Finite Element Analysis of Twin Screw Extruder

1.1 Twin screw extruders
Twin screw extruders are used in various parts in polymer industry. In most applications, solid polymer pellets are fed to the extruder, which gives three different zones in twin screw extruders. These zones are:

• A solid transport zone, which is filled with polymer pellets from the hopper;
• A melt zone, where the polymer melts;
• A pump zone, which is completely filled with material. In this zone pressure is built up to overcome the die resistance.

In a polymerisation process with a liquid starting material, only two zones can be distinguished in the extruder (figure 1):

• a partially filled zone, where the screws are not completely filled with material;
• a pump zone or fully filled zone, where pressure is built up. The transition from the partially filled zone to the fully filled zone takes place within one pitch length. When the extruder is used as a polymerisation reactor, also a reaction zone can be identified. The position of the reaction zone depends on product properties and on process parameters such as barrel temperature, throughput, screw speed and die resistance. The optimum situation in reactive extrusion is obtained when the reaction takes place in the whole extruder volume. However, in many applications, the extruder used is larger than necessary, which implies that the extruder is starved fed, resulting in a partially filled zone as illustrated in figure 1.

1.2 Analysis of Flow in Extruder
As discussed in the previous section, it is convenient to consider the output from the extruder as consisting of three components – drag flow, pressure flow and leakage. The derivation of the equation for output assumes that in the metering zone the melt has a constant viscosity and its flow is isothermal in a wide shallow channel. These conditions are most likely to be approached in metering zone.

a) Drag Flow Consider the flow of the melt between parallel plates as shown in Fig.2 (a) for the small element of fluid ABCD the volume flow rate dQ is given by

\[ dQ = V \cdot dy \cdot dx \]  

Assuming the velocity gradient is linear, then

\[ V = V_d \left( \frac{y}{H} \right) \]  

Substituting in above eq. and integrating over the channel depth, the total drag flow is given by

\[ Q_d = \int_{0}^{H} V_d \cdot dy \cdot dx \]  

\[ Q_d = \frac{1}{2} THV_d \]  

This may be compared to the situation in the extruder where the fluid is being dragged along by the relative movement of screw and barrel. Fig. 3 shows the position of the element of fluid and equ. (5) may be modified to include terms relevant to the extruder dimension.

\[ V_d = \pi DN \cos \Phi \]
In both cases, \( AB = dz \), element width = \( dx \) and channel width = \( T \)

**Fig. 2** Material flow between parallel

Where \( N \) is the screw speed (in revolution per unit time).

\[
T = \left( \frac{\pi D}{2} \tan \phi - e \right) \cos \phi
\]  
\[
Q_d = \frac{1}{2} \left( \pi D \tan \phi - e \right) \left( \pi D \cos^2 \phi \right) T
\]

In most cases the term, \( e \), is small in comparison with \((\pi D \tan \phi)\) so this expression is reduced to

\[
Q_d = \frac{1}{8} \pi^2 D^2 N H \sin \phi \cos \phi
\]

Note that the shear rate in the metering zone will be given by \( \frac{V_s}{H} \)

**Fig. 3** Detail of Extruder Screw

**(b) Pressure flow:** Consider the element of fluid shown in fig. forces are

\[
F_1 = \left( \frac{P}{2} \right) dz \quad (10)
\]

\[
F_2 = P \cdot dy \quad dz \quad (11)
\]

\[
F_3 = \tau_y dz dx \quad (12)
\]

Where \( P \) is pressure and \( d\tau \) is the shear stress acting on the element. For steady flow these forces are in equilibrium so they may be equated follows:

\[
F_1 = F_2 + 2F_3 \quad (13)
\]

Which reduces to

\[
\frac{P}{dz} \frac{dy}{dz} = \tau_y \quad (14)
\]

Now for a Newtonian fluid, the shear stress, \( \tau \), is related to the viscosity \( \eta \) and the shear rate \( \gamma \), by equation

\[
\tau_y = \eta \gamma = \eta \frac{dy}{dz} \quad (15)
\]

Using this equation in above

\[
\frac{P}{dz} \frac{dy}{dz} = \eta \frac{dy}{dz} \quad (16)
\]

Integrating

\[
\int_0^y dy = \frac{1}{2} \frac{dz}{dy} \int_{y/2}^{y/2} y dy \quad (17)
\]

\[
\frac{y}{2} = \frac{1}{2} \frac{dz}{dy} \left( \frac{y^2}{2} - \frac{H^2}{8} \right) \quad (18)
\]

Also, for the element of fluid of depth \( dy \), at distance \( y \) from the centre line (and whose velocity id \( V \)) the element flow rate \( dQ \), is given by

\[
dQ = VT \quad dy \quad (19)
\]

This may be integrated to give the pressure flow \( Q_p \)

\[
Q_p = 2 \int_0^y \frac{1}{\eta} \frac{dz}{dy} \cdot T \left( \frac{y^2}{2} - \frac{H^2}{8} \right) dy \quad (20)
\]

\[
Q_p = \frac{1}{12\eta} \frac{dz}{dy} \cdot TH^3 \quad (21)
\]

Referring to the element of fluid between the screw flights as shown in Fig. 3, this equation may be rearranged using the following substitutions.

Assuming \( e \) is small,

\[
T = \pi D \tan \phi \cos \phi \quad (22)
\]

Also, \( \sin \phi = \frac{dl}{dz} \) so \( \frac{dl}{dz} = \frac{dl}{dz} \sin \phi \)

Thus the expression for \( Q_p \) becomes

\[
Q_p = \pi D H^3 \sin \phi \cos \phi \quad (24)
\]

**(C) Leakage:**

The leakage flow may be considered as flow through a wide slit which has a depth, \( \delta \), a length \( (e \cos \phi) \) and a width of \( \pi D \cos \phi \). Since this is a pressure flow, the derivation is similar to that described in (b). For convenience therefore the following substitution may be made …

\[
h = \delta \quad (25)
\]

\[
T = \pi D \cos \phi \quad (26)
\]

\[
Pressure \ gradient = \frac{\Delta P}{e \cos \phi} \quad (27)
\]

So the leakage flow, \( Q_L \), is given by

\[
Q_L = \frac{\pi D D^2 H^3}{12\eta} \tan \phi \frac{dl}{dz} \frac{dl}{dz} \quad (28)
\]

A factor is often required in this equation to allow for eccentricity of the screw in barrel. Typically this increase the leakage flow by about 20%
Fig. 5 - Variation of drag flow and pressure flow

Where ‘L’ is the length of the extruder. In practice the length of an extruder screw can vary between 17 to 30 times the diameters of the barrel. Maximum output would be obtained if the screw flight angle was about 35°. In practice a screw flight angle of 17.7° is frequently used because

i) this is the angle which occurs if the pitch of the screw is equal to the diameter and so it is convenient to manufacture.

ii) For a considerable portion of extruder length, the screw is acting as a solids conveying device and it is known that the optimum angle in such cases is 17° to 20°.

1.3 Extruder /Die Characteristics:

From equation (29) it may be seen that there are two interesting situation to consider. One is case of free discharge where there is no pressure build up at the end of the extruder so

\[ Q = Q_{\text{max}} = \frac{1}{2} \pi D^2 N H \sin \Phi \cos \Phi \]  

(30)

The other case is where the pressure at the end of the extruder is large enough to stop the output. From (29) with \( Q = 0 \) and ignoring the leakage flow

\[ P = P_{\text{max}} = \frac{6 \sigma_D L N H}{R^2 \tan \Phi} \]  

(31)

In Fig. 4.12 these points are shown as the limits of the screw characteristic. It is interesting to note that when a die is coupled to the extruder their requirements are conflicting. The extruder has a high output if the pressure at its outlet is low. However, the outlet from the extruder is the inlet to the die and the output of the latter increases with inlet pressure. As will be seen later the output, \( Q \),of a Newtonian fluid from a die is given by a relation of the form

\[ Q = K P \]  

(32)

Where \( K = \frac{\pi R^4}{8 \eta L_d} \) for a capillary die of radius \( R \) and length \( L_d \)

Equation (32) enables the die characteristics to be plotted on Fig. and the intersection of the two characteristics is the operating point of the extruder. This plot is useful in that it shows the effect which changes in various parameters will have on output. For example, increasing screw speed, \( N \), will move the extruder characteristic upward. Similarly an increase in the die radius, \( R \), would increase the slope of the die characteristic and in both cases the extruder output would increase.

The operating point for an extruded die combination may also be determined

from equations (32) and (29) - ignoring leakage flow

\[ Q = \frac{1}{2} \pi D^2 N H \sin \Phi \cos \Phi - \frac{\pi D H^2 \sin^2 \Phi P}{12 \eta} - \frac{\pi R^4}{8 \eta L_d} P \]  

(33)

1.4 Experimental setup:

In this experiment we have Twin screw of following specification which consider as

- MOC – SS 304
- Screw Dia – 50 mm
- Flit height – 13 mm
- Flit thickness – 10 mm
- Pitch of screw – 15 mm
- RPM of screw – 55 rpm
- Flit angle – 17.7°

As per analysis of flow of extruder we can calculate pressure generated in screw at the time of drag flow, pressure flow & leakage flow. Here we are considering max. pressure generated due to over material feed & which is transferred towards die.

Finite Element Analysis Results:

Here we created a model of given dimension in CATIA software which is export to Hypermesh software for analysis.

<table>
<thead>
<tr>
<th>Description</th>
<th>Calculated Value</th>
<th>Analysis Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Stress Value</td>
<td>Average stress</td>
<td>Min. 149 N/mm²</td>
</tr>
<tr>
<td></td>
<td>599 N/mm²</td>
<td></td>
</tr>
</tbody>
</table>

According to screw rpm pressure generated & out put of extruder changes. Observed values of screw rpm and related output values are tabulated in following figure.

Fig. 7 – Hyperview of vonMises Stresses in Twin Screw

Fig. 8 – Influence of Screw speed
Conclusion :-
We have taken the screw material SS 304 & its youngs modulus of elasticity is 2.1 x 10^5 N/mm².

As we considering condition of max. pressure generated at extruder die and according to that we calculate stress developed.

We calculated stress value of 599 N/mm² with given formule & with finite element analysis we get that value in range between 149 N/mm² to 6422 N/mm².

Thus calculated value & finite element analysis value are under the limit value for selected material of screw, design of screw is safe.

REFERENCE