, 0 9.188, 1078.66, 554.92, 0
i11,i12,i13 i21,i22,i23	0, 1, 0 194.38, 0, 0	0, 1.76, 22.95 63.49, 0, 4.92

Table 2: Solution for Proposed Fuzzy model

	Proposed Fuzzy model
$Min\ z_1$ $[481816.7, 368756.2]$	399497.1
$Max\ z_2$ $[558150, 268950]$	479517.2
$\mu_1(c_1x)$	0.7281
$\mu_2(c_2x)$	0.7281
$\mu_1(a_1x)$	1
$\mu_2(a_2x)$	1
$\mu_3(a_3x)$	1
$\mu_4(a_4x)$	1
$\mu_5(a_5x)$	1
$\mu_6(a_6x)$	1
p111,p121,p211,p221 p112,p122,p212,p222 p113,p123,p213,p223	900,0,46.79, 620.93 0,1360.47, 382.27,0 4,1095, 550,0
i11,i12,i13 i21,i22,i23	0, 310.47, 0 217.73, 0, 0

Here achievement level for fuzzy maximization objective is greater than 0.75, the achievement level for fuzzy minimization objective is equal to one and fuzzy constraint for demand level is greater than 0.75. The solution obtained by both objective and constraint are satisfied by the decision maker. The achievement level obtained by the proposed method is a balanced solution and it is greater than other methods.

5. Conclusion:

Fuzzy linear programming (FLP) is simple and suitable tool for multi objective problems compared to other methods. The model can be extended to any number of objectives by incorporating only one additional constraint in the constraint set for each additional objective function. The tolerances are introduced by decision maker to accommodate the vagueness. By adjusting these tolerances a range of solution with different aspiration levels are obtained from which decision maker chooses one that best meets his satisfactory level within given tolerances. The proposed model gives preference to the decision maker achievement level. It can be employed for any number of fuzzy goal and fuzzy constraints. FLP can effectively handle the vagueness and imprecision of input data and the varying importance of criteria in production management model.

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