



## Determination of Influence Lines of a Portal Frame Adopting Matrix Method of Structural Analysis

### KEYWORDS

Bridge, Columns, Portal Frame, Matrix Method of Structural Analysis, Influence Line, Vibrations.

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**ABSTRACT** The objective of the investigation is to mathematically simulate dynamics and vibrations of a portal frame subjected to a concentrated load moving on its horizontal member with a certain constant velocity. This portal frame is a basic structure of a low length single span bridge. In an earlier paper [1] similar investigation is performed applying basic concept of structural analysis encountering elastic deformation of upper beam portion of the portal frame. As against that in this paper Matrix Method of Structural Analysis [2] is applied. The findings by these two approaches are also detailed. The emphasis is on an approach to model forced vibrations of the vertical members of the portal frame.

### 1. CONSTRUCTION OF A SHORT LENGTH BRIDGE

Fig. 1 is a schematic presentation of a short length bridge. The length is so short that the basic structure of a bridge is a simple one span portal frame  $O_1ABO_2$ . The width of the bridge is also fairly small so that it could be considered as a particular case of a girder bridge [3]. The material of the frame is Mild Steel (M.S.). The philosophy of the analysis is explained through a representative small scale structure with dimensions length of AB = 1005 mm, width = 50 mm and thickness is = 0.01m. The vertical members  $O_1A$  and  $O_2B$  are geometry wise identical. The material of  $O_1A$  & that of  $O_2B$  is also M.S. A vehicle with total weight  $W$  is moving on AB with a constant velocity.

The objective of the investigation is to estimate vibration response of  $O_1A$  &  $O_2B$ .

### 2. ANALYSIS OF A PORTAL FRAME BY MATRIX METHOD

The specifications of the portal frame under consideration are as follows :

Material for all the members viz.  $O_1A$ , AB,  $O_2B$  is M.S. with modulus of Elasticity  $E = 2.0 \times 10^8$  kN/m<sup>2</sup>; Lengths of  $O_1A = AB$  and  $O_2B$  are equal each in turn equals to 1m, cross section rectangular 50 mm x 10mm; For member  $O_1A$ ,  $I = (bd^3)/12.0$  where  $b = 0.01$ m and  $d = 0.05$ m. Dimension  $b$  is in the plane of the paper where as dimension  $d$  is at right angles to the plane of the paper. Geometry wise members  $O_1A$  &  $O_2B$  are identical. For member AB the thickness = 0.01m where as width = 0.05m.

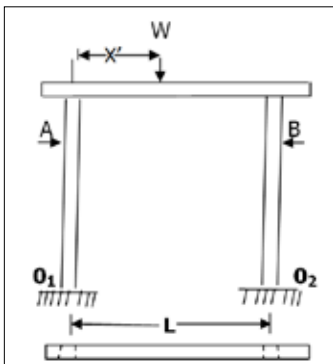


Figure 1 : schematics of a portal frame for a short bridge

As far as joints  $O_1, A, B, O_2$  are concerned the Horizontal Force components vertical force components and rotation in vertical plane about axis, passing through  $O_1$  and perpendicular to the plane of the paper are respectively  $U_{01}, V_{01}$  and  $q_{01}$  (positive C.W.). Similarly, the same quantities at remaining joints A, B and  $O_2$  are concerned are as follows.

For A :  $U_A, V_A, q_A$   
For B :  $U_B, V_B, q_B$   
And  
For  $O_2$  :  $U_{02}, V_{02}, q_{02}$

For a concentrated load  $W = 1$  kN acting at distance  $x = x' = 0.25$ m from A, these quantities (i.e.  $U_{01}, V_{01}, q_{01}, U_A, V_A, q_A, U_B, V_B, q_B, U_{02}, V_{02}, q_{02}$ ) are estimated adopting Matrix Method of Structural Analysis [2].

Similar approach for the same load and the same location is applied for the members AB &  $O_2B$  also. The findings are as shown in Table 1. Similar approach is adopted for the analysis of entire structure when the same load  $W = 1$  kN changed its position  $x$  to 0.5m, 0.8m. The complete findings are shown in Table-1. The graphic presentation of  $V_{01}$  and  $V_{02}$  are decided ofcourse adopting the concept of interpolation [4] These graphic plots are shown in Figure 3.

### 3. DETERMINATION OF EQUIVALENT AXIAL LOAD FOR $O_1A$ AND $O_2B$ .

This is detailed in this article for the case when  $x = 0.25$ m &  $W = 1$ kN.

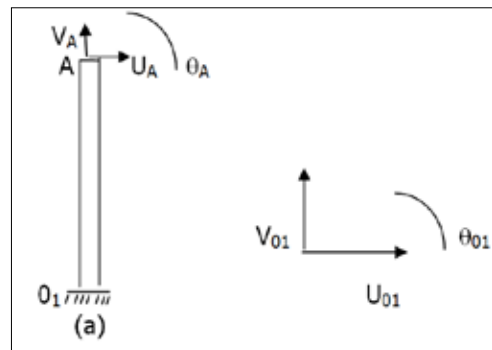


Figure 2 : Results of Force Analysis of Various Members of Portal Frame

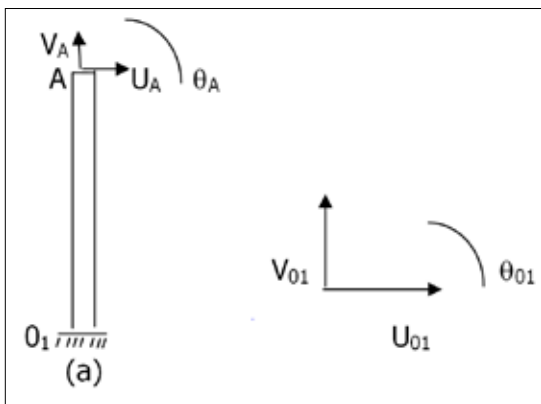
Effect of V01 & VA is to create an axial compressive load equal to 0.879 x 103 kN

Effect of U01 & UA is to create a c.c.w. couple = 0.195 x 103 kN-m

This stands to reason because of 1 kN load on AB at x = 0.25m a c.w. moment is created on AB of 0.25 kNm by 01A to which 01A should experience an anti clockwise moment.

This moment 0.195 x 103 kN-m c w on 01A will change its magnitude as the load 1 kN changes its position on AB. Thus, it will induce bending vibrations in 01A. Similarly, the net rotation of cross section at A with respect to cross section at O1 due to ΔA = 0.1263 radians c.c.w. and Δ0 = 0.0622 radians c.c.w. is 0.0641 radians c.c.w. which is also time variant has effect only to the extent of inducing bending vibrations in 01A.

On the same lines Fig. 2(c) shows the Free Body Diagram (FBD)



- UB = -101.99 N
- VB = 0.0458 kN = 45.8 N
- θB = -0.0299 radians
- U(02) = 101.99 N
- V(02) = -0.0459 kN = 45.8 N
- θ(02) = 0.0153 radians

Effect of VB = 45.8 N & V02 4.8 kN is to create tension in the member 02B when x = 0.25m and W = 1kN. Similarly, the effect of U(02), +101.99 N and UB = -101.99 N is to create c.c.w. moment on 02B. The magnitude of this moment is to change with position of W. Thus, it is going to create bending vibrations in the 02B. Similarly, net rotation of Section B w.r.to 02 is (-0.0299) + (0.0153) = -0.0146 c.w. which also changes with time and creates time varying bending vibrations in 02B.

As stated earlier, in this paper as emphasis on longitudinal vibrations of 01A and 02B is only considered, bending vibrations is not detailed further.

**4.0 INFLUENCE LINE FOR 01A AND 02B:**

Figure 3(a) & 3(b) respectively shows the variation of V01 = VA and V02 = VB as the load i.e. the weight of the moving vehicle changes its position on AB. This can be considered as the influence lines of member AB of the portal frame. This is ofcourse by the analysis of the concept INFLUENCE LINE OF A SIMPLY SUPPORTED BEAM [5]. These figures 3 (a) and 3 (b) shows nonlinear variation of longitudinal load on the members 01A & 02B respectively of the portal frame 01AB02 under consideration. In other words as per matrix method of Structural Analysis one gets nonlinear variation of compressive load on members 01A & 02B of the portal frame as some concentrated constant load changes its position on portion AB of the portal frame. As stated earlier in this paper the single span short length bridge is treated analoging

to a single span portal frame the loading on columns of the bridge would also be nonlinear. This nonlinearity is in terms of changed position of concentrated load. It is obvious that this is also interms of time because the load is nothing but the dead weight of the vehicle which is varying with constant velocity but the position of the vehicle is changed with respect to time line. Hence, as per this approach of Structural Analysis [2], it is concluded that the column reactions change nonlinearly with time as against the linear variation as obtained in the previous analysis [1].

**5.0 APPROACH TO ESTIMATION OF VIBRATION RESPONSE:**

Figure 3(a) shows influence line of 01A. This variation is having exponential decay form. Thus it can be stated as under if F(01A) stands for external longitudinal load acting on the column 01A.

$$F(01A) = \theta + K1t^{-n}, \dots\dots\dots (I)$$

Whereas if F02B stands for external column load acting on the column 02B then it is as described in Figure 3(b). Figure 3(b) shows the variation of F02B in exponential rising form. Hence it can be stated as under

$$F02B = K2tn^2 \dots\dots\dots (II)$$

The parameters K1, n1, k2, and n2, can be decided by plotting these variations on log-log graph [4]

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If one considers the entire column represented by SDOF system, in which M, K, C represents the mass, elastic stiffness and damping co-efficient of the structure of the column then the governing equation of forced vibration phenomena of the column is presented as under.

$$M\ddot{x} + c\dot{x} + kx = K_1 t^{-n1} \dots\dots\dots (III)$$

For the column 0,A where as that for the column 02B would

$$M\ddot{x} + c\dot{x} + kx = K_{21} t^{-n2}$$

The initial conditions are ofcourse as under,  
at t = 0; x = 0  
and at t = 0; x = 0

A more realistic solution could be by treating the entire column as a multidegree of freedom system represented by 34 identical masses, stiffness of the springs and damping co-efficient.

On the same lines an entire column can be considered as a distributed mass, distributed elasticity and distributed damping system. Further all other possible configurations of vehicular traffic should be considered as detailed in the earlier paper.

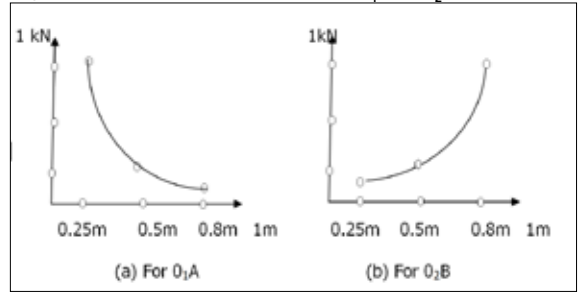
**6.0 CONCLUSION**

The conclusion essentially is the comparison of present approach with the earlier one which was based on visualisation of likely elastic deformation of the portal frame, and using the concept of modulus of elasticity of the material. In the earlier approach of arriving at the influence lines [5] was based on applying basic concepts of deformation of the structure under the influence of vehicular traffic resulted in giving straight line pattern of the influence line whereas in the present approach of MATRIX METHOD it has turned out to be nonlinear shape of the influence line as shown in the Figure 3(a) & 3(b). This difference is certain to deduce substantial difference in the vibration response of the members 0,A & 02B which are analogous to the end supports of a single span bridge.

**TABLE – 1**  
**FINDINGS OF ANALYSIS OF A PORTAL FRAME 01AB02,**  
**FOR W = 1kN & x = 0.25,05, 0.8m**

Sr. No.	Loading Condition X, W	$q_{(01)}$	$U_{(01)}$	$V_{(01)}$	$q_A$	$U_A$	$V_A$	$q_B$	$U_B$	$V_B$	$q_{02}$	$U_{02}$	$V_{02}$
1	X = 0.25M W=1kN	0.0622 radians	$0.195 \times 10^3$	$-0.879 \times 10^3$	0.1263 radians	$-0.195 \times 10^3$	$0.879 \times 10^3$	-0.0299 radians	-101.99 N	0.0458 kN	0.0153 radians	101.99 N	-0.0459 kN
2	X=0.5M W=1kN	-0.0611	$0.469 \times 10^3$	$-0.182 \times 10^3$	-0.122 raduabs	$-0.489 \times 10^3$	$0.182 \times 10^3$	0.121	$-0.489 \times 10^3$	$0.180 \times 10^3$	0.060	$0.489 \times 10^3$	$-0.180 \times 10^3$
3	X=0.8M W=1kN	$0.042 \times 10^{-6}$	0.0103 kN	-0.044 kN	$0.30 \times 10^{-1}$	-0.103 kN	0.44 kN	0.124	+0.878 kN	0.185	0.0621	-0.185 kN	+0.878 kN

**Figure 3.0 : INFLUENCE LINES OF  $0_1A$  &  $0_2B$**



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