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Distributed Lags: A New Estimator

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ABSTRACT *'Lagged variables' is one of the basic problems in Econometrics. Under Standard assumptions of a general linear model, straight forward application of Ordinary Least Squares (OLS) procedure to obtain estimates of the parameters of the distributed lag model has two main disadvantages, they are (i) If the number of lags is large and the sample size is small, then the parameters may not be estimable; (ii) Since the successive values of the same explanatory variables are highly correlated, the problem of multicollinearity creeps in. Therefore, certain weight patterns are suggested to reduce the number of lagged variables and to alleviate the severity of multicollinearity (Lankipalle, 1977). Though there is good number of weight generating mechanisms in the literature, often Almon's polynomial type weight pattern is used. Some Monte Carlo studies revealed that the OLS is supposed to be better than Almon's procedure from the point of view of bias, and variance. In view of the above, in this paper an attempt has been made for estimation of parameters in Lagged variables linear models. A new estimator has been suggested and it is shown that the new estimator has smaller variance than Almon's estimator and provides a remedy for multicollinearity.*

INTRODUCTION
The concept of distributed lag is based on the principle that any cause produces a supposed effect after some lag in time. Further, this effect is felt, not all at a single point of time, but is distributed over a number of pints of time.

In short, distributed lag theory, in many cases, states that a dependent variable y, is determined by a weighted sum of past values of an independent variables X. Therefore, without less of generality, the distributed lag model may be represented in the usual notation, as

$$y_t = \sum_{i=0}^{s} W(i) X_{t-1} + \in_t \qquad (1)$$

t= 1, 2, …, n

Under Standard assumptions of a general linear model, straight forward application of Ordinary Least Squares (OLS) procedure to obtain estimates of the parameters of the distributed lag model has certain disadvantages, primarily because

(i) If the number of lags is large and the sample size is small, then the parameters may not be estimable;
(ii) Since the successive values of the same explanatory variable are highly correlated, the problem of multicollinearity creeps in.
Therefore, certain weight patterns are suggested to reduce the number of lagged variables and to alleviate the severity of multicollimearity (Lankipalle, 1977).

Though there is good number if weight generating mechanisms in the literature, often Almon's polynomial type weight pattern is used. The practical preference to Almon's scheme is that it is very flexible and easy to estimate. Since Almon's technique is based on the mathematical theorem that any function can be approximated by a polynomial of an appropriate degree, the weight pattern defined by Almon has some theoretical basis. However, estimators of parameters obtained by Almon lag technique have certain serious drawbacks. Therefore, in this article a new estimator is proposed which has lesser variance than the Almon's estimator.

In the next section, Almon's procedure to estimate the parameters of the model (1) is described briefly and different drawbacks of the method are clearly pointed out. In the last section a new estimator with Almon's weight pattern is proposed.

Almon's Procedure
Almon(1965), proposed interpolation distribution for weights ie. W (i) are the values at x=0,1,…, S of a polynomial W(x) of degree r (r < s). Its estimation is based on the fact that once (r+1) points on the curve are known, all the W(i) can be calculated as a linear combination of these known values by,

$$W(i) = \sum_{j=0}^{r} f_j(i)\alpha_j \qquad (2)$$

where $f_j(i)$ are the values at x = i of the Lagrange interpolation polynomial . Therefore,

$$y = \sum_{j=0}^{r} \alpha_j \sum_{i=0}^{s} f_j(i) x_{t-1} + \in_t \qquad (3)$$

Again, by writing,

$$z_{tj} = \sum_{i=0}^{s} f_j(i) x_{t-i} \qquad (4)$$

$$y_t = \sum_{j=0}^{r} \alpha_j z_{tj} + \in_t$$

and (5)

According to Almon, α_j is estimated by OLS and hence an estimator of β_i.

Almon procedure assumes lag length (s) and degree of the polynomial (r). However, correct specification of (r) and (s) is a big problem in practice. With the help of data if one chooses that combination of (r) and (s) which minimizes residual variance, then, Frost (1975) has shown that the estimated lagged weights are biased. Further, in the process of selection, mis-

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specification of (r) and (s) is possible. For further discussion, (Terasirta,1976).

Monte Carlo evidence, reported by Cargill and Meyer (1974), is also not in favour of Almon technique. In fact, the performance of OLS is supposed to be better than Almon procedure from the point of view of bias, and variance.

To take the discussion more precise, we rewrite (5) as

$$y_{n.1} = z_{n.r+1}\,\alpha_{r+1.1} + \in_{n-1} \qquad (6)$$

where,

$$Z_{n.r+1} = X_{n.s+1} A_{S+1.\,r+1} \qquad (7)$$

$$A = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 2^2 & \cdots & 2^r \\ \vdots & & & & \\ 1 & s & s^2 & \cdots & s^r \end{bmatrix} \qquad (8)$$

$$\beta = A\alpha$$

and (9)

The OLE of α is

$$\hat{\alpha} = \left(Z'Z\right)^{-1} Z'X \text{ and} \qquad (10)$$

$$V(\hat{\alpha}) = \sigma^2 \left(Z'Z\right)^{-1} \qquad (11)$$

Therefore Almon's Estimator is

$$\hat{\beta} = A\hat{\alpha} \qquad (12)$$

and $V(\hat{\beta}) = \sigma^2 A \; V(\hat{\alpha}) A'$ (13)

Thus under standard assumptions of linear model and correct specification of (r) and (s), Almon's estimator is unbiased, but biased when the specifications are not correct or when they are selected by 'trial and error' method. Since Almon's estimator is theoretically interpreted as restricted least squares estimator, its variance is less than the variance of OLE of β. But, since each Zi is a linear combination of all the x_{t-i}s, whether there are linear dependencies among x_{t-i}s or not, Z_js are linearly dependent unless A is an orthogonal matrix, which is not. In fact, the degree of dependency of the transformed variables Z_j may be more than the dependency of the original explanatory variables x_{t-i} and hence estimator's obtained by Almon's procedure exhibits 'large biases' and 'inflatory variances'. Therefore, Almon's procedure does not 'break' the problem of multicollinearity and is not a remedial measure of multicollinearity. The only advantage of Almon's procedure is since r < s, in the analysis of data, one uses less number of explanatory variables in the process of estimation.

New Estimator

In the general context of linear models, for the problem of multicollinearity, one recent remedial measure suggested by Hoerl and Kennard (1970a, 1970b) and often discussed, is the Ridge Estimator. However, direct application of Ridge Estimator may not be of much use in the case of distributed lags, since no weight pattern is taken into account.

Now, we propose a new estimator, which takes into account both weight pattern and multicollinearity. Consider Almon's weight pattern, for a pre-specified polynomial of degree (r) and lag length (s). Then from (7) we have

$$Z'Z = A'X'XA = T \qquad (14)$$

Since, T is a symmetric positive definite matrix of order (r+1), there exists a matrix P such that

P' T P = Λ (diagonal),

whose elements are Eigen roots, say, $\lambda_0 > \lambda_1 \dots \dots > \lambda_m \dots\dots > \lambda_r$.

Let P_0, P_1, ….., P_m…..,P_r, be the orthogonal latent vectors corresponding to the latent roots. It is believed that multicollinearity is not one of choice but degree. Small latent roots and latent vectors indicate multicollinearities.

Now it can be shown that OLE of α is

$$\hat{\alpha} = \sum_{j=0}^{r} \lambda_j^{-1} Q_j P_j$$

Where, $Q_j \quad P_j'Z'Y$ **(15)**

and it is unbiased for α,

$$V(\hat{\alpha}) = \sigma^2 \sum_{j=0}^{r} \lambda_j^{-1} P_j P_j'$$

also (16)

Since small latent roots indicate multicollinearities, so delete the terms $\lambda_{m+1,}\dots\dots,\lambda_r$, and define

$$\bar{\alpha} = \sum_{j=0}^{m} \lambda_j^{-1} Q_j P_j$$

Here we impose the restriction that $\lambda_{m+1} = \dots. = \lambda_r = 0$ and the corresponding ortho-normal vectors to be null vectors.

Let

$$= (\bar{\alpha}_1, \dots.\bar{\alpha}_m, o, o, \dots., o)' \quad \alpha\square \qquad (17)$$

is essentially a principal component estimator.

$$= \sigma^2 \sum_{j=0}^{m} \lambda_j^{-1} P_j P_j'$$

V (α□) (18)

and the remaining P_j's are null vectors.

It is clear that α□ is biased; but from (16) and (18)

$$< V(\hat{\alpha})$$

V (α□) (19)

If we use $\hat{\alpha}$ to obtain an estimator of α, we have a new estimator as

b= A α□ (20)

The variance of the new estimator is

$V(b) = A\,V$ (α□) A' (21)

We note that the variance of new estimator is less than the variance of Almon's estimator. Even if the rank of T is m, (m < r), α□ can be had and hence the new estimator. Thus the new estimator, though biased, provides a remedy for multicollinearity and possesses smaller variance.

If (r) and (s) are selected by trial and error method, the combination of (r) and (s) for which the residual variance is minimum, then Almon's estimator is also biased. Even in this situ-

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ation the new estimator has smaller variance than Almon's estimator and both the estimators are biased.

AMOUNT OF BIAS IN THE NEW ESTIMATOR

For the new estimator proposed, bias can be derived.

$$E(b) = A\,E_{(\alpha\square)}$$

$$= \alpha - \sum_{j=m+1}^{r} (P_j{}'\alpha) P_j$$

E (α□)

$$E(b) = A\alpha \; - A\left(\sum_{j=m+1}^{r} (P_j{}'\alpha) P_j\right)$$

Hence the amount of bias in the new estimator is:

$$A \sum_{j=m+1}^{r} (P_j{}'\alpha) P$$

b is biased unless $P_{jj}{}' \; \alpha$ = 0.

If the bias is not too large, good amount of reduction in the variance of new estimator over Almon's estimator can be achieved.

SUMMARY

1. From the above results, the new estimator has smaller variance than Almon's estimator and provides a remedy for multicollinearity.
2. If (r) and (s) are selected by trial and error method, the combination of (r) and (s) for which the residual variance is minimum, then Almon's estimator is also biased. Even in this situation the new estimator has smaller variance than Almon's estimator and both the estimators are biased.
3. Instead of principal component estimator of α , if the ridge estimator is used, this also serves as a solution to the multicollinearity problem. But ridge estimator has its own limitations and difficulties from practical application point of view.
4. Finally, in distributed lag models, with different weight patterns the new estimators can be defined in a similar way, where multicollinearity is to be resolved.

REFERENCE 1. Almon, S. (1965), "The Distributed Lag Between capital Appropriations and Expeniture", Econometrica, 33, 178-196. | 2. Cargill, T.F. and Meyer, R.A (1974), "Some Time and Frequency Domain Distributed Lag Estimators: A Comparitive Monte Carlo Study" Econometrica, 42, 1031-1044. | 3. Dhrymes, P.J (1971), " On the Strong Consistency of Estimators for certain Distributed Lag Models with Auto correlated Errors" International Economic Review, 12, 329-342. | 4. Frost, P.A. (1975), "Some properties of Almon Lag Techniques when one searches for Degree of Polynomial and Lag: Journal of the American Statistical Association, 70, 606-612. | 5. Griliches, Z. (1967), "Distributed Lags: A survey," Econometrica, 35, 16-49. | 6. Gunst, R.F., and Mason, R.L(1977) " Biased Estimation in Regression ; An | evaluation using Mean Square Error", Journal of the American Statistical Association, 72 , 616-628. | 7. Hoerl, A.E. and Kennard, R.W. (1970 a), " Ridge Regression Biased Estimation for Non – Orthogonal Problems" , Technometrics , 12, 55-67 | 8. Hoerl, A.E. and Kennard, R.W (1970b), "Ridge Regression: Applications to Non-orthogonal Problems", Technometrics, 12, 69-92. | 9. Johnston, J. (1972), "Econometric Method", Mc Graw Hill Book Company, New York, 2nd Edition. | 10. Lankipalle, K.N. (1977), "Distributed Lags", Proceedings of the Seventeenth | Indian Econometric Society Conference. | 11. V.B.Naidu & V.K.Reddy (2006), "Estimation of Parameters in the presence of Measurement Errors and missing data: A New Estimator" Asian – African Journal on Economics and Econometrics, Vol.6, No.2, 2006; 219-225. | 12. Shiller, R.J (1973), "A Distributed Lag Estimator Derived from Smoothness Priors", Econometrica, 41, 775-788. | 13. Terasvirta, T. (1976), "A Note on Bias in the Almon Distributed lag estimator, | Econometrica, 44, 1317 – 1321. |