RESEARCH PAPER | Mathematics | Volume : 3 | Issue : 7 | July 2013 | ISSN - 2249-555X

An outline of Vacation Model of Two-Phase Queueing System

KEYWORDS: Busy cycle, Service cycle, Gated service

S.Palaniammal	C.Vijayalakshmi	K. Ramya
Department of Mathematics, Sri Krishna College of Engineering, Coimbatore, India	Department of Mathematics, VIT University, Chennai, India.	Department of Mathematics, Kingston Engineering College, Vellore, India.

ABSTRACT *A two phase service queueing model with gated service vacation. In this gated service vacation model, only those customers who are present in the queue when the server starts a batch mode service are taken into the batch. The service to further arrivals is deferred until the server completes the second phase individual service to the batch.*

1. Introduction

In this paper we study the two phase queueing system with gated service and multiple server vacations. The gated service policy to be considered in the present model under consideration is exceptional in the sense that the server's gated mechanism will last until the system becomes empty after which the server is allowed to take vacations.

In certain situations it is necessary to perform batch mode service to those customers only who are present in the system at the commencement of the batch service. Customers first enter into a waiting room. They are served in the service room. When the service room becomes empty, all the customers in the waiting room are transferred to service room when they served individually. At such epochs of transfer of customers from the waiting room, a random number of overhead customers are also added to the service room.

2. The Model

Consider a single server two phase service vacation queueing system in which customers arrive according to a Poisson process with parameter λ. The customers are served in batch mode at the first phase following the gated service policy.

On the completion of the batch mode service, the server starts servicing the customers of the same batch following the first in first out queue discipline. After exhausting the batch the server returns immediately to the first phase and starts the next gated batch service provided at least one customer waiting in the queue.

On the other hand, if the system is empty the server starts taking a sequence of vacations. The vacation sequence is terminated only when the system is non-empty at the end of vacation, from which point of time the service sequence starts.

Busy Cycle

The time interval from the instant of commencement of a batch service at the end of a vacation to the instant of starting the next batch service after availing at least one vacation is a Busy cycle.

Service Cycle

The time interval between the commencements of two consecutive batch services when the later starts immediately at the end of the second phase service to the former batch.

Notations

N = System size at the commencement of a batch service.

N_1 = System size at the end of the batch service.

N_2 = System size when the server completes the second phase of service to the batch.

M = Number of customer arrivals during batch service.

X = Total number of arrivals when the server is in the individual service mode during a service cycle.

Y = Number of arrivals ≥ 1 during a vacation.

B(t) = Batch service time.

F(t) = Individual service time.

V(t) = Vacation time.

The system is in steady state.

$\gamma = \lambda E(B)$

$\rho = \lambda E(S) < 1$

$N_1 = N + M$

$N_2 = X + M$

$$N = \begin{cases} N_2 & \text{when } N_2 > 0 \\ Y & \text{when } N_2 = 0 \end{cases}$$

$P(N_2 = 0) = P_{20}$

Let $P^*(z)$, $P_1^*(z)$ and $P_2^*(z)$ denote the PGFs of N, N_1 and N_2 respectively.

$$\begin{aligned} P^*(z) &= E(z^N) \\ &= E[E(z^N/N_2)] \\ &= E(z^N/N_2 = 0)P(N_2 = 0) + \sum_{k=1}^{\infty} E(z^N/N_2 = k)P(N_2 = k) \\ &= E(z^Y)P_{20} + \sum_{k=1}^{\infty} E(z^k)P(N_2 = k) \\ &= E(z^Y)P_{20} + \sum_{k=1}^{\infty} z^k P(N_2 = k) \\ &= \{[\bar{V}(\lambda - \lambda z) - \bar{V}(\lambda)]/[1 - \bar{V}(\lambda)]\}P_2 + P_2^*(z) - P_{20} \\ &= P_2^*(z) - P_{20}\{1 - [\bar{V}(\lambda - \lambda z) - \bar{V}(\lambda)]/[1 - \bar{V}(\lambda)]\} \end{aligned}$$

Volume : 3 | Issue : 7 | July 2013 | ISSN - 2249-555X

Let

$$b_k = P\{\text{K customers arrive during a vacation}\}$$
$$= \int_0^\infty e^{-\lambda t}\left[\frac{(\lambda t)^k}{k!}\right] dV(t)$$

P (K customers arrive during the j-th vacation) = $(\overline{V}(ë))^{j-1} b_k$, j = 1, 2, 3, … .

Hence P(K customers are present in the system at the end of vacation)

$$= \sum_{j=1}^{\infty} \overline{V}(ë)^{j-1} b_k$$
$$= \frac{b_k}{1-\overline{V}(ë)}$$

$$E(z^Y) = \sum_{k=1}^{\infty} z^k [b_k/(1-\overline{V}(\lambda))]$$
$$= [1/(1-\overline{V}(\lambda))] \sum_{k=1}^{\infty}\left[z^k \int_0^\infty e^{-\lambda t}\left[\frac{(\lambda t)^k}{k!}\right] d(V(t))\right]$$
$$= [1/(1-\overline{V}(\lambda))] \int_0^\infty e^{-\lambda t} \sum_{k=1}^{\infty}\left[\frac{(\lambda t)^k}{k!}\right] d(V(t))$$
$$= (\overline{V}(\lambda-\lambda z) - \overline{V}(\lambda)/(1-\overline{V}(\lambda))$$

$$P_1^*(z) = E(z^{N+M}) = E(z^N)E(z^M) = P^*(z)\overline{B}(\lambda-\lambda z) \quad (2)$$

$$P_2^*(z) = E(z^{X+M}) = E(z^X)E(z^M)$$

$$E(z^X) = E[E(z^X/N)] = P^*(\overline{F}(\lambda-\lambda z))$$

$$P_2^*(z) = \overline{B}(\lambda-\lambda z)P^*(\overline{F}(\lambda-\lambda z)) \quad (3)$$

Combine all the equations

$$P^*(z) = \overline{B}(\lambda-\lambda z)P^*(\overline{F}(\lambda-\lambda z)) - P_{20}\{1-[(\overline{V}(\lambda-\lambda z)-\overline{V}(\lambda))/1-\overline{V}(\lambda)]\}$$

Define

$$g(z) = \overline{F}(\lambda-\lambda z)$$
$$h(z) = \overline{B}(\lambda-\lambda z)$$
$$L(z) = 1-1-[(\overline{V}(\lambda-\lambda z)-\overline{V}(\lambda))/1-\overline{V}(\lambda)]$$

$$P^*(z) = P*(g(z))h(z) - P_{20}L(z) \quad (4)$$

Let

$$h^{(0)}(z) = z$$
$$h^{(1)}(z) = h(z) = \overline{B}(\lambda-\lambda z)$$

$$g^{(0)}(z) = z$$
$$g^{(1)}(z) = g(z) = \overline{F}(\lambda-\lambda z)$$
$$g^{(n)}(z) = g(g^{(n-1)}(z)) = g^{(n-1)}(g(z)), \quad n > 1$$

By repeat, we get

$$P^*(z) = -P_{20}\left\{\sum_{k=0}^{\infty}\left(\prod_{j=0}^{k-1} h(g^{(j)}(z))L(g^{(k)}(z))\right) + \prod_{j=0}^{\infty} h(g^{(j)}(z))\right\}$$

Since $P^*(0) = 0$

$P^*(g(0))\, h(0) - P_{20} = 0$

where

$$g(0) = \overline{F}(\lambda)$$
$$h(0) = \overline{B}(\lambda)$$

$$P_{20} = \frac{h(0)\prod_{j=1}^{\infty} h(g^{(j)}(0))}{1 + h(0)\sum_{k=0}^{\infty}\left(\prod_{j=1}^{\infty} h(g^{(j)}(0)\right)L(g^{(k+1)}(0))}$$

Let the random variable Q_S denote the system size when a random customer leaves the system after his service completion.

Let $Q_S^*(z)$ denote the PGF of Q_S. K-denote the position of the random customer in his batch.

X^1 – number of customers left behind the random customer.

X_2 – number of customers arrivals that occur during the K individual service completions in the second phase.

$Q_S = M + X_1 + X_2$.

M is independent of X_1 and X_2 where as X_1 and X_2 are dependent random variables.

$$Q_S^*(z) = E(z^{Q_s}) = E(z^{M+X_1+X_2}) = E(z^M) + E(z^{X_1+X_2})$$

$$E(z^{X_1+X_2}) = E[E(z^{X_1+X_2}/N,K)]$$
$$= \sum_{n=1}^{\infty}\sum_{k=1}^{n} z^{n-k}(\overline{F}(\lambda-\lambda z))^k P(N=n, K=k)$$
$$= \sum_{n=1}^{\infty}\sum_{k=1}^{n} z^{n}(g(z)/z)^k (1/n)(nP(N = n/E(n))$$
$$= \{g(z)/E(N)\}\{[P^*(z) - P^*(g(z))]/[z-g(z)]\}$$

$$Q_S^*(z) = g(z)\,[P^*(z)\,(1-h(z)-P_{20}\,L(z)]\,/\,E(N)\,(g(z)-z)]$$

When the server is not allowed to take any vacation and there is no batch service so that $\overline{V}(0)=1$ and $\overline{B}(0)=1$

The System Characteristics

We obtain the first two moments of the system size and system time as follows.

Differentiate $Q_S^*(z)$ with respect to z at z = 1.

$$E(Q_S) = ñ + [2(1-ñ)]^{-1} ë^2 \{E(S^2) + 2E(B)\} + E(B^2)/E(N) + P_0\, E(V^2)E(N)(1-\overline{V}(ë))\}$$

Differentiate $Q_S^*(z)$ with respect to z twice at z = 0

$$E(Q_S^2) = \rho + \frac{\lambda^2(1+2\rho)}{2(1-\rho)}\{E(S^2) + 2E(B)/\lambda + E(B^2)/E(N) + P_{20}E(V^2)/[E(N)(1-\overline{V}(\lambda))]\} + \frac{\lambda^3}{3(1-\rho)}\{E(S^3) + E(B^3)/E(N)$$

$$+ P_{20}E(V^3)/[E(N)(1-\overline{V}(\lambda)))\}$$

$$+ \frac{\lambda^4 E(S^2)}{2(1-\rho)^2}\{E(S^3) + 2E(B)E(S^3)/\lambda + E(V^2)/E(N)\}$$

$$+ \frac{\gamma E(N(N-1))}{E(N)(1-\rho)} + \frac{\lambda^2 E(B^2)}{1-\rho} + \frac{P_{20}\lambda^3 E(S^2)E(V)}{E(N)(1-\rho)(1-\overline{V}(\lambda))}$$

$$E(N) = \gamma/1-\rho + P_{20}\lambda E(V)/(1-\rho)(1-\overline{V}(\lambda))$$

Similarly obtain E(N(N−1))

$$V(Q_S) = E(Q_S^2) - (E(Q_S))^2$$

Server Utilization

Server utilization in the model under consideration is defined as the long-run proportion of time the server is busy in serving the customers in the system during any busy cycle.

T_1 and T_2 represent the duration of time the server is busy in the system and on vacations respectively.

$\overline{T}_1(è)$ and $\overline{T}_2(è)$ be the LSTs of T_1 and T_2 respectively.

$$\overline{T}_1(\theta) = \frac{P_{20}\overline{B}(\theta)P^*(\overline{F}(\theta))}{1-\{1-P_{20}\}\overline{B}(\theta)P^*(\overline{F}(\theta))}$$

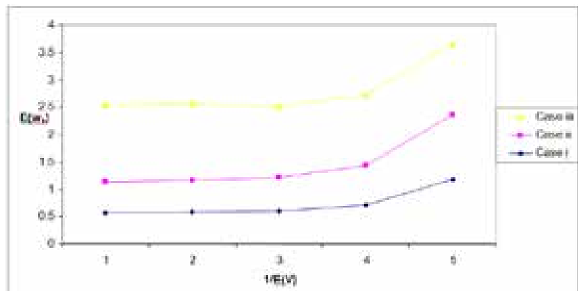
$$\overline{T}_2(\theta) = \frac{(1-\overline{V}(\lambda))\overline{V}(\theta)}{1-\overline{V}(\lambda)\overline{V}(\theta)}$$

4.0 NUMERCIAL CALCULATIONS

Table 1

Case (i) λ = 5, E(B) = 1/15		Case (ii) λ = 5, E(B) = 1/10		Case (iii) λ = 5, E(B) = 1/2	
$\frac{1}{E(V)}$	$E(w_s)$	$\frac{1}{E(V)}$	$E(w_s)$	$\frac{1}{E(V)}$	$E(w_s)$
5	0.56	5	0.58	5	1.41
4	0.58	4	0.59	4	1.4
3	0.6	3	0.61	3	1.31
2	0.71	2	0.72	2	1.3
1	1.18	1	1.18	1	1.3

GRAPHICAL REPRESENATAIONS



5.0 Conclusion

In this paper we analyzed a two phase service vacation queueing model with gated service policy. The mean and variance of the system size are derived. This type of modeling helps analyzing certain situations in which customers require service in two stages of which the first stage of service is essentially of gated type.

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