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# An Inventory Model for Weibull Distributed Deteriorating Items with Inventory Returns and Special Sales

KEYWORDS: Inventory, Stochastic Process, Heuristic, Deterioration, Newton Raphson Method.

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**ABSTRACT** *In this paper we reconsider the order level Inventory model with inventory returns and special sales discussed by several Authors. Here the items deteriorate with time. The main stress is on the discussion that the situation where the optimal stock level of an Inventory system is smaller than the amount on hand. Author has considered this problem in case of EOQ Inventory system. It has extended this model for the case of order level inventory system. However, this model is presented in a novel manner by considering shortages with prescribed scheduling period for deterministic demand. In these two models the assumption is that the order level inventory is less than the on-hand inventory. This type of situation may arise in any wholesale or retail business. The demand of a particular product decreases due to launching of a new product, which is cheaper and or superior, due to the effects of new budget such as price increase or due to any, other market fluctuations. In any such instances the optimum amount to be retained or sold, if any should be determined by minimizing the losses due to various costs involved in the inventory system.*

## INTRODUCTION

Inventories are an asset to the firm and from a financial stand point, inventories represent capital investment and must, therefore compete with other asset form for the firm's limited capital funds. The objective of inventory is to maximize the profit and minimize the cost of the firm. Most of the organizations particularly in India are tend to attempt to lower inventory using non-analytical approach with lower service levels. Various process changes can be suggested and be modeled to verify their impact on the inventory levels and service levels. It would be apt to consider real world constraints prior to deciding on the appropriate changes. The theory of inventory control took its roots from the Scientific Management Movement during early 1900's. Since World War II the inventory control discipline has developed further, using the tools of Operations Research. Over hundred years, still many are exploring on inventory models, as this subject has become an interesting research area.

## ASSUMPTIONS AND NOTATIONS :

The models are developed under the following assumptions

1. Demand is deterministic at a constant rate of 'R' units per unit time.
2. Scheduling period is a prescribed constant, T.
3. Replenishment size is constant and its rate is infinite. The fixed lot size '$q_p$' rises the inventory level in each scheduling period to the order level 'S'.
4. Shortages are allowed and completed backlogged.
5. The inventory carrying cost $C_1$ per unit time, the shortage cost $C_2$ unit per unit time, the cost of each deteriorated unit $C_3$ and the returning or selling cost $C_4$ per unit are known and constant during the period under consideration.
6. The system starts with an amount of 'Q' units on-hand of which only 'P' units are retained after returning or selling the rest the problem is to determine Optimal value of 'P'.
7. The deterioration rate functions for two parameter Weibull Distribution is

$$\theta(t) = \alpha\beta t^{\beta-1}, 0 < \alpha < 1, \beta > 0, t > 0.$$

Where b = 1, q(t) becomes a constant which is the case of an exponential decay. When b<1, the rate of deterioration is decreasing with time and b >1, is increasing with time't'.

## MATHEMATICAL FORMULATION

Consider the period 't' with the initial inventory level of 'Q' units and final inventory is assumed to be zero. This assumption is meaningful since (Q-P) units are sold with special sales price i.e. $C_4$. The retained 'P' units are to be exhausted during the time $t_1$ < T, during the remaining period (T – $t_1$) the optimal order level system will be operated. Now $Q_1(t)$ denotes the inventory position at time t $(0 \le t \le t_1)$ then the differential equation governing the system for the Weibull distributed deteriorating items is given by

$$\frac{d}{dt}Q(t) + \theta(t)Q(t) = -R; \quad 0 \le t \le t_1 \qquad \ldots (1)$$

$$\frac{d}{dt}Q(t) = -R \quad ; \quad t_1 \le t \le T \qquad \ldots (2)$$

where $\theta(t) = \alpha\beta t^{\beta-1}$ , $0 < \alpha < 1$, $\beta > 0, t > 0.$

The boundary conditions are $Q_1(0) = P$ and $Q_1(t_1) = 0$ . . . (3)

when $0 < \alpha < 1$, we ignore the terms of O($\alpha^2$) and use the conditions (3), then the solutions of the above equations are

$$Q(t) = \{-Rt - \frac{R\alpha}{\beta+1}t^{\beta+1}\}(1+\alpha t^{\beta})^{-1} + P(1-\alpha t^{\beta})^{-1}; \; 0 \le t \le t_1 \qquad \ldots (4)$$

And $Q(t) = -R(t - t_1)$ ; $t_1 \le t \le T$ . . . (5) &

Since $Q(t_1) = 0$ at $t = t_1$ we get $P = Rt_1 + \frac{R\alpha}{\beta+1}t_1^{\beta+1}$ . . . (6)

From ( 2.4 ) and ( 2.6 ) the total inventory carried during the period '$t_1$' is

$$Q(t_1) = \int_0^{t_1} Q(t)dt = \frac{Rt_1^2}{2} + \frac{R\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} \qquad \ldots (7)$$

The total cost of the system during the period 'T' is given by

$$K(P) = C_4(Q-P) + C_1Q(t_1) + C_1Q(t_1) + (T-t_1)C(t_1) \qquad \ldots (8)$$

where $C(t_1)$ is the average total cost per unit of optimum or-

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der level operating system during

( T – $t_1$ ) and is given by

$$C(t_1)=\frac{C_3R\alpha}{T(\beta+1)}t_1^{\beta+1}+\frac{C_1}{T}[\frac{Rt_1^2}{2}+\frac{R\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}]+\frac{C_2}{T}\frac{R(T-t_1)^2}{2} \quad ...(9)$$

The $t_1$* of $t_1$ can be obtained by differentiating the above equation with respect to $t_1$ and equating to zero. However one should ensure that the second derivative must be greater than zero to get optimum value of $t_1$* of $t_1$ i.e. $t_1$* is the solution of the following equation.

$$C_3\alpha t_1^{\beta-1}+C_1(1+\frac{\alpha\beta t_1^{\beta+1}}{\beta+1})-C_2(T-t_1)=0 \quad . . . (10)$$

Proceeding in similar fashion of equation ( 6 ), we get the optimum order level $S^0$ of S as

$$S^0=R_1t_1^*+\frac{R\alpha}{\beta+1}t_1^{*\beta+1} \quad . . . (11)$$

again the total amount of back order at the end of the cycle is R(T – $t_1$). Therefore the optimum value of $q_p$* of $q_p$ is given by $q_p$* = $S^0$ + R ( T – $t_1$*)

$$q_p^*=\frac{R\alpha}{\beta+1}t_1^{*\beta+1}+RT \quad . . . (12)$$

and the minimum value of the average total cost C($t_1$) is C($t_1$*).

**RESULTS IN THE ABSENCE OF DETERIORATIONS**

If the deterioration of the item is switched off (a = 0), the equation (10) for the optimum value of t1 reduces to linear equation. $C_1t_1-C_2T+C_2t_1=0 \quad \Rightarrow \quad t_1=\frac{C_2T}{C_1+C_2}$

and corresponding value of t1 = t1* $t_1^*=\frac{C_2T}{C_1+C_2}$ . . . (13)

Moreover the expressions for $S^0$ of q* obtained by puta = 0 in the equation (11) & (12) $S^0$ = Rt1* (14) & qp*=RT . . . (15)

which agrees with Naddor . Using the equation ( 6 ), (7),(9) in (8) we get

$$K(P)=C_4[Q-(Rt_1+\frac{R\alpha}{\beta+1}t_1^{\beta+1})]+C_1[\frac{Rt_1^2}{2}+\frac{R\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}]+(T-t_1)C(S) \quad . . . (16)$$

Since the above equation is a function of $t_1$, it is denoted as K(p,$t_1$) again P is a function of $t_1$ as in (6), the necessary condition for the minimization of the cost K($t_1$) $\frac{d}{dt}K(t_1)=0$ is After little simplification, the condition can be written as

$$C_1(\alpha\beta t_1^{\beta+1}+(\beta+1)t_1)-C_4(1-\alpha t_1^{\beta})(\beta+1)-C(S)R(\beta+1)=0 \quad . . . (17)$$

The solution of the above equation gives the optimal value of $t_1$ say $t_1$*. The above equation in $t_1$ can be solved by using Newton Rapshson method or any other search method Substituting $t_1$* in (6) we get the optimum value of $P^0$ of P, the sufficient condition of minimum total cost is $\frac{d^2}{dt^2}K(t_1)$ at $t_1=t_1^*$ = $-C_4\alpha\beta t_1^{*\beta-1}+C_1+C_1\alpha\beta t_1^{*\beta}>0$ should be satisfied.

Note that the maximum Quantity that can be returned or sold if ever is Q i.e. the optimum value of P must be less than or equal to Q. However P depends on $t_1$ and therefore the optimum solution of the present inventory system should be represented as follows.

$$P^0=R_1^*+\frac{R\alpha}{\beta+1}t_1^{*\beta+1} \text{ ; if } 0\le C_4<\frac{C_1(1+\alpha\beta t_1^{*\beta})}{\alpha\beta t_1^{*\beta-1}} \text{ = Q; otherwise . . . (18)}$$

The above equation (18 ) gives the optimal value of P i.e. the optimal Quantity to be retained.

**NUMERICAL ILLUSTRATIONS:**

Let the hypothetical values of parameters of the inventory models be $C_1$=3, $C_2$ = 15, $C_3$ =5, $C_4$ = 4 R = 100, T=1. All the parameters are expressed in consistent units per month. For different values of 'a' and 'b', we have determined the optimal Quantity to be retained and the associated costs are portrayed in the following table. To do this, at first we solved the (17) by using Newton Raphson method and t1* is substituted in equations (18),(16) to get optimum quantity to retained and associated minimum cost respectively.

Sensitivity of the Model with respect to Deterioration rates i.e. 'a' and 'b'.

| α- values | β-values | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | 113 | 116 | 118 | 118 | 118 | 118 | 118 | 117 | 116 | 115 |
| 0.01 | 173 | 174 | 175 | 175 | 176 | 176 | 175 | 175 | 175 | 175 |
| | 97 | 145 | 245 | 449 | 831 | 1492 | 2534 | 4043 | 6133 | 8878 |
| | 121 | 127 | 131 | 132 | 132 | 132 | 131 | 129 | 127 | 126 |
| 0.02 | 178 | 180 | 182 | 183 | 183 | 183 | 183 | 183 | 183 | 184 |
| | 117 | 225 | 454 | 916 | 1737 | 3038 | 4913 | 7445 | 10781 | 15072 |
| | 130 | 138 | 143 | 146 | 146 | 145 | 143 | 141 | 139 | 136 |
| 0.03 | 184 | 187 | 189 | 190 | 190 | 190 | 190 | 190 | 191 | 193 |
| | 139 | 318 | 706 | 1465 | 2744 | 4644 | 7248 | 10662 | 15086 | 20756 |
| | 138 | 149 | 156 | 159 | 159 | 158 | 156 | 153 | 150 | 146 |
| 0.04 | 189 | 193 | 196 | 197 | 197 | 197 | 197 | 198 | 199 | 202 |
| | 165 | 427 | 997 | 2079 | 3812 | 6273 | 9549 | 13780 | 19227 | 26210 |
| | 146 | 160 | 168 | 171 | 172 | 171 | 168 | 164 | 160 | 156 |
| 0.05 | 194 | 199 | 202 | 204 | 204 | 204 | 204 | 205 | 208 | 211 |
| | 194 | 550 | 1323 | 2742 | 4916 | 7908 | 11819 | 16833 | 23267 | 31527 |
| | 154 | 170 | 180 | 184 | 185 | 183 | 180 | 176 | 171 | 166 |
| 0.06 | 200 | 205 | 209 | 210 | 210 | 211 | 211 | 213 | 216 | 221 |
| | 225 | 688 | 1681 | 3439 | 6042 | 9541 | 14063 | 19836 | 27236 | 36751 |
| | 162 | 181 | 191 | 196 | 197 | 195 | 192 | 187 | 181 | 175 |
| 0.07 | 205 | 212 | 216 | 217 | 217 | 217 | 218 | 220 | 224 | 230 |
| | 260 | 838 | 2064 | 4162 | 7177 | 11166 | 16282 | 22798 | 31149 | 41904 |
| | 169 | 191 | 202 | 208 | 209 | 207 | 203 | 198 | 191 | 185 |
| 0.08 | 211 | 218 | 222 | 223 | 223 | 224 | 225 | 228 | 232 | 239 |
| | 298 | 1002 | 2469 | 4902 | 8317 | 12780 | 18478 | 25725 | 35016 | 47001 |
| | 177 | 200 | 213 | 219 | 220 | 218 | 214 | 208 | 201 | 194 |
| 0.09 | 216 | 224 | 228 | 230 | 230 | 230 | 232 | 235 | 240 | 248 |
| | 339 | 1178 | 2891 | 5653 | 9456 | 14382 | 20651 | 28621 | 38842 | 52050 |
| | 184 | 210 | 224 | 231 | 232 | 230 | 225 | 219 | 211 | 203 |
| 0.1 | 222 | 230 | 235 | 236 | 236 | 237 | 239 | 243 | 248 | 257 |
| | 383 | 1364 | 3328 | 6411 | 10592 | 15972 | 22804 | 31632 | 42634 | 57057 |

The first row values are corresponding 'a' are obtained using equation (9) i.e. the optimal cost during the period (t – $t_1$) and the second row values for corresponding 'a' are the optimum quantities to be retained i.e.$P^0$ of P using equation (6). The third row values for corresponding 'a' are obtained from equation (16).

From the above table we observe that as 'a' increases optimum quantity to be retained will increase and there is a marginal change in the cost even though 'P' increases. Whereas in case of 'b' the associated cost will increase drastically. This can be noted from the third row values of the table 1 When a

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= 0.01. Similar observation can be made for different values of 'b' and 'a'. Hence the model is very sensitive for changes in 'b' rather than changes in the 'a' values. However, it would be interesting to check this sensitivity of the model with respect to the changes in the special sales i.e. $C_4$. To do so, the pertinent computations are summarized in table 2.

| SENSITIVITY OF THE MODEL WITH RESPECT TO CHANGES IN THE 'α' AND '$C_4$'. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| α-values | $C_4$-values | | | | | | | | |
| | 0.2 | 0.24 | 0.28 | 0.32 | 0.36 | 0.4 | 0.44 | 0.48 | 0.52 |
| 0.01 | 44 | 46 | 47 | 48 | 50 | 51 | 52 | 54 | 55 |
| | 284 | 322 | 360 | 398 | 436 | 474 | 512 | 573 | 607 |
| 0.02 | 47 | 49 | 50 | 51 | 53 | 54 | 55 | 57 | 58 |
| | 289 | 327 | 365 | 403 | 441 | 479 | 517 | 578 | 612 |
| 0.03 | 50 | 51 | 53 | 54 | 56 | 57 | 58 | 60 | 61 |
| | 294 | 332 | 370 | 408 | 446 | 484 | 522 | 582 | 616 |
| 0.04 | 53 | 54 | 56 | 57 | 58 | 60 | 61 | 62 | 64 |
| | 299 | 337 | 375 | 413 | 451 | 489 | 526 | 586 | 621 |
| 0.05 | 56 | 57 | 58 | 60 | 61 | 62 | 64 | 65 | 67 |
| | 304 | 342 | 380 | 418 | 455 | 493 | 531 | 590 | 625 |
| 0.06 | 58 | 60 | 61 | 62 | 64 | 65 | 67 | 68 | 69 |
| | 308 | 346 | 384 | 422 | 460 | 498 | 536 | 595 | 630 |
| 0.07 | 61 | 62 | 64 | 65 | 66 | 68 | 69 | 71 | 72 |
| | 313 | 351 | 389 | 427 | 465 | 503 | 540 | 599 | 634 |
| 0.08 | 63 | 65 | 66 | 68 | 69 | 71 | 72 | 73 | 75 |
| | 318 | 356 | 394 | 432 | 469 | 507 | 545 | 603 | 639 |
| 0.09 | 64 | 67 | 69 | 70 | 72 | 73 | 75 | 76 | 77 |
| | 322 | 361 | 399 | 436 | 474 | 512 | 550 | 608 | 644 |
| 0.1 | 69 | 70 | 71 | 73 | 74 | 76 | 77 | 79 | 80 |
| | 328 | 365 | 403 | 441 | 479 | 517 | 555 | 566 | 649 |

From the above table we note that as $C_4$ values increase there is a marginal change in the values of 'P' and K (P) i.e. the optimum quantity to be retained and the associated costs.

**CONCLUSIONS:**

In this chapter we have considered that the inventory is depleted not only by demand but also by Weibull distribution deterioration. Moreover we find that the feasibility condition for working of this model is proposed in equation 18. Also the above sensitivity tables show that the influence of the parameter 'b' is more significant than the changes in the other parameter like 'a' and $C_4$ values. It would be interesting to deal with this model in the context of finite Horizon Model. It also very interesting to deal with this situation in probabilistic demand. However, one cannot expect a closed form solution for the optimum quantity to be retained. In such situation one can use any search method like Genetic Algorithm.

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