RESEARCH PAPER | Statistics | Volume : 3 | Issue : 6 | June 2013 | ISSN - 2249-555X

Maximum Allowable Proportion Defective (MAPD) --- A Quality Measure

KEYWORDS: Maximum Allowable Proportion Defective , Point of inflexion, OC curve, Marginal Decrease, Probability of acceptance.

Ramkumar.T.B

Department of Statistics, St.Thomas' College, Thrissur, Kerala, India. 680001

ABSTRACT *This is a summery of MAPD as quality measure with substantiating evidences and properties. How it will be superior to other measures was stated based on papers. Also the derived sampling plans with respect to MAPD was mentioned. Some interesting results were stated. Graphs are prepared to depict the inflection property as well as the maximizing concept. Tables can be used to assess the other quality levels of any sampling plan.*

Introduction

Paul .L Mayer (1956) in the paper on "Some properties of the inflection point on OC curve for a single sampling plan" introduced the concept of Maximum Allowable Proportion Defective (MAPD). Formerly it was considered only as the inflection point of OC curve. Attention was focused to assess to what degree the inflection point empowers the OC curve to discriminate between good and bad lots in a single sampling plan using Poisson model. MAPD is the first incoming quality index which is not in terms of prefixed level of probability of acceptance of the lot but it was a logical formation of probability. Also it is a direct comparison of proportion of defective ($d/n \leq c/n$) to the maximum allowable level of acceptance.

Properties of MAPD

The following were the properties of MAPD as an incoming quality measure which imperate the usefulness, admissibility and efficiency of this measure.

1. A layman can understand the logic of acceptance of a lot inspected, even without knowing the theory of probability, since $d \leq c$ leads to MAPD.
2. Technicians and Engineers, who are not conversant with theoretical background, welcome MAPD due to the fact that $d/n \leq c/n$, leading to the acceptance of lot, where d is the number of defectives observed in the sample.
3. Engineers with little idea of Statistics can start with the initial quality at MAPD and later Statisticians can translate this to AQL or IQL or LTPD.
4. AQL, LTPD, AOQL, IQL are purely mathematical while the concept of MAPD is logically simple, at the same time highly mathematical.
5. Misleading interpretations were possible in the measures like AQL, AOQL, and LTPD unless the services of well trained engineers and experienced inspectors were at our disposal, due to probability statements. The use of such measures may leads to acceptance of undesirable quality or rejection of usable material. But such a situation is less seen in the case of MAPD.
6. $p^* = c/n$ for any single sampling plan such that MAPD is directly obtained from the Sampling Plan Parameters. (without any probability statement)
7. MAPD can be geometrically interpreted being the highest slope - tangent of OC curve.
8. MAPD is unique for a sampling plan, at the same time for a MAPD, many sampling plans does exist.
9. p* represents the point of inflection of OC curve so that probability of descent is only marginal up-to p* and steep decline occurs beyond p* so that probability of acceptance reduces quickly and only a few bad lots were expected to accept beyond it.
10. It is the utmost representative quality on an OC curve since maximum negative tangent occurs at that point. (Fig:1)
11. For matching OC curves, MAPD can be better than AQL or LTPD or IQL.
12. An OC curve can be characterized by a single point MAPD (with specification of probability of acceptance at p* (Probability at Allowable Risk), as two points were essential in all other situations.
13. MAPD can be determined graphically by drawing curves (1) Pa(p),(2) d/dp(Pa (p)),and (3) d^2/dp^2 (Pa (p)). It is the minimum point of second curve and point of intersection of third curve with p-axis. The second derivative of probability of acceptance function crosses the p axis at MAPD. The third derivative of probability of acceptance function is positive at MAPD and hence rate of decrease is minimum at MAPD (Fig:1).
14. The y coordinate PAR (Pa(p*)) of MAPD is a function of c alone so that it is independent of the sample size.
15. In some practical situations where the quality improves, MAPD is used to represent the quality, since OC curve can show the difference without affecting the probability of acceptance.
16. MAPD is both consumer protective (as sudden decline is expected for $p>p^*$) and producer protective.(marginal decrease is expected at better quality).
17. There is no prefixed risk figure required to define MAPD while they are kept at constant level in AQL and LTPD.
18. MAPD and p_t (point of inflection tangent crossing the p-axis) together determine a sampling plan unique, where p_t can be considered as a measure of degree of sharpness of OC curve.
19. The pair (MAPD, MAAOQ) provides a unique sampling plan compared to AOQL.
20. The pair (MAPD, AOQL) is not capable of providing a unique sampling plan.
21. The ratio of MAPD to AOQL is no unique and it is minimum at c=5 deciding an optimum sampling plan for given MAPD.
22. (p*, Pa(p*) determines Sampling Plan uniquely.
23. The pair (MAPD, tanA) determines a better discriminating sampling plan, where A is the angle subtended by the inflection tangent with p-axis so that tanA is the slope of the OC curve at MAPD and the plan is unique.
24. The ratio $(p_2 - p_1) / (p^* - p_1)$ uniquely determines a SSP. MAPD uniquely determines the minimum tangent and hence a unique sampling plan.
25. MAPD with discriminant distance D provide a unique sampling plan
26. (MAPD,AQL) provide a unique sampling plan
27. (MAPD,LTPD) suggests a attribute sampling plan uniquely.
28. MAPD-Min ATI provide better protection compared to Dodge LTPD Min ATI plan.

Volume : 3 | Issue : 6 | June 2013 | ISSN - 2249-555X

29. The relation AQL<MAPDx(100c/n) found in attribute sampling and MIL-STD-414 for variable sampling plan is a real eye opener for Engineers and Statisticians to apply a maximum allowable quality level instead of 95% quality level confidently.
30. AOQ curve also has the point of inflection nearby MAPD and both coincide for large sample size .
31. The ''sliding scale'' effect of other measures leads to higher acceptability for larger lots with big sample size, but this is not the case with MAPD as it is a direct ratio of c and n.
32. MAPD is less than or equal to the point of control, while for large sample size both coincide.
33. The inequality $p^* < p_0 < p_t$ holds for any sampling plan and they will coincide for large sample size.
34. The second derivative of probability of acceptance Pa (p) is zero at MAPD and it is positive for p > p* and negative for p <p*.(Fig:2)
35. For SSP, let p_m be the p coordinate of AOQL then p_m <p* for c ³3 and p_m <p* for c £ 2.
36. When the difference between p* and p_t decreases the discriminating power of OC curve increases.
37. Even though the difference between p* and p_t decreases to zero for large sample sizes the difference between probabilities of acceptance at p* and p_t is a constant.
38. The relative slope at MAPD h*=p*/ (p_t –p*) is an index showing the discriminating power .
39. When c is fixed p_t and p* are linearly related and relation is

$$p_t = p^*.\left[1+\frac{c!}{c^{(c+1)}}.\sum_{r=0}^{c}\frac{c^r}{r!}\right]$$

40. Empirical Relation between AOQL and MAPD

$$AOQL = MAPD.\frac{a.c^k}{b^c}$$

where a=0.45832955, b=1.8366067, k=0.12802105

AOQL ³ 0.6336 p* and MAPD > AOQL for all sampling plans

0.6336p* £ AOQL £ 0.84p* for c=1,2,3,............40.

0.6336p* £ AOQL £ 0.744325p* for c=5……….40.

41. MAPD/Tan A or MAPD* Tan A will not give unique sampling plan where A is the steepness angle subtended by the tangent at MAPD on the OC curve.
42. For SSP, AQL+LTPD³ 2*MAPD where c=1,2,……….58.
43. MAPD and IQL is approximately related linearly by 0.6728 ie IQL≈MAPD+0.6728.
44. LTPD ³3MAPD-2AQL in the case c=1,2,3.
45. For process controlled data, the process average $\bar{p}$ < p*

REFERENCE

1.Mayer. (1956). Some properties of inflection point of an OC curve for a Single Sampling Plan | Proceedings of the 2nd Statistical Engineering Symposium, Published in Chemical Corps. Engineering Command. | 2.Norman Bush,E.J.Leonard, Marvin.Q.M, Marchant.J.R. (1953). A method of discrimination for Single and Double Sampling OC curves utilising tangent of the point of inflection, ENASR Report No.P.R.-7.Engineering agency. | 3.Mandelson. (1962). The Statistician, Engineer, Sampling Plans, Industrial Quality Control, 19,12-15. | 4.Soundararajan.V. (1975) Maximum Allowable Percent Defective (MAPD) | Single Sampling Inspection by Attributes Plan. Journal of Quality Technology. 7, 4, 173-182. | 5.Muthuraj and Soundararajan.V. (1989). Single Sampling Plan indexed by point of control and inflection point. Communication in Statistics, Theory and Methods,14,10,2393-2410. | 6.Suresh.K.K and Ramkumar.T.B. (1994). A Consumer protected indexing procedure | for selection of Single Sampling Plan. Seminar on 25th year annual conference, Bangalore University. | 7.Suresh.K.K and Balamurali.S. (1994).Construction and solution of TNT Plans indexed by MAPD, Journal of Applied Statistics, Vol 21,6,587-595. | 8.Ramkumar and Suresh.(1996). Selection of sampling plan indexed with maximum allowable average outgoing quality. Journal of Applied Statistics, 23, 6, 645-654. | 9.Radhakrishnan (2008) Selection of SSP using conditional weighted Poisson Distribution. Journal of Statistics and systems Vol:3 Issue:1 | 10.Ramkumar.T.B. (2009) A Result on Certain characteristics of Average Outgoing Quality Limit, International Transactions in Applied Sciences V1, N4. | 11.Ramkumar.T.B. (2009). Design of SSP by Discriminant at MAPD, Probstatforum, 104-114. |