

Effects of Heat Source on MHD flow of an Elasto-viscous Fluid through a Porous Medium in Presence of Chemical Reaction

KEYWORDS	Free convection, MHD, mass transfer, elasto-viscous, heat source and chemical reaction		
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ABSTRACT The magnetic effect on unsteady free convective hydrodynamic elasto-viscous fluid past an infinite vertical			

plate through porous medium in the presence of heat source and chemical reaction is studied while considering mass transfer .The dimensionless governing equations are solved in closed form by Laplace-transform technique. The results are obtained for velocity, temperature and concentration for different parameters like phase angle, chemical reaction parameter, Schmidt number, time and magnetic number. The flow characteristics are presented by graphs.

1.INTRODUCTION

Free convection flows are of great interest in number of industrial applications such as granular insulation, geothermal system etc .MHD has attracted the attention of many scholars due to its diverse application in geophysics and astrophysics. It is applied to study the stellar and solar structure, interstellar matter, radio propagation through the ionosphere etc. The phenomena of mass transfer is very common in theory of stellar structure ,burning a pool of oil, spray drying ,adsorption, leaching and mass transport process in animal and plant life. The effect of chemical reaction depends whether the reaction is homogeneous or heterogeneous, the rate of reaction depends on the concentration of species itself . The heat source/sink effects in thermal convection, are significant where there may exist a high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reactions.

Mohapatra and Senapati [1] have been analyzed magneto hydrodynamic free convection flow with mass transfer past a vertical plate. Biswal, and Sahoo [2]have studied the Hall effect on oscillatory hydromagnetic free convective flow of a visco-elastic fluid past an infinite vertical porous flat plate with mass transfer. Soundalgekar et.al.[3]have studied MHD flow past a vertical oscillating plate . Kim [4] discussed unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Muthucumaraswamy et al[5] have analyzed the effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate . Two-dimensional MHD oscillatory flow along a uniformly moving infinite vertical porous plate bounded by porous medium is presented by Ahmed et al[6]. Muthucumaraswamy et.al[7] have studied mass transfer effect on exponentially accelerated isothermal vertical plate.Senapati et.al[8] have studied magnetic effect on mass and heat transfer of a hydrodynamic flow past a vertical oscillating plate in presence of chemical reaction. Senapati et.al[9,10] also discussed the chemical effects on mass and heat transfer on MHD free convection flow of fluids in vertical plates and in between parallel plates for slip flow regions and poiseuille flow respectively.

This paper deals with the study of effects of heat source on MHD flow of an elasto-viscous fluid through a porous medium in presence of chemical reaction.

2. FORMULATION OF PROBLEM

The unsteady free convection and mass transfer in an electri-

cally conducting incompressible elasto-viscous fluid past an infinite vertical plate through porous medium in the presence of heat source and chemical reaction has been considered. The X'-axis is taken along the plate in vertical upward direction and Y'-axis is taken normal to it. A magnetic field of uniform strength H₀ is applied normal to the plate .Initially the plate and surrounding fluid are at rest and are in same temperature $T_{\infty}^{'}$ and having mass concentration $C_{\infty}^{'}$ at all points. At time t'>0, the plate is exponentially accelerated with a velocity u'=uOe^{a't'}) in its own plane and the plate temperature and the level of concentration are raised linearly with time t near the plate. The effect of viscous dissipation is assumed to be negligible. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial z'} = g\beta(T' - T_{\infty}') + g\beta_c(C' - C'_{\infty}) + v \frac{\partial^2 u'}{\partial v'^2 \partial z'} - \frac{\sigma B_s' u'}{\sigma} - \frac{v u'}{\kappa'}$$
(1)

$$C_p \frac{\delta T'}{\delta t'} = k \frac{\delta^2 T'}{\delta w^2} + Q(T' - T_{\infty}') \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K_l(C' - C'_{\infty})$$
(3)

where electromagnetic induction $B_0 = H_0 \mu_e$

Using following initial and boundary conditions

$$t' \leq 0 : u' = 0, T' = T'_{w}, C' = C'_{w} \text{ for all } y'$$

$$t' \geq 0 : u' = u_0 e^{u't'}, T' = T'_{w} + (T'_w - T'_w)At', C' = C'_w + (C'_w - C'_w)At' \text{ at } y' = 0$$

$$u' \rightarrow 0, T' \rightarrow T'_{w}, C' \rightarrow C'_{w} \text{ as } y' \rightarrow \infty$$
(4)

where $A = \frac{u_0^2}{2}$

On introducing the following non-dimensional quantities

$$u = \frac{u^{i}}{u_{0}}, t = \frac{t^{i}u_{0}^{2}}{v}, y = \frac{y^{i}u_{0}}{v}, \theta = \frac{T^{i} - T_{m}^{i}}{T_{m}^{i} - T_{m}^{i}}, Gr = \frac{\beta\beta v(T_{m}^{i} - T_{m}^{i})}{u_{0}^{2}}, C = \frac{C^{i} - C_{m}^{i}}{C_{\mu}^{i} - C_{m}^{i}},$$

$$, Gm = \frac{\beta\beta_{v}v_{i}^{i}c_{v}^{i} - c_{m}^{i}}{u_{v}^{2}}, Pr = \frac{\mu\xi_{2}}{k}, Sc = \frac{v}{\theta}, M = \frac{a\pi_{v}^{k}v}{\mu u_{v}^{2}}, F = \frac{qv^{k}}{ku_{v}^{k}}, S = \frac{\kappa_{2}u_{0}^{k}}{\mu v^{2}},$$

$$K = \frac{u_{0}^{k}x_{i}^{i}}{v^{k}}, a = \frac{av}{u_{0}^{k}}, R = \frac{\kappa_{i}v}{u_{0}^{k}},$$
(5)

where D is mass diffusion, Gr is Grashof number ,Gm modified Grashof number, M magnetic number, Pr is prandtl number, Sc is Schmidt number , β is thermal expansions co-efficient , β_c is concentration expansion co-efficient, R is chemical reaction parameter, F is source /sink parameter, S is elastic parameter and a is exponential parameter.

With the help of equation (5) the equations (1) to (3) reduce to

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$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - S \frac{\partial^3 u}{\partial y^2 \partial t} - Mu - \frac{u}{\kappa} + Gr\theta + GmC \quad (6)$$

$$Pr\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} - F\theta \tag{7}$$

$$Sc\frac{\partial c}{\partial t} = \frac{\partial^2 c}{\partial y^2} - ScRC \tag{8}$$

The initial and boundary conditions in non-dimensional form are

$$t \le 0 : u = 0, \theta = 0, C = 0 \text{ for all } y$$

$$t > 0 : u = e^{at}, \theta = t, C = t \quad at \ y = 0$$

$$u \to 0, \theta \to 0, C \to 0 \quad as \ y \to \infty$$
(9)

3. METHOD OF SOLUTION

To find the solution of equations (6) to (8) under initial and boundary condition (9) we have to use of equation $u=U_0+SU_1$ in equation (6) and then by the Laplace Transform technique, we get following equations

$$\frac{d^2 \overline{U_0}}{dy^2} - \left(M + \frac{1}{K} + p\right) \overline{U_0} = -Gr\bar{\theta} - Gm\bar{C} \quad (10)$$

$$\frac{d^2 \overline{U_1}}{dy^2} - \left(M + \frac{1}{K} + p\right) \overline{U_1} = -\frac{d^2 \overline{U_0}}{dy^2} \quad (11)$$

$$\frac{d^2\bar{\theta}}{dy^2} - (F + Prp)\bar{\theta} = 0$$
(12)

$$\frac{d^2\bar{C}}{dy^2} - (ScR + Scp)\bar{C} = 0$$
(13)

with the boundary conditions

$$\overline{U_0} = \frac{1}{p-a}, \overline{U_1} = 0, \overline{\theta} = \frac{1}{p^2}, \overline{C} = \frac{1}{p^2} \quad at \quad y = 0$$

$$\overline{U_0} \to 0, \overline{U_1} \to 0, \overline{\theta} \to 0, \overline{C} \to 0 \quad as \quad y \to 0$$

$$(14)$$

where p is the Laplace transform parameter.

By solving equations (10) to (13) using (14) we get

$$\tilde{\theta} = \frac{e^{-\sqrt{2\tau p + 2\gamma}}}{p^2}$$
(15)

$$\widetilde{C} = \frac{1}{p^{2}}$$

$$\widetilde{U}_{0} = \left[\frac{1}{p^{-\alpha}} + \frac{Cr}{(Pr-1)(p-c)} + \frac{Cm}{(Sc-1)(p-b)}\right]e^{-\sqrt{N'+py}}$$
(16)

$$-\frac{6r}{(p_{r-1})(p-c)}e^{-\sqrt{p_{rp}+p_{y}}}-\frac{6m}{(se_{r-1})(p-b)}e^{-\sqrt{se_{rp}+RSe_{y}}}$$
(17)

$$\overline{U}_{1}^{c} = \left[\frac{Gr(J^{c}p + F)}{(Pr - 1)^{2}(p - c)^{2}} + \frac{Gm(ScR + Scp)}{(Sc - 1)^{2}(p - b)^{2}}\right] e^{-\sqrt{M' + py}} \\ + \frac{y}{2\sqrt{M' + p}} \left[\frac{M' + p}{p - a} + \frac{Gr(M' + p)}{(Pr - 1)(p - c)} + \frac{Gm(M' + p)}{(Sc - 1)(p - b)}\right] e^{-\sqrt{M' + py}} \\ - \frac{Gr(Prp + F)}{(Pr - 1)^{2}(p - c)^{2}} e^{-\sqrt{(Prp + F)}y} - \frac{Gm(ScR + Scp)}{(Sc - 1)^{2}(p - b)^{2}} e^{-\sqrt{(ScR + Scp)}y}$$
(18)

where
$$M' = M + \frac{1}{\kappa}$$
, $c = \frac{M' + \delta}{Pr - 1}$, $b = \frac{M' + \delta R}{\delta c - 1}$
king the inverse Laplace transform of Equat

Taking the inverse Laplace transform of Equation (15) to (18), we get the following solutions

$$\theta = \frac{\epsilon}{2} \left[\exp\left(-2\eta\sqrt{Ft}\right) \operatorname{erfc}\left(\eta\sqrt{pr} - 2\sqrt{\frac{Ft}{pr}}\right) + \exp\left(2\eta\sqrt{Ft}\right) \operatorname{erfc}\left(\eta\sqrt{pr} + 2\sqrt{\frac{Ft}{pr}}\right) \right]$$

$$- \frac{gFr}{2}\sqrt{\frac{F}{r}} \left[\exp\left(-2\eta\sqrt{Ft}\right) \cdot \operatorname{erfc}\left(\eta\sqrt{Fr} - 2\sqrt{\frac{Ft}{Pr}}\right) - \exp\left(2\eta\sqrt{Ft}\right) \cdot \operatorname{erfc}\left(\eta\sqrt{Fr} + 2\sqrt{\frac{Ft}{Pr}}\right) \right] \qquad ... (19)$$

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 $C = \frac{t}{c} \left[\exp\left(-2\eta\sqrt{ScRt}\right) \cdot erfc\left(\eta\sqrt{Sc} - \sqrt{tR}\right) + \exp\left(2\eta\sqrt{ScRt}\right) \cdot erfc\left(\eta\sqrt{Sc} + \sqrt{tR}\right) \right]$

 $-\frac{\eta}{2}\left[\frac{tSc}{2}\left[\exp\left(-2\eta\sqrt{ScRt}\right), erfc(\eta\sqrt{Sc}-\sqrt{tR})-\exp\left(2\eta\sqrt{ScRt}\right)erfc(\eta\sqrt{Sc}+\sqrt{tR})\right]\right] (20)$

 $u = \left| \frac{1}{2} e^{at} \left(exp \left(-2\eta \sqrt{t(M'+a)} \right) . erfc \left(\eta - \sqrt{t(M'+a)} \right) + exp \left(2\eta \sqrt{t(M'+a)} \right) . erfc \left(\eta - \sqrt{t(M'+a)} \right) . erfc \left$

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\sqrt{t(M'+a)}+
     \frac{cr}{2(nr-1)}e^{ct}\left(exp\left(-2\eta\sqrt{t(M'+c)}\right).erfc\left(\eta-\sqrt{t(M'+c)}\right)+exp\left(2\eta\sqrt{t(M'+c)}\right).erfc\left(\eta+\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(nr-1)}\right)e^{ct}\left(\frac{cr}{2(
     \sqrt{t(M'+c)}+
     \frac{Gm}{2(5c-1)}e^{bt}\left(exp\left(-2\eta\sqrt{t(M'+b)}\right).erfc\left(\eta-\sqrt{t(M'+b)}\right)+exp\left(2\eta\sqrt{t(M'+b)}\right).erfc\left(\eta+\frac{Gm}{2(5c-1)}\right)\right)
     \sqrt{t(M'+b)}
     \frac{cr}{2(p-z)}e^{et}\left(exp\left(-2\eta\sqrt{t(F+Prc)}\right).erfc\left(\eta\sqrt{Pr}-\sqrt{\frac{p(F+Prc)}{Pr}}\right)+\right)
     exp\left(2\eta\sqrt{t(F + Prc)}\right). erfc\left(\eta\sqrt{Pr} + \sqrt{\frac{t(F+Prc)}{Pr}}\right)
     \frac{Gm}{2(\pi c_{-1})}e^{bt}\left(exp\left(-2\eta\sqrt{t(ScR+bSc)}\right).erfc\left(\eta\sqrt{Sc}-\sqrt{t(R+b)}\right)+\right)
     exp\left(2\eta\sqrt{t(ScR+bSc)}\right).erfc\left(\eta+\sqrt{t(R+b)}\right)
       +S \left[\left(\frac{GrPr}{2(Pr-1)^6} + \frac{rGr(F+Pre)}{(Pr-1)^6}\right)\left(e^{-2\eta\sqrt{t(M'+e)}}erfc\left(\eta - \sqrt{t(M'+c)}\right) + e^{2\eta\sqrt{t(M'+e)}}erfc\left(\eta + e^{-2\eta\sqrt{t(M'+e)}}erfc\left(\eta + e^{-2\eta\sqrt{t(M'+e)}}\right)\right)\right]
       \sqrt{t(M'+c)}
     \left(\frac{\eta\sqrt{i}Cr(P+Pre)e^{\theta}}{2(Pr-1)^{\theta}\sqrt{N'+e}}-\frac{\eta\sqrt{i}Cr\sqrt{N'+e}}{2(Pr-1)}\right)\left(e^{-2\eta\sqrt{i}(N'+e)}erfc\left(\eta-\sqrt{t(M'+e)}\right)-e^{2\eta\sqrt{i}(N'+e)}erfc\left(\eta+\frac{1}{2}e^{-2\eta\sqrt{i}(N'+e)}\right)\right)
     \sqrt{t(M'+c)}+
   \left(\frac{Gm(ScR+Scb)t}{2(Sc-1)^8} + \frac{GmSc}{2(Sc-1)^8}\right)\left(e^{-2\eta\sqrt{t(M'+b)}}erfc\left(\eta - \sqrt{t(M'+b)}\right) + e^{-2\eta\sqrt{t(M'+b)}}\right)
 e^{2\eta\sqrt{t(M'+b)}}e^{rfc}\left(\eta+\sqrt{t(M'+b)}\right)\right) - \left(\frac{\eta\sqrt{t}Gm(btB+ttb)e^{H}}{2(to-1)^{2}\sqrt{t'+b}} - \frac{\eta\sqrt{t}Gm\sqrt{M'+b}}{2(to-1)}\right)\left(e^{-2\eta\sqrt{t(M'+b)}}e^{rfc}\left(\eta-t(t)-t(t)-t(t)\right)\right)
   \sqrt{t(M'+b)} - e^{2\eta\sqrt{t(M'+b)}}erfc\left(\eta + \sqrt{t(M'+b)}\right) + \left(\eta e^{-M!t} + \frac{\eta dre^{-M!t}}{(Br-1)} + \frac{\eta dre^{-M!t}}{(Sr-1)}\right) \frac{e^{-\eta}}{\sqrt{\pi}}
 \left(\frac{\eta\sqrt{t(M'+a)}}{a^{at}}\right)\left(e^{-2\eta\sqrt{t(M'+a)}}erfc\left(\eta-\sqrt{t(M'+a)}\right)-e^{2\eta\sqrt{t(M'+a)}}erfc\left(\eta+\sqrt{t(M'+a)}\right)\right)+
 \frac{\eta \sqrt{2\beta \tau \phi r} \sqrt{\beta \theta + 2\beta r}}{z(pr-1)^2} e^{zt} \left( e^{-2\eta \sqrt{x(\beta + c\beta r)}} erfc\left(\eta \sqrt{\beta r} - \frac{\sqrt{z(\beta + c\beta r)}}{\beta r}\right) - e^{2\eta \sqrt{x(\beta + c\beta r)}} erfc\left(\eta \sqrt{\beta r} + \frac{\sqrt{z(\beta + c\beta r)}}{\beta r}\right) \right) - e^{2\eta \sqrt{x(\beta + c\beta r)}} e^{-2\eta \sqrt{x(\beta + c\beta
 \frac{drPr-2ddr(Pre+F)}{2(Pr-1)^2}e^{rt}\left(e^{-2\eta\sqrt{n(P+cPr)}}erfc\left(\eta\sqrt{Pr}-\frac{\sqrt{n(P+cPr)}}{2r}\right)+e^{2\eta\sqrt{n(P+cPr)}}erfc\left(\eta\sqrt{Pr}-\frac{\sqrt{n(P+cPr)}}{2r}\right)\right)
 (F(F+cFr)))+
\frac{p_{F}}{\eta(2tcom/5d+50)}e^{be}\left(e^{-2\eta\sqrt{t(5cR+5cb)}}e^{\eta f}c\left(\eta\sqrt{5c}-\sqrt{t(R+b)}\right)-e^{2\eta\sqrt{t(5cR+5cb)}}e^{\eta f}c\left(\eta\sqrt{5c}+\frac{1}{2}\right)\right)
   \sqrt{t(R+b)} -
 \frac{a_{mSC-25}a_{m}(z_{\ell}x_{\ell}+z_{\ell}b)}{2(z_{\ell}-z)^{2}}a^{bc}\left(a^{-2\eta\sqrt{\gamma(5\,\ell R+5\,\ell b)}}a_{m}fc\left(\eta\sqrt{5\,c}-\sqrt{t(R+b)}\right)+a^{2\eta\sqrt{\gamma(5\,\ell R+5\,\ell b)}}a_{m}fc\left(\eta\sqrt{5\,c}+\frac{1}{2}a_{m}fc\left(\eta\sqrt{5\,c}+\frac{1}{2}a_{m}fc\left(\eta\sqrt{5\,c}+\frac{1}{2}a_{m}fc\left(\eta\sqrt{5\,c}+\frac{1}{2}a_{m}fc\right)\right)\right)+a^{2\eta\sqrt{\gamma(5\,\ell R+5\,\ell b)}}a_{m}fc\left(\eta\sqrt{5\,c}+\frac{1}{2}a_{m}fc\left(\eta\sqrt{5\,c}+\frac{1}{2}a_{m}fc\right)\right)
 \sqrt{t(R+b)}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              (21)
 The non-dimensional shearing stress at the plate from the equations (21) is given by
 \tau = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} =
 \frac{1}{2}e^{at}\left[2\sqrt{t(M'+a)}\left(erfc(\sqrt{t(M'+a)}-erfc(-\sqrt{t(M'+a)})\right)+\right.
 \frac{4}{\sqrt{a}} e^{-t(M'+a)} +
\frac{Gr}{2(Pr-1)}e^{ct}\left[2\sqrt{t(M'+c)}\left(erfc(\sqrt{t(M'+c)}-erfc(-\sqrt{t(M'+c)})+\right)\right]
 \frac{4}{\sqrt{\pi}}e^{-t(M'+c)} +
\frac{Gm}{2(G_{m-1})}e^{bt}\left[2,\left|\overline{t(M'+b)}\right|\left(erfc(\left|\overline{t(M'+b)}-erfc(-\left|\overline{t(M'+b)}\right)+\frac{4}{2^{d}}e^{-t[M'+b]}\right|\right)-\frac{4}{2^{d}}e^{-t[M'+b]}\right]
\frac{Gr}{2(Pr-1)}e^{ct}\left[2\sqrt{t(F+Prc)}\left(erfc(\sqrt{\frac{t(F+Prc)}{Pr}}-erfc(-\sqrt{\frac{t(F+Prc)}{Pr}})+\right.\right.
\frac{4\sqrt{Pr}}{\sqrt{\pi}} = e^{\frac{-t(P+Prc)}{Pr}} \left[ -\frac{Gm}{2(Sc-1)} e^{bt} \left[ 2\sqrt{t(ScR+Scb)} \right] \left( erfc(\sqrt{t(R+b)} - \frac{Gm}{2(Sc-1)} + \frac{Gm}{2(Sc-1)} \right) \right]
erfc(-\sqrt{t(R+b)}) + \frac{4\sqrt{5c}}{\sqrt{\pi}} e^{-t(R+b)} +
S\left[\left(\frac{GrPr}{2(Pr-1)^2}+\right)\right]
     \frac{tPr(F+Prc)}{(Pr-1)^2}\left(2\sqrt{t(M'+c)}\left(erfc\left(\sqrt{t(M'+c)}\right)-erfc\left(-\sqrt{t(M'+c)}\right)\right)+\right)
     \frac{4}{\sqrt{\pi}} e^{-t(M'+c)} - \left( \frac{Gr(F+Prc)\sqrt{t}e^{at}}{2(Pr-1)^2\sqrt{M'+c}} - \frac{Gr\sqrt{t(M'+c)}}{2(Pr-1)} \right) \left( erfc \left( -\sqrt{t(M'+c)} \right) - \frac{Gr}{\sqrt{t}e^{at}} \right) + \frac{Gr}{\sqrt{t}e^{at}} \left( -\frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} \right) + \frac{Gr}{\sqrt{t}e^{at}} \left( -\frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} \right) + \frac{Gr}{\sqrt{t}e^{at}} \left( -\frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} \right) + \frac{Gr}{\sqrt{t}e^{at}} \left( -\frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} \right) + \frac{Gr}{\sqrt{t}e^{at}} \left( -\frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} \right) + \frac{Gr}{\sqrt{t}e^{at}} \left( -\frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{\sqrt{t}e^{at}} - \frac{Gr}{
     erfc\left(-\sqrt{t(M'+c)}\right)+
     \left(\frac{Gm(ScR+Scb)t}{2(Sc-1)^2}+\right.
     \frac{GmSc}{2(Sc-1)^2}\left(2\sqrt{t(M'+b)}\left(erfc\left(\sqrt{t(M'+b)}\right) - erfc\left(-\sqrt{t(M'+b)}\right)\right)\right) - erfc\left(-\sqrt{t(M'+b)}\right)\right)
     \left(\frac{Gm(ScR+Scb)\sqrt{t}e^{bt}}{2(Sc-1)^2\sqrt{N'+b}}-\frac{Gm\sqrt{t(M'+b)}}{2(Sc-1)}\right)\left(erfc\left(-\sqrt{t(M'+b)}\right)-\frac{Gm\sqrt{t(M'+b)}}{2(Sc-1)}\right)=0
     erfc\left(\sqrt{t(M'+b)}\right) + \left(1 + \frac{Gr}{Pr-1} + \frac{Gm}{Sr-1}\right) \frac{e^{-M/t}}{\sqrt{\pi}} +
```

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$$\begin{split} & \left(\frac{\sqrt{t(M'+a)}}{2}e^{at}\right)\left(erfc\left(-\sqrt{t(M'+a)}\right) - erfc\left(\sqrt{t(M'+a)}\right)\right) + \\ & \left(\frac{PrGr\sqrt{t(F+Prc)}}{2(Pr-1)^2}\right)e^{at}\left(erfc\left(-\frac{\sqrt{t(F+Prc)}}{Pr}\right) - erfc\left(\frac{\sqrt{t(F+Prc)}}{Pr}\right)\right) - \\ & \left(\frac{GrPr-2Gr(Prc+F)}{2(Pr-1)^2}\right)e^{ct}\left(2\sqrt{t(F+Prc)}\left(erfc\left(\frac{\sqrt{t(F+Prc)}}{Pr}\right) - \\ & erfc\left(-\frac{\sqrt{t(F+Prc)}}{Pr}\right)\right) + \frac{4\sqrt{Pr}}{\sqrt{\pi}}e^{-\frac{t(F+Prc)}{Pr^2}}\right) + \\ & \left(\frac{ScGm\sqrt{t(ScR+Scb)}}{2(Sc-1)^2}\right)e^{bt}\left(erfc\left(-\sqrt{t(R+b)}\right) - erfc\left(\sqrt{t(R+b)}\right)\right) + \\ & \left(\frac{GmSc-2tGm(ScR+Scb)}{2(Sc-1)^2}\right)e^{bt}\left(2\sqrt{t(ScR+Scb)}\left(erfc\left(\sqrt{t(R+b)}\right) - \\ & erfc\left(-\sqrt{t(R+b)}\right)\right) + \frac{4\sqrt{Sc}}{\sqrt{\pi}}e^{-t(R+b)}\right) \end{bmatrix} \end{split}$$

The dimensional rate of heat transfer/ Nusselt Number,

$$Nu = -\left(\frac{2\pi}{\delta r}\right)_{q=0} = \frac{1}{2}\left[2\sqrt{Ft}\left(\operatorname{orfe}\left(2\sqrt{\frac{Ft}{pr}}\right) - \operatorname{orfe}\left(-2\sqrt{\frac{Ft}{pr}}\right)\right) - 4\sqrt{\frac{Ft}{\pi}}e^{-4\left(\frac{Ft}{pr}\right)}\right] + \frac{Ft}{2}\sqrt{\frac{Ft}{\pi}}\left(\operatorname{orfe}\left(-2\sqrt{\frac{Ft}{pr}}\right) - \operatorname{orfe}\left(2\sqrt{\frac{Ft}{pr}}\right)\right) \\ \dots (23)$$

The dimensionless rate of mass transfer/ Sherwood Number,

$$Sh = -\left(\frac{\delta \pi}{\delta \psi}\right)_{\eta=0} = \frac{1}{2} \left[2\sqrt{SeRt} \left(orf \varepsilon \left(-\sqrt{Rt} \right) - orf \varepsilon \left(\sqrt{Rt} \right) \right) - 4\sqrt{\frac{f\varepsilon}{\pi}} e^{-Rt} \right] + \frac{1}{2} \sqrt{\frac{f\varepsilon t}{R}} \left(orf \varepsilon \left(-\sqrt{tR} \right) - orf \varepsilon \left(\sqrt{Rt} \right) \right) \right)$$
(24)

where
$$\eta = \frac{\gamma}{2\sqrt{t}}$$

4.Result and Discussion

In this paper we have studied Heat source and mass transfer effects on MHD flow of an elasto-viscous fluid through a porous medium in presence of chemical reaction. The effect of Gr, Gm, M, K, Sc,Pr, F,S,R,a and t on flow characteristics have been studied and shown by means of graphs .

Figure-(1) illustrates the effect of the parameters Pr and Sc on velocity profile (u) at any point of the fluid when M=2,K=2,Gr=2,Gm=2, F=2,S=1,a=1 and R=1. It is noticed that the velocity decreases with the increase of Prandtl number (Pr) and Schmidt number (Sc).

Figure-(2) illustrates the effect of the parameters M and K on velocity profile (u) at any point of the fluid, when Sc=2,Pr=2,Gr=2,Gm=2,F=2,S=1,a=1 and R=1.It is noticed that the velocity increases with the increase of modified magnetic parameter (M) and decreases with the increase of porous parameter (K).

Figure-(3) illustrates the effect of the parameter Gr ,Gm and a on velocity profile (u) at any point of the fluid ,when M=2,K=2,Sc=2,Pr=2, F=2,S=1 and R=1. It is noticed that the velocity increases with the increase of Grashof number (Gr), modified Grashof number (Gm) and exponential parameter(a).

Figure-(4) illustrates the effect of the parameters R, S and F on velocity profile (u) at any point of the fluid when M=2,K=2,Gr=2,Gm=2, Sc=2,a=1 and Pr=2.It is noticed that the velocity increases with the decrease chemical reaction parameter(R) and source /sink parameter(F) and in the increase of elastic parameter(S).

Figure-(5) illustrates the effect of the parameters Sc and Pr on skin friction at the plate ,when M=2,K=2,R=2,F=2,Gr=2,Gm=2 and a=2. It is noticed that the skin friction increases with the increase of Prandtl number (Pr) and Schmidt number (Sc) for t>0.05 .

Figure-(6) illustrates the effect of the parameters M and K on skin friction at the plate ,when Sc=2,Pr=2,R=2,F=2,Gr=2,Gm=2 and a=2. It is noticed that the skin friction decreases with the increase of modified magnetic parameter (M) and increases with the increase of porous parameter (K) for t>0.05.

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Figure-(7) illustrates the effect of the parameters R and F on skin friction at the plate ,when M=2,K=2,Sc=2,Pr=2,Gr=2,Gm=2 and a=2. It is noticed that the skin friction increases with the increase of chemical reaction parameter(R) and source /sink parameter (F) for t>0.05.

Figure-(8) illustrates the effect of the parameters Gr ,Gm and a on skin friction at the plate ,when M=2,K=2,R=2,P=2, and Sc=2. It is noticed that the skin friction decreases with the increase of Grashof number (Gr), modified Grashof number (Gm) and exponential parameter(a) for t>0.05.

Figure-(9) illustrates the effect of the parameters Sc,R, and t on mass concentration in the absence of other parameters. It is noticed that the mass concentration decreases with the increase of chemical reaction parameter (R) and Schmidt number (Sc), whereas increases with the increase of time (t).

Figure-(10) illustrates the effect of the parameters Pr,F, and t on Heat in the absence of other parameters. It is noticed that the heat falls with the increase of source /sink parameter(F), and Prandtl number (Pr),whereas rises with the increase of time (t).

Figure-(11) illustrates the effect of the R and Sc on Sherwood number at the plate ,when t=1 .It is noticed that the Sherwood number increases with the increase chemical reaction parameter (R) and Schmidt number (Sc).

Figure-(12) illustrates the effect of the F and Pr on Nusselt number at the plate ,when t=1 .lt is noticed that the Nusselt number decreases with the increase source /sink parameter(F), and Prandtl number (Pr).

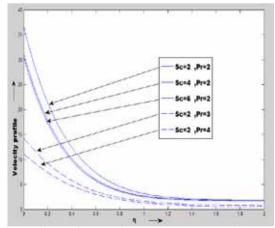


Fig-(1)-Effect of Sc and Pr on velocity profile ,when $M{=}2,K{=}2,Gr{=}2,Gm{=}2$,F=2,S=1,a=1 and R=1

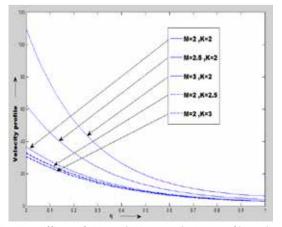


Fig-(2)- Effect of M and K on velocity profile ,when Sc=2,Pr=2,Gr=2,Gm=2 ,F=2,S=1,a=1 and R=1

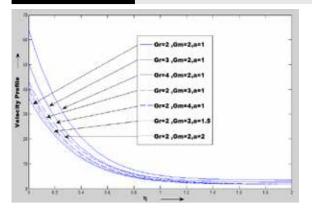


Fig-(3)- Effect of Gr,Gm and a on velocity profile ,when M=2,K=2,Sc=2,Pr=2 ,F=2,S=1 and R=1

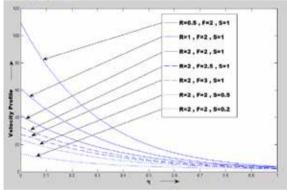


Fig-(4)- Effect of R,S and F on velocity profile ,when $M{=}2,K{=}2,Gr{=}2,Gm{=}2$,Sc=2,a=1 and Pr=2

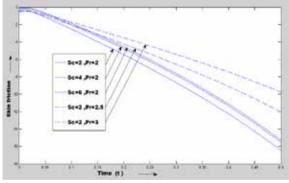


Fig-(5)-Effect of Sc and Pr on skinfriction, when $M{=}2,K{=}2,R{=}2,G{=}2,G{m=}2$ and $a{=}2$

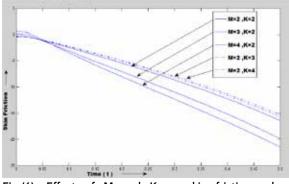


Fig-(6)- Effect of M and K on skin friction, when Sc=2,Pr=2,R=2,F=2,Gr=2,Gm=2 and a=2 $\,$

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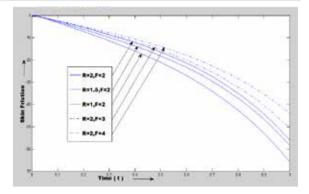


Fig-(7)- Effect of R and F on skin friction ,when M=2,K=2,Sc=2,Pr=2,Gr=2,Gm=2 and a=2

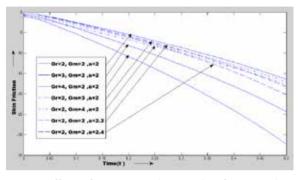


Fig-(8)- Effect of Gr, Gm and a on skin friction, when $M{=}2{,}K{=}2{,}R{=}2{,}P{=}2{,}P{=}2$ and Sc=2

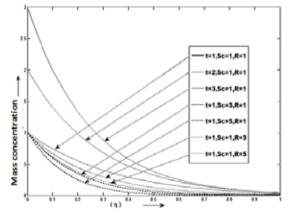


Fig-(9)-Effect of Sc,R, and t on mass concentration in the absence of other parameters

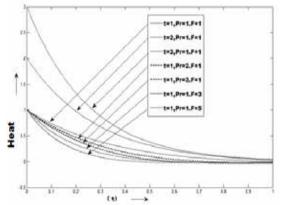


Fig-(10)- Effect of Pr,F, and t on Heat in the absence of other parameters

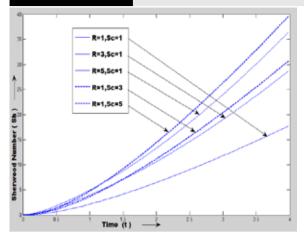


Fig-(11)- Effect of Sc, and R on Sherwood Number in the absence of other parameters

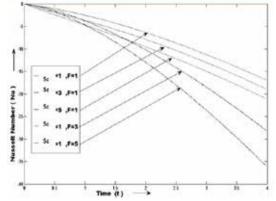


Fig-(12)- Effect of Pr, and F on Nusselt Number in the absence of other parameters

5.conclusions:-

- The velocity increases when increase of convective i. amount of heat and mass concentration, magnetic force dominant over viscus force, whereas it decreases when kinetic viscosity dominant over mass diffusion. Kinetic viscosity dominant over thermal diffusion and increase of void space in porous.
- ii. Enhance of chemical spices and heat source decreases the concentration of mass and heat source/ sink in the fluid.

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