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Cost Optimization in Manufacturing of toys using Linear Programming

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ABSTRACT *Cost has been always a major concern in all human activities. It receives more and more importance and attention as long as society and technology advances. Concepts of cost are universally applicable. A visitor in strange place or in a distant country can bargain the prices by using simple economic principles. Cost is a simple phenomenon but a harsh fact which is understood everywhere in the world. This paper addresses the issues related to application of linear programming in cost optimization of toys especially in the area of manufacturing. This paper also discusses the methodology for optimal resource allocation and cost-effective toy production which would be beneficial for both people and other enterprise environments.*

I. INTRODUCTION TO COST OPTIMIZATION

The recognition of alternatives and decision, constitute optimization. Optimization can be qualitative (judge by human preference) or quantitative (detected by exact mathematical means). Optimization in a general sense involves the determination of a highest or lowest value over some range.

The toy is manufactured at minimum possible cost and sold at maximum possible price. This invokes managers to go deep down in the ocean of the cost and come up with a notion of the cost in general and optimal cost in particular. In advancing technology of today an accurate estimate is necessary to win and compete with others. An under estimate is a threat to the survival of company itself. Economic optimisation where qualitative factors are introduced in quantitative way, for example a toy designer cannot ignore aesthetics and beauty during engineering optimization of toy design for maximum reliability.

II. LINEAR PROGRAMMING METHODOLOGY

Suppose that Bob, Alice and Harry are three friends. They own a toy manufacturing machine M_1, M_2 and M_3 respectively and installed in their own shop in a market. After passage of time they evolved an empirical relationship that they mostly receive two types of toys i.e. "A" and "B" respectively, their machine capacity and number of products produced X_1 of type A and X_2 of type B are governed by following equations:

Toy Machine M_1 $X_1+2X_2<720$ Eq.1
M_2 $2X_1+X_2< 780$...... Eq.2
M_3 $X_1 < 320$........ Eq.3

The inspection of these equations show that individual machine can be run in full capacity just by adjusting X_1 and X_2 in such a manner that satisfy three equations separately. The inspection of first relation shows that chances of full capacity utilization is more for machine M_1 if X_2 is produced more as compared to X_1.The maximum number of X_2 can be produced is 360 and this is possible when production of X_1 is nil.

The inspection of second equation shows that X_1 should be produced more as compared to X_2.The maximum number of X_1 can be produced is 390 and this is possible when production of X_2 is nil.

The inspection of third equation shows that X_1 should be produced up to maximum limit irrespective of X_2.The maximum number of X_1 can be produced is 320 and has no bearing on the production of X_2.

To have a congenial relationship it is obligatory upon the syndicate that every toy machine owner should get some job. The ideal condition will be that the entire machine runs at full capacity.

Now it is to be examined that, what is maximum production capacity of a particular product of each machine, only then decision can be taken regarding fixation of number of units produced of product X_1 and X_2 by any machine as shown in Table 1.

Table 1 Maximum Production capacity

Maximum Production Capacity Of a Machine	Product X_1	Product X_2
M_1	720	360
M_2	390	780
M_3	320	X_2 does not need any operation on machine M_3

An entrepreneur entered in the market and said that he has capacity to sell as much as all the three could produce. Entrepreneur realizes from his business experience that his profit is guided by the relation

$Z=60X_1 + 40X_2$

Naturally entrepreneur will adjust the value of X_1 and X_2 in such a manner that his profit is maximum. Such situation will happen when production of X_1 and X_2 happens to full capacity, as he has said that he has potential to sell any number of units X_1 and X_2 produced. Thus problem reduces to adjustment of production X_1 and X_2.

Approach to the solution is very simple. There are three equations and unknowns are only two. So any two equations will give the value of unknowns but these values have to satisfy the third equation. So the strategy will be, determine the value of X_1 and X_2 and test these values in third equation. Now two things will happen either the values of X_1 and X_2 will satisfy the third equation or third relation is violated. If third relation is satisfied then solution will be called a feasible solution for the given set of constraint equation.

If third relation is not satisfied then value of X_1 and X_2 should be so adjusted that third relation is satisfied.

Since constraint equations are three and unknowns are two i.e. three pairs of simultaneous equation, will be formed. Solution of these equations will give 3 sets of X_1 and X_2 and all these three sets are to be tested and modified on third relation which was not the part of the pair. The extremum values of X_1 and X_2 are adopted that will keep all the three machines working.

Now let us have look on the graphical representation of the given equations.

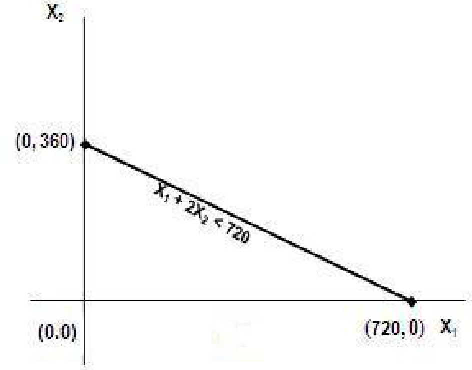

Fig1. Graphical representation of Eq. 1

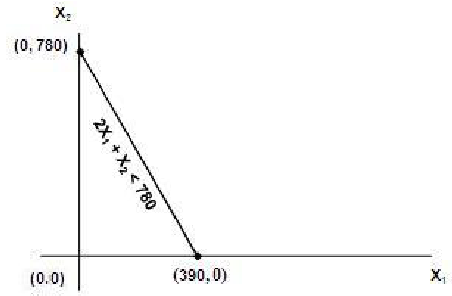

Fig2. Graphical representation of Eq. 2

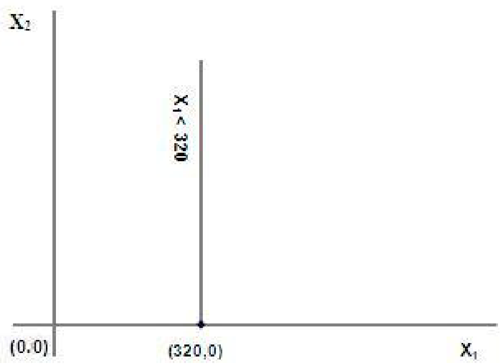

Fig3. Graphical representation of Eq. 3

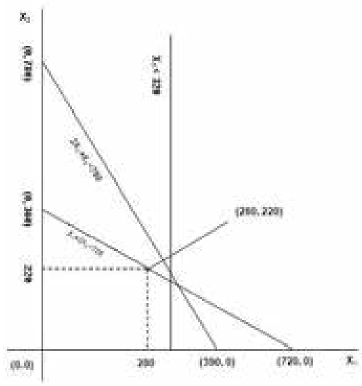

Fig 4 Extremum solution at point of Intersection

III. FEASIBLE SOLUTION

Solution of Equation 1 and 2 yields

$X_1+2X_2 =720$

$2X_1+X_2 =780$

$2X_1 + 4X_2 =1440$

$-2X_1 - X_2 = -780$

$3X_2 = 660$

$X_2 =220$

$X_1=280$ $X_2=220$ (First Set)

In first set X_1 is less than 320 so third constraint is satisfied Now find the value of X_1 and X_2 with the help of equation 2 and 3

$2X_1 + X_2 =780$

$X_1 =320$

This yields $X_1=320$ $X_2=140$

Now test these values on equation 1

$X_1+2X_2 =720$

L H S

320 + 280

600

L H S is less than 720. So first constraint is satisfied Thus $X_1=320$ and $X_2=140$ is also a feasible solution Now work on equation 3 and 1

$X_1 =320$

$X_1 + 2X_2 =720$

This yields $X_1= 320$ and $X_2= 200$

Test these values on equation 2

$2X_1 + X_2 =780$

This yields L H S

+ 200

Which is more than780. Thus this solution is not a feasible solution.

Now the value of X_1 and X_2 so adjusted that II constraint is satisfied. Now whatever solution we have got analytically that can also be obtained graphically by superimposing Fig. 1 and 2, Fig 2 and 3, Fig 3 and 1. The point of intersection gives an extremum solution as shown in Fig.4. Intersection of two lines is a pointer of satisfaction of two constraints.

If all the three constraints intersect at one point i.e. all the constrains have been satisfied this will be the ideal situation.

Every constraint equation divides the first quadrant into two parts one is triangular which can be called intra the line or below the line. Another part is trans the line or across the line. What these lines tell that Solution Point is below this line. Every line will form a right angle triangle with X and Y axes as base, height and hypotenuse. The hypotenuse is the line of constraint. On joining line of constraint a polygon will be formed and feasible solution will lie inside the polygon which has X and Y axis as two sides and rest of the side will represent constraints. The polygon thus formed having minimum area will contain feasible solution. The co-ordinates of any point falling in this region is a feasible solution. Now the point of vertices will give optimum solution which will maximize the objective function. The set which maximizes the objective function is the optimal solution.

With the advent of computers what is desired is an iterative solution of simultaneous linear equation in the case of linear programming. There is no dearth of iterative methods but this solution does only satisfy the simultaneous equations. But in case of linear programming we need a solution which maximizes or minimizes the objective function or one can say small and above the simultaneous linear equation one more equation with one more variable (profit, revenue, cost etc.) is given that has to be satisfied and value of this variable is not known before hand .

Up till now we were assuming that machines should be utilized to full capacity i.e. idle time is zero. Now give a different thought that every machine has idle time represented by S_1, S_2, S_3 then constraint equation reduces to

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$X_1 + 2X_2 + S_1 = 720$
$2X_1 + X_2 + S_2 = 780$
$X_1 + S_3 = 320$

Now three more variables have been added. These variables are called slack variable. Value of these slack variables have to be determined at subsequent stages. Now three things will happen with the values of S_1, S_2, S_3.

First possibility if the value of slack variable is positive i.e. machine has spare capacity or has idle time.

Second possibility if the value of slack variable is zero i.e. machine is running at its full production potential. This is ideal condition for any particular machine.

Third possibility is value of slack variable is negative i.e. jobs are in waiting and machine hours are short. In this situation either reduce the number of jobs or increase number of machines. In this particular case number of machines cannot be increased, so we are forced to reduce number of jobs.

Finally we are in search of those values of X_1 and X_2 which will satisfy constraint equations as well as maximize the objective function. Since equations are linear so solution is not very tedious. Since computer lacks wisdom and common sense most of the time. So it is necessary to develop a system on the basis of which computer can do the work. Computer is very happy with iterative work. So let us develop an iterative method. The first step in this direction is to write inequality in the form of equality equation. This can be done by adding variables S_1, S_2 and S_3 on LHS from the pocket. These artificially introduced variables are called slack variable.

$Z = 60X_1 + 40X_2$
$X_1 + 2X_2 + S_1 = 720$………….. Eq.4
$2X_1 + X_2 + S_2 = 780$………… .Eq.5
$X_1 + S_3 = 320$……………… .Eq.6

IV. CONCLUSION

For manufacturing of toy, cost optimization is important factor because if toy is economical and reliable, it's sales market would be good and also children of middle class family can play and enjoy the toy and will not feel deprived. Therefore, this paper suggests a method of linear programming for optimal resource allocation and cost-effective toy production which would be beneficial for both people and other enterprise environments. On inspection of equations 4, 5 & 6 if one unit of X1 and X2 are to be produced simultaneously then resource needed are S1 three units, S2 three units and S3 one unit and in turn will yield ₹100 as profit.

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