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Analysis of Vortex Around Bluff Body

KEYWORDS: Circular Cylinder, Vortex, Vibration, Boundary Condition.

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ABSTRACT *Flow around circular cylinder is studied here. In this flow situation there are two free shear layers that in the flow bounding the wake region. Many mathematical models have been developed to characterize the response in the lock-in region or wake region. The objective of this unsteady flow theory is to determine the structural response at vortex shedding. The theory may be considered to be a single degree of freedom model. By the help of mathematical model for vortex induced vibration of circular cylindrical structures are determined by unsteady flow theory. The phenomenon of structural response to vortex shedding has received wide attention. When the natural frequency of a structure is close to the vortex shedding frequency lock in resonance may occur depending on the mass damping.*

INTRODUCTION

Vortex shedding across a bluff body has been studied for more than 100 years. The phenomenon of structural response to vortex shedding has received wide attention. When the natural frequency of a structure is close to the vortex shedding frequency, Lock-in resonance may occur depending on the mass damping parameter and structure may collapse. Many mathematical models have been developed to characterize the response in the lock-in region or wake region. The objective of this unsteady flow theory is to determine the structural response at vortex shedding. The theory may be considered to be a single degree of freedom model.

UNSTEADY FLOW THEORY

By the help of mathematical model for vortex induced vibration of circular cylindrical structures are determined by unsteady flow theory. Motion dependent fluid forces are measured in a water channel by Chen. From the measured fluid forces fluid stiffness and fluid damping coefficients are calculated as a function of reduced flow velocity and oscillation amplitude. Once these coefficients are known the mathematical model can be applied to predict the structural response to vortex shedding including response amplitude lock in frequency and stability characteristics.

The phenomenon of structural response to vortex shedding has received wide attention. When the natural frequency of a structure is close to the vortex shedding frequency lock in resonance may occur depending on the mass damping.

Fluid excitation forces and their effects are included in the equation of a single degree of freedom system. The fluid effects are characterized in terms of fluid damping, fluid stiffness or the other fluid force coefficients that are a function of system parameters and may be linear or nonlinear.

Consider a tube vibrating in a flow as shown in Fig. 1. The axes are parallel to x and y direction as shown in figure 1. says tube 1. The radius of tube is r, and the fluid is flowing with a gap flow velocity U. The displacement components of tube in the x and y directions are u and v respectively. The motion-dependent fluid-force components acting on tube in the x and y directions are, respectively, and are given by Chen (1987).

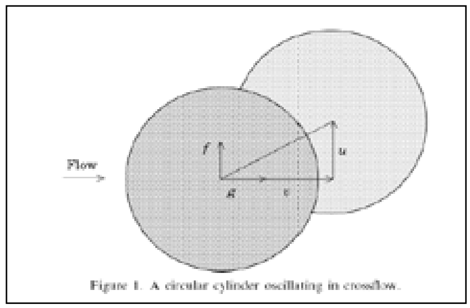

Figure 1. A circular Cylinder oscillating in cross flow

$$f = -\rho\pi R^2\left(\alpha\frac{\partial^2 u}{\partial t^2} + \sigma\frac{\partial^2 v}{\partial t^2}\right) + \frac{\rho U^2}{\omega}$$
$$\left(\alpha'\frac{\partial u}{\partial t} + \sigma'\frac{\partial v}{\partial t}\right) + \rho U^2\left(\alpha'' u + \sigma'' v\right), \quad (1)$$

$$g = -\rho\pi R^2\left(\tau\frac{\partial^2 u}{\partial t^2} + \beta\frac{\partial^2 v}{\partial t^2}\right) + \frac{\rho U^2}{\omega}$$
$$\left(\tau'\frac{\partial u}{\partial t} + \beta'\frac{\partial v}{\partial t}\right) + \rho U^2\left(\tau'' u + \beta'' v\right), \quad (2)$$

Where f and g are force in X and Y direction, ρ is fluid density, t is time, ω circular frequency of tube oscillations a, b, s and t and are added mass coefficients a¢, b¢, s¢ and t¢ are fluid damping coefficients, a², b², s² and t²are fluid-stiffness coefficients. Motion-dependent fluid forces depend on deviation from a reference state.

Various methods can be used to measure fluid force coefficients. The unsteady flow theory is used. Fluid force coefficients can be determined by measuring the fluid forces acting on the cylinder as a result of its oscillations. If the cylinder is excited in the x direction, its displacement in the x direction is given by

$$u = d_0 \cos\omega t \quad (3)$$

Where d_0 is the oscillation amplitude. The motion dependent fluid force components acting on the cylinder in the x and y directions are

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$$f = \frac{1}{2}\rho U^2 e_l \cos(\omega t + \phi_t) d_0,$$
$$g = \frac{1}{2}\rho U^2 e_d \cos(\omega t + \phi_d) d_0 \quad (4, 5)$$

Where el and ed are the fluid force amplitudes fl and fd are the phase angles by which the fluid forces acting on the cylinder lead the displacement of the cylinder. By using equations (1)-(3) we can also write the fluid force components as

$$f = (\rho\pi R^2\omega^2\alpha + \rho U^2\alpha'')d_0\cos\omega t - \rho U^2\alpha' d_0 \sin\omega t \quad (6)$$

$$g = (\rho\pi R^2\omega^2\tau + \rho U^2\tau'')d_0\cos\omega t - \rho U^2\tau' d_0 \sin\omega t \quad (7)$$

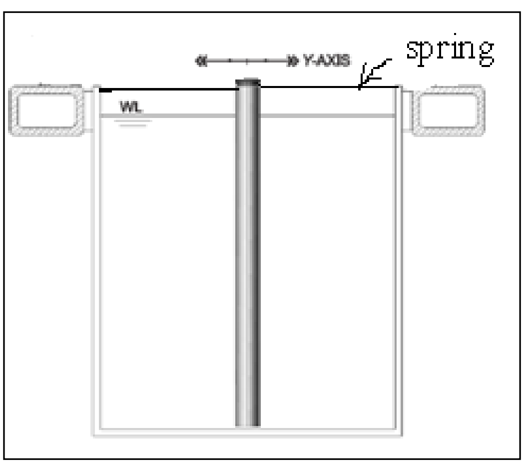
Use of equations (4) and (6) and equations (5) and (7) yields

$$\alpha'' = \frac{1}{2}e_l\cos\phi_l - \frac{\pi^3}{U^2}\alpha \quad \tau'' = \frac{1}{2}e_d\cos\phi_d - \frac{\pi^3}{U^2}\tau,$$
$$,$$
$$\alpha' = \frac{1}{2}e_l\sin\phi_l, \quad \tau' = \frac{1}{2}e_d\sin\phi_d, \qquad (8)$$

Where U is the reduced flow velocity (U=pU/wR). The added mass coefficients a and b can be calculated from the potential flow theory. Other fluid force coefficients b¢, b², s¢ and s², can be obtained in a similar manner by exciting the cylinder in the y direction.

EXPERIMENT SETUP

We prepare the experiment setup in laboratory as shown in figure 2. The channel is shown in figure 3. Water is pumped into an input tank. The flow passes through a series of screens and honeycombs and then into a rectangular flow channel. The water level is controlled by standpipes in the output tank and the flow is controlled by the running speed of the pump motor. Flow velocity is measured with a Pitot tube. The rate of propeller rotation is directly proportional to stream velocity. The relatively rigid main bodies of the tubes are constructed from stainless plastic tubing with a 2.54 cm (1-inch), and a 45 cm (17.71-inch) length. A water channel was used to measure motion dependent fluid forces.



As shown in figure 2. we can see the rectangular channel of 50 cm and length is 10 meter. When water is released from pump then at velocity 0.15 m/s we see the vibration and vortex around circular cylinder as shown in figure 3. Figure 2. Sketch diagram of experiment setup

Figure 3.Vortex flow around circular cylindrical

By help of the figure 4. we find that we can determine the value of force in x direction and y direction at given reduced velocity and from figure 5 and figure 6 we can determine the value of fluid damping coefficient. Chen determine the value of Fluid-damping coefficients Fluid-stiffness coefficients at

$$U_r = \frac{\pi U}{\omega r}$$

different reduced velocity (U_r) Where

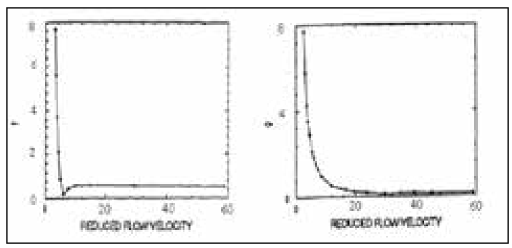


Figure 4. Magnitude of fluid force in the x direction and y direction

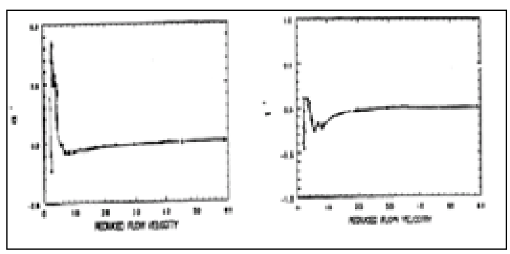


Figure 5. Fluid-damping coefficients α ζ

By help of figure 7 and 8 we can determine the value fluid stiffness coefficient at different reduced velocity. By figure 3.9 we can find out Fluid-stiffness coefficients. This is all use in equation 10.

The experimentally determined fluid force coefficients for the three cases shown in Figure1 are shown in figure All fluid force coefficients were plotted as a function of reduced flow velocity U_r (U/fD) where U=gap velocity, f=ω/2∏, D=cylinder diameter.

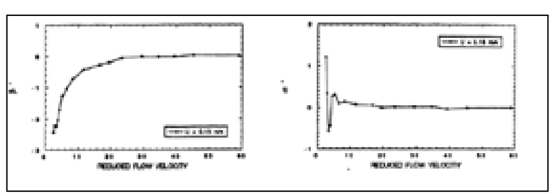


Figure6. Fluid-damping coefficients β σ

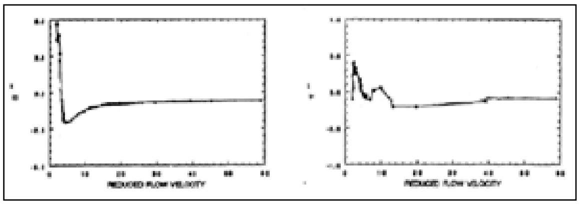


Figure7. Fluid-stiffness coefficients α² ζ²

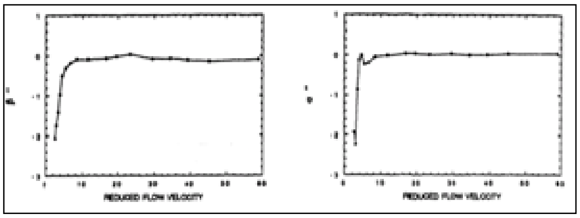


Figure8. Fluid-stiffness coefficients β² σ²

1. At high reduced flow velocity the coefficients were almost independent of reduced flow velocity and excitation amplitude. This characteristic is similar to that observed in other general tube. Therefore, at high reduced flow velocity the fluid force coefficients are much easier to quantify they can be determined for various values of flow velocity excitation frequency and excitation amplitude.
2 The coefficients ζ¢ and ζ² were relatively small this means that the
3 Cylinder motion in the lift direction induces a very small effect on the drag direction.
4 Drastic changes in the fluid force coefficients occurred in the region corresponding to vortex shedding.

EQUATION OF MOTION FOR VORTEX INDUCED VIBRATION

Once the fluid excitation forces and motion dependent fluid forces are known, the response of the cylinder can be predicted by equation 9. As an example consider a single tube supported by springs. The tube is subjected to a cross flow uniformly along its length. Then equation is used this is the most important equation in this equation we determine the value of Fluid-stiffness coefficients and Fluid-damping coefficients by figure 5 figure 6 ,figure 7 and figure 8.

$$m\frac{d^2u}{dt^2} + C\frac{du}{dt} + Ku + \frac{\rho\pi D^2}{4}\alpha\frac{d^2u}{dt^2} - \frac{\rho U^2}{w}\alpha'\frac{du}{dt} - \rho U^2\alpha' u = \frac{1}{2}\rho U^2 D C'_L\cos(\omega_s t) \qquad (9)$$

Where
K= the spring constant.
C = damping coefficient.
M = tube mass per unit length.
C'_L = fluctuating lift coefficient.
ωs = circular frequency of vortex shedding.

The variables i.e. natural frequency wv and modal damping ratio ζv can be calculated from the equation of motion and appropriate boundary conditions or from an in test practically in air

$$u(z,t) = D\varphi(t)\quad U_r = U/fD\ ,\quad \gamma = \rho\pi D^2/4m \qquad (10)$$

Where f is oscillation frequency and U is the gap velocity. Substituting equations in equation one obtains we put the value equation 10 then we find out equation 11.

$$\frac{d^2q}{dt^2} + 2\varsigma\omega\frac{dq}{dt} + \omega^2 q = \frac{1}{2(1+\gamma\alpha)}\left(\frac{\rho U^2 C'_L}{m}\right)\cos(\omega_s t), \qquad (11)$$

Where, $\omega = \omega_v(1+\gamma C_M)^{-0.5}$

$$\varsigma = \frac{\varsigma_v}{1+\gamma\alpha}\left[(1+\gamma C_M)^{0.5} - \frac{\gamma U^2_r\alpha'}{2\varsigma\pi^3}\right],\quad C_M = \alpha + \frac{U^2_r\alpha''}{\pi^3} \qquad (12)$$

RESULT

The unsteady flow theory can form the basis for calculating a complete response due to flow the calculation will require an iteration technique theoretically it will depend on as an approximation the data for a specific u may be used for this purpose.

By the help of this study and this experiment velocity of flowing water channel at U = 0.15 and by the help of this we can determine the value of reduced velocity by using this formula.

$$Ur = \frac{U}{fD} = \frac{0.15}{.2\times.0265} = 28.30$$

$\gamma = \rho\pi D^2/4m$ (From eq. 10)

$\gamma = \frac{1000\times3.14\times(.0265)^2}{4\times.5}$ Where m is the mass of tube per meter m= $\frac{.200}{.40} = .500$ kg/m

$\gamma = 1.1025$

w_v = natural Frequency= $\sqrt{\frac{m}{k}} = \sqrt{\frac{.500}{.0003}} = 40.82$ where k=stiffness of plastic

$C_M = \alpha + \frac{U^2_r\alpha''}{\pi^3}$ (From eq. 12)

$$C_M = 1.114 + \frac{(28.30)^2\times1.0}{3.14^3} = 26.98$$

$\omega = \omega_v(1+\gamma C_M)^{-0.5}$ (from eq. 12)

$$\omega = 40.82(1+1.1025\times26.98)^{-0.5} = 1.3276$$

$$\varsigma = \frac{\varsigma_v}{1+\gamma\alpha}\left[(1+\gamma C_M)^{0.5} - \frac{\gamma U^2_r\alpha'}{2\varsigma\pi^3}\right]$$

(from eq. 12)

$$\varsigma = \frac{.02}{1+1.1025\times1.113}\left[(1+1.1025\times26.98)^{0.5} - \frac{1.}{}\right.$$

$\varsigma = -3.151$

$$\frac{d^2q}{dt^2} + 2\varsigma\omega\frac{dq}{dt} + \omega^2 q = \frac{1}{2(1+\gamma\alpha)}\left(\frac{\rho U^2 C'_L}{m}\right)\cos($$

(From eq. 12)

$$\frac{d^2q}{dt^2} + 2\times-3.151\times1.3276\frac{dq}{dt} + (1.3276)$$
$$\frac{1000\times(28.30)^2\times.1}{2(1+1.113\times1.1025)\times.5}\cos(40.82t)$$

(From eq. 12)

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$$\frac{d^2q}{dt^2} + 2\times -3.151\times 1.3276\frac{dq}{dt} + (1.3276)$$

$$\frac{1000\times(28.30)^2\times .1}{2(1+1.113\times 1.1025)\times .5}\cos(40.82t)$$

$$\frac{d^2q}{dt^2} - 8.366\frac{dq}{dt} + 1.76252q = 35.92\times 10^{-3}\cos(40$$

By solving this equation we determine the

$$\frac{d^2q}{dt^2} + 2\lambda\frac{dq}{dt} + \mu^2 q = p\cos nt$$

Where $2\lambda=-8.366$, $\lambda=-4.183$, $\mu^2=1.76252$ $p=35.92\times10^{-3}$, $n=40.82$

The auxiliary equation is $D^2+2\lambda D+\mu^2=0$ where $D=-\lambda\pm(\lambda^2-\mu^2)^{-1/2}$

$$complementry.function = e^{-\lambda t}\left[c_1 e^{t\sqrt{\lambda^2-\mu^2}} + c_2 e^{-}\right.$$

It represents the free oscillation of the system which die out as $t\rightarrow\infty$

$$Particular.integral = p\frac{1}{D^2+2\lambda D+\mu^2}\cos nt$$

$$= p\frac{1}{-n^2+2\lambda D+\mu^2}\cos nt$$

$$= p\frac{(\mu^2-n^2)-2\lambda D}{(\mu^2-n^2)^2-4\lambda^2 D^2}\cos nt =$$

$$p\frac{(\mu^2-n^2)^2\cos nt + 2\lambda n\sin nt}{(\mu^2-n^2)^2+4\lambda^2 n^2}$$

Putting $\mu2-n2=R\cos\theta$ and $2\lambda n=R\sin\theta$

$$P.I. = \frac{p}{\sqrt{(\mu^2-n^2)^2+4\lambda^2 n^2}}\cos(nt-\theta)$$

Which represent the forced oscillation of the system having

$$amplitude = \frac{35.92}{\sqrt{([1.7252 - 40.82^2]^2 + 4\times 4.183^2\times 40.86^2)}}$$

$$= .002\times 10^{-3}\,mm$$

(b) Period=$2\pi/n$

$$period = \frac{2\times 360}{40.82} = 17.63$$

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