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Buoyancy Driven Convection Due to Thermal and Salinity Gradients in a Tilted Porous Slot

KEYWORDS: Double-diffusive convection • Buoyancy-Driven convection • Convection in an inclined porous slot

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ABSTRACT *The stabilityof double diffusive convection driven by temperature and salinity gradients in a tilted porous slot saturated by porous media is investigated analytically using linear stability analysis. Here the boundaries are infinite and assumed to be free and isothermal, tilted at a small angle of inclination with respect to the horizontal. A second order perturbation method is employedin terms of small angle of inclination is used to determine the critical Rayleigh number and wave number at the critical point and is supported by numerical solution of the resulting differential equation at higher order of approximation for velocity, temperature and salinity. The method of solution and the principal results obtained are somewhat identical for horizontal boundaries and for slightly tilted slot as for as nature of flow and convective instability is concerned. But, however the expression shows a considerable deviation in the basic flow that imparts definite structure with the convection pattern under investigation.*

NOMENCLATURE

Cp_0: heat capacity, evaluated at the temperature T_0
d: depth of fluid layer
κ_0: thermalconductivity of fluid, evaluated at temperature T_0
P': pressure associated with the basic flow, i.e, undisturbed flow

$$P = \frac{P'}{\rho_0 U_c^2}$$

T_1 , T_2 : the temperatures at respectively, the lower and upper plane
T_0 : the arithmetic mean temperature $\frac{T_1+T_2}{2}$
x,y,z: dimensionless Cartesian coordinates
u , v , w: dimensionless velocity components in the x , y , z direction respectively
U_c : characteristic velocity $= \frac{\kappa_f}{\rho_0 cp_0 d}$
U(y): the basic; i.e, velocity profile(dimensionless)
R_T : Thermal Rayleigh number $= \frac{\rho c_p g \alpha_T (T_1 - T_0) d^3}{\nu\kappa}$
g: gravitational acceleration
$\alpha_T$is the coefficient of volumetric expansion of temperature
R_S : Solutal Rayleigh number $= \frac{\rho c_p g \alpha_S (S_1 - S_0) d^3}{\nu\kappa}$
$\alpha_S$is the coefficient of volumetric expansion of concentration
α& β are wave numbers in the x & z direction, respectively
Da: Darcy number = $\frac{k}{d^2}$
θ: dimensionless temperature= $\frac{T-T_0}{T_1-T_0}$
ф: dimensionless concentration $= \frac{S-S_0}{S_1-S_0}$
μ: the fluid viscosity
ν: is the kinematic viscosity
k: permeability of the porous matter
ρ: fluid density
σ: Prandtl number = $\frac{c_p \rho \nu}{\kappa_T}$ (physical property evaluated at T_0)
γ : ratio of solutal diffusivity to thermal diffusivity $\frac{\kappa_S}{\kappa_T}$

Introduction

The problem of double diffusive convection i porous media has attracted considerabl interest during the last few decades becaus of its wide range of applications, from th solidifications of binary mixtures to th migration of solutes in water saturated soils The other examples include geophysica systems, electrochemistry, and the migratio of moisture through air contained in fibrou insulation. It is well known that the buoyanc driven convection in porous media hav variety of engineering applications, such a geothermal reservoirs agriculture produc storage system, packed bed catalytic reactors the pollutant transport in underground an heat removal of nuclear power plants Because of this the study is under activ investigation and is extensively studied.

Early studies on the phenomena of doubl diffusive convection in porous media ar mainly concerned with problem of convectiv instability in a horizontal layer heated an salted from below. A comprehensive reviev of the literature concerning double diffusiv natural convection in a fluid saturated porou medium may be found in the book by Niel and Bejan(2006). The study of doubl diffusive convection in porous medium is firs undertaken by Nield(1968) on the basis c linear stability theory for various thermal an solutal boundary conditions. The onset c double diffusive convection in a horizonta porous layer has been investigated b Rudraiah et al.(1982) using nonlinea perturbation theory. The linear stabilit analysis of the thermosolutal convection i carried out by Poulikakos (1986) using th Darcy-Brinkman model. The double diffusiv convection in porous media in the presence c cross diffusion effects is analysed by Rudraia and fluid regions has also been analysed by Taslim and Narasawa (1989).Chen (1990) has implemented a linear stability analysis to investigate the effect of throughflow on the onset of thermal convection in a fluid layer overlaying a porous layer with an idea of understanding the control of convective instability by the adjustment of throughflow. Murray and Chen (1989) have extended the linear stability theory, taking in to account the effects of temperature-dependent viscosity and volumetric expansion coefficients and a nonlinear basic salinity profile.McKay (1998) has considered the onset of buoyancy-driven convection in superposed reacting fluid and porous layers. The problem of double diffusive convection in fluid saturated porous layer was later on investigated by many authors (Taunton et al. 1972 , Trevisan and Bejan 1986).

Straughan and Hutter (1999) have investigated the double diffusive convection with Soret effect in a porous layer using Darcy-Brinkman model and derived a priori bounds. An analytical and numerical study of double diffusive parallel flow in a horizontal sparsely packed porous layer under the influence of constant heat and mass flux is performed using a Brinkman model by Amahmid et al. (1999). Mamou and Vasseur (1999) have studied the double diffusive instability in a horizontal rectangular porous enclosure subject to vertical temperature and concentration gradients. Double diffusive convection in a vertical enclosure filled with anisotropic porous media has been studied numerically by Bennacer et al. (2001). Mamou et al. (2001) performed both analytical and numerical stability analysis of double diffusive convection in a confined horizontal rectangular enclosure based on Galerkin and finite element methods respectively. Using the Darcy-Brinkman model Bennacer et al. (2002) have studied thermosolutal convection in a two dimension rectangular cavity filled with saturated homogenous porous medium that is thermally anisotropic. They have presented an analytical and numerical study of combined heat and mass transfer driven by buoyancy, due to temperature and concentration variation. Bahloul et al. (2003) have carried out an analytical and numerical study on the double diffusive convection in a shallow horizontal porous layer under the influence of Soret effect.

Hill (2005) performed linear and nonlinear stability analysis of double diffusive convection in a fluid saturated porous layer with a concentration based internal heat source using Darcy's law. Double diffusive natural convection with in a multilayer anisotropic porous medium is studied numerically and analytically by Bennacer (2005). Mansour et al. (2006) have investigated the multiplicity of solutions induced by thermosolutal convection in a square cavity heated from below and subjected to horizontal concentration gradient in the presence of Soret effect.

Liang &Acrivos(1969) investigated the buoyancy driven convection in a slot and in a fluid layer bounded by the infinite parallel surfaces, tilted at a small angle φ, with respect to the horizontal. Here the instability sets in whenever the temperature difference between the two planes exceeds a certain critical value. The similarity between this and the usual case in which the planes are exactly horizontal is of course evident; in fact, both the method of solution and some of the principal results of the linear stability analysis are almost identical. However, it will be seen that, although the critical wave number will remain unaffected by tilting the planes by a small amount, a preferred mode will emerge in the form of rolls having their axes along the direction of the mean motion. Hence, owing to the existence of this basic flow which imparts a definite structure to the undisturbed system, the degeneracy usually

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associated with convection problems of this type will be removed.

In this paper the investigation is to study double diffusive buoyancy driven convection driven by both temperature and salinity gradients in an inclined porous slot bounded by two infinite parallel plates inclined at an angleφ to the horizontal. The critical Rayleigh number expression, using the linear stability analysis and the perturbation method, is obtained on the lines of weakly nonlinear theory. Within the transition range of the angle of inclination when it is small but finite lead to a rather complicated dependence of the critical Rayleigh numbers R_T and R_S on σ , γ and Da which leads to longitudinal rolls with their axis aligned in the direction of the mean flow. This is in contrast to the problem in a vertical slot in which the secondary flow pattern is known to consist of transvers rolls, their axes normal to the mean motion.

Mathematical formulation

We consider two-component Newtonian fluid-saturated horizontal porous layer confined between two boundary surfaces at $y^{'} = 0\,, d$ are taken to be free and isothermal and tilted at a small angle φ with respect to the horizontal. The layer is heated and salted from below. The configuration is as shown in figure 1. The boundary surfaces are maintained at constant temperatures T_1 and T_2 and solutal concentrations S_1 and S_2 respectively. The basic governing equations of the problem under the Boussinesq approximations are the following.

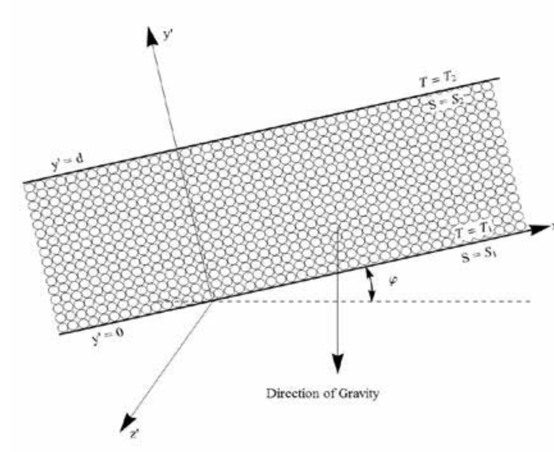

Fig1. Physical configuration of the system

The governing basic equations are

Conservation of mass:

$$\nabla \cdot \vec{q} = 0 \qquad (1)$$

Momentum equations:

$$\rho_0 c p_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q}\right] = -\nabla \mathrm{p} + \rho \mathrm{g} + \mu \nabla^2 \vec{q} - \frac{\mu}{k}\vec{q} \qquad (2)$$

Energy equation:

$$\rho_0 c p_0 \left[\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T\right] = \kappa_T \nabla^2 T \qquad (3)$$

Concentration equation:

$$\rho_0 c p_0 \left[\frac{\partial S}{\partial t} + (\vec{q}\cdot\nabla)S\right] = \kappa_S \nabla^2 S \qquad (4)$$

Equation of State:

$$\rho = \rho_0[1 - \alpha_T(T - T_0) + \alpha_S(S - S_0)] \quad (5)$$

The following non-dimensional quantities are introduced to equations (1) to (5) and using Boussinesq approximations

$$(x , y , z) = \left(\frac{x^{'}}{d}, \frac{y^{'}}{d}, \frac{z^{'}}{d}\right), \quad \vec{q} = (u\,,\ v\,,\ w) = \left(\frac{u^{'}}{U_c}, \frac{v^{'}}{U_c}, \frac{w^{'}}{U_c}\right) = \left(\frac{\vec{q^{'}}}{U_c}\right), \quad P = \frac{P^{'}}{\rho_0 U_c^2}$$

$$t = \frac{t^{'}U_c}{d}, \theta = \frac{T - T_0}{T_1 - T_0}, \emptyset = \frac{S - S_0}{S_1 - S_0}, \ T_0 = \frac{T_1 + T_2}{2} Da = \frac{d^2}{k}$$

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in which a prime refers to a dimensional variables and a script '0' to a physical quantity evaluated at the temperature T_0.

$$\nabla \cdot \vec{q} = 0 \quad (6)$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} = -\nabla p + \sigma R_T \theta g - \sigma R_S \phi g + \sigma \nabla^2 \vec{q} - \frac{\sigma}{Da}\vec{q} \quad (7)$$

$$\frac{\partial \theta}{\partial t} + (\vec{q} \cdot \nabla)\theta = \nabla^2 \theta \quad (8)$$

$$\frac{\partial \phi}{\partial t} + (\vec{q} \cdot \nabla)\phi = \gamma \nabla^2 \phi \quad (9)$$

where $R_T = \frac{\rho c_p g \alpha_T (T_1 - T_0) d^3}{\nu \kappa}$ is the Thermal Rayleigh number , $R_s = \frac{\rho c_p g \alpha_S (S_1 - S_0) d^3}{\nu \kappa}$ Solutal Rayleigh number $\gamma = \frac{k_S}{k_T}$ is the ratio of diffusivities i.e Lewes number and $Da = \frac{k}{d^2}$ is the Darcy number

Basic State Solution of the Problem

Using the basic state u = U(y) , v = w = 0 and $\theta = \theta(y)$ and boundary conditions $T = T_1 \ at\ y = 0\ and\ T = T_2 \ at\ y = 1$ in equation (8) we get

$$\theta = 1 - 2y \quad (10)$$

and $\phi = \phi(y)$ and boundary conditions $S = S_1 \ at\ y = 0\ and\ S = S_2 \ at\ y = 1$ in equation (9)

we get

$$\phi = 1 - 2y \quad (11)$$

The two boundary surfaces are free we have

$$\frac{\partial U(y)}{\partial y} = 0 \ at\ y = 0\ \&\ 1$$

$$U(y) = Da(R_T - R_S)\sin\varphi + 2\sqrt{Da}\left\{\frac{\sin h\left(\frac{y}{\sqrt{Da}}\right) - \sin h\left(\frac{y-1}{\sqrt{Da}}\right)}{1 + \cos h\left(\frac{1}{\sqrt{Da}}\right)}\right\} \quad (12)$$

$$P = P_0 + \sigma(R_T - R_S)\cos\varphi\,(y - y^2) = P(y) \quad (13)$$

where P_0 is a constant, R_T is the thermal Rayleigh number, R_S is the solute Rayleigh number, σ is the Prandtl number, γ is the Lewis number and *Da* is the Darcy number. This solution indicates that no matter how small the inclined angle φ, a shear-like flow in the x-direction [u = U(y)] will always be established, and that even in the presence of such a motion, the transport of heat from lower to the upper plane will be due to conduction alone provided no lateral boundaries exist.

Linear Stability Analysis

On the basic state, we superpose small perturbations around the basic solutions in the form, $u = U(y) + \hat{u}(x, y, z)$, $v = \hat{v}(x, y, z)$, $w = \hat{w}(x, y, z)$, $P = P(y) + \hat{p}(x, y, z)$, $\theta = -2y + \hat{\theta}(x, y, z)$ $and\ \phi = 1 - 2y + \hat{\phi}(x, y, z)$ (14)

where the caret quantities indicate small perturbations. Substituting Eq. (14) into Eqs. (6)-(9), and neglecting the non-linear terms and dropping carets yields

$$\frac{\partial u}{\partial t} + U(y)\frac{\partial u}{\partial x} + vDU(y) = -\frac{\partial p}{\partial x} + \sigma R_T \theta \sin\varphi - \sigma R_S \phi \sin\varphi + \sigma \nabla^2 u - \frac{\sigma}{Da} u \quad (15)$$

$$\frac{\partial v}{\partial t} + U(y)\frac{\partial v}{\partial x} = -\frac{\partial p}{\partial y} + \sigma R_T \theta \cos\varphi - \sigma R_S \phi \cos\varphi + \sigma \nabla^2 v - \frac{\sigma}{Da} v \quad (16)$$

$$\frac{\partial w}{\partial t} + U(y)\frac{\partial w}{\partial x} = -\frac{\partial p}{\partial z} + \sigma \nabla^2 w - \frac{\sigma}{Da} w \quad (17)$$

$$\frac{\partial \theta}{\partial t} + U(y)\frac{\partial \theta}{\partial x} - 2v = \nabla^2 \theta \quad (18)$$

$$\frac{\partial \phi}{\partial t} + U(y)\frac{\partial \phi}{\partial x} - 2v = \gamma \nabla^2 \phi \quad (19)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (20)$$

where$D=\frac{d}{dy}$ $\quad and\ \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

Cross differentiating to eliminate the pressure term and furthereliminating u and w from the equations give

$$\sigma\nabla^4 v-\frac{\partial}{\partial t}(\nabla^2 v)-U(y)\frac{\partial}{\partial x}(\nabla^2 v)+\frac{\partial v}{\partial x}D^2U(y)—\frac{\sigma}{Da}\nabla^2 v=-\sigma cos\varphi(R_T\nabla_1^2\theta-R_S\nabla_1^2\phi)+$$

$$\sigma sin\varphi\left(R_T\frac{\partial^2\theta}{\partial x\partial y}-R_S\frac{\partial^2\phi}{\partial x\partial y}\right) \qquad (21)$$

$$\left\{\nabla^2-\frac{\partial}{\partial t}-U(y)\frac{\partial}{\partial x}\right\}\theta+2v=0 \qquad (22)$$

$$\left\{\gamma\nabla^2-\frac{\partial}{\partial t}-U(y)\frac{\partial}{\partial x}\right\}\phi+2v=0 \qquad (23)$$

where $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$

Using the normal mode analysis, the dependent variables are assumed in the following form

$$v(x,y,z,t)=V(y)\exp\{i(\alpha x+\beta z-ct)\}$$

$$\theta(x,y,z,t)=\theta(y)\exp\{i(\alpha x+\beta z-ct)\} \qquad (24)$$

$$\phi(x,y,z,t)=\phi(y)\exp\{i(\alpha x+\beta z-ct)\}$$

whose real parts represent the actual physical quantities. The wave numbers $\alpha\ and\ \beta$ are real and the growth rate c, is generally complex. Substituting equation (24) into equations (21) to (23) yields the following equations

$$\left(\sigma(D^2-\alpha^2-\beta^2)^2-i\alpha\left[\left\{U(y)-\frac{c}{\alpha}\right\}(D^2-\alpha^2-\beta^2)-D^2U(y)\right]-\frac{\sigma}{Da}(D^2-\alpha^2-\beta^2)\right)v(y)$$

$$=\sigma sin\varphi\ i\alpha\{R_T D\theta(y)-R_S D\phi(y)\}+\sigma(\alpha^2+\beta^2)cos\varphi\{R_T\theta(y)-R_S\phi(y)\} \qquad (25)$$

$$[(D^2-\alpha^2-\beta^2)+ic-i\alpha U(y)]\theta(y)+2v(y)=0 \qquad (26)$$

$$[\gamma(D^2-\alpha^2-\beta^2)+ic-i\alpha U(y)]\phi(y)+2v(y)=0 \qquad (27)$$

The boundary conditions of the problem are

$$v(y)=D^2v(y)=\theta(y)=\phi(y)=0\ \ at\ y=0,1 \qquad (28)$$

Equations (25) to (27) are the familiar Orr-Sommerfeld equation coupled with the energy and concentration equation. Here, since the present study is restricted to small inclined angles φ, the above equation will be solved by using perturbation technique using *sin*φ as a small parameter perturbation quantity. Thus, expanding all the following quantities in terms of the perturbation expansions,

$$\begin{Bmatrix} c\\ \alpha\\ \beta\\ R_T\\ R_S\\ V(y)\\ \theta(y)\\ \phi(y)\end{Bmatrix}=\begin{Bmatrix} c_0\\ \alpha_0\\ \beta_0\\ R_{T0}\\ R_{S0}\\ V_0(y)\\ \theta_0(y)\\ \phi_0(y)\end{Bmatrix}+\begin{Bmatrix} c_1\\ \alpha_1\\ \beta_1\\ R_{T1}\\ R_{S1}\\ V_1(y)\\ \theta_1(y)\\ \phi_1(y)\end{Bmatrix}sin\varphi+\begin{Bmatrix} c_1\\ \alpha_1\\ \beta_1\\ R_{T1}\\ R_{S1}\\ V_1(y)\\ \theta_1(y)\\ \phi_1(y)\end{Bmatrix}sin^2\varphi+\cdots\cdots\cdots\cdots\cdots \qquad (29)$$

At zeroth order the system of equations are

$$\left[\sigma\left(D^2-\alpha_0{}^2-\beta_0{}^2\right)^2+ic_0\left(D^2-\alpha_0{}^2-\beta_0{}^2\right)-\frac{\sigma}{Da}\left(D^2-\alpha_0{}^2-\beta_0{}^2\right)\right]V_0(y)$$

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$= \sigma(\alpha_0{}^2 + \beta_0{}^2)(R_{T0}\theta_0 - R_{S0}\phi_0)$ (30)

$[(D^2 - \alpha_0{}^2 - \beta_0{}^2) + ic_0]\theta_0(y) + 2V_0(y) = 0$ (31)

$[(D^2 - \alpha_0{}^2 - \beta_0{}^2) + ic_0]\phi_0(y) + 2V_0(y) = 0$ (32)

Since the principle of the exchange of stabilities applies, ic_0 is real and the marginal state is characterized by $c_0 = 0$.
The solution for the system of equations (29) to (31) is readily available and is given by

$V_0(y) = sin\pi y\,, \quad \theta_0(y) = \frac{4}{3\pi^2} sin\pi y\,,$

$\phi_0(y) = \frac{4}{\gamma 3\pi^2} sin\pi y\,,$

$\alpha_0{}^2 + \beta_0{}^2 = \frac{\pi^2}{2}\,, R_{T0} = \frac{1}{\gamma} R_{S0} + \frac{27\pi^4}{8} + \frac{1}{Da}\frac{9\pi^2}{4}$ (33)

Prior to solving the higher order equations, it is first necessary to solve the homogeneous adjoint problem.

$\left[\sigma(D^2 - \alpha_0{}^2 - \beta_0{}^2)^2 - \frac{\sigma}{Da}(D^2 - \alpha_0{}^2 - \beta_0{}^2)\right] V^*(y) + 2\theta^*(y) + 2\phi^*(y) = 0$ $(D^2 - \alpha_0{}^2 - \beta_0{}^2)\theta^*(y) - \sigma(\alpha_0{}^2 + \beta_0{}^2)R_{T0}V^*(y) = 0$ (34)

$\gamma(D^2 - \alpha_0{}^2 - \beta_0{}^2)\phi^*(y) + \sigma(\alpha_0{}^2 + \beta_0{}^2)R_{S0}V^*(y) = 0$

And that the boundary conditions are the same as

$V(y) = D^2V(y) = \theta(y) = \phi(y) = 0 \quad at \;\; y = 0\,, 1$

Hence

$V^*(y) = V_0(y) = sin\pi y\,, \quad \theta^*(y) = -\frac{\sigma}{3} R_{T0} sin\pi y\,, \;\; \phi^*(y) = \frac{\sigma}{3\gamma} R_{S0} sin\pi y$ (35)

Substituting (29) in to (25) to (28), we next obtain for the first order equations

$$\left[\sigma(D^2 - \alpha_0{}^2 - \beta_0{}^2)^2 + ic_0(D^2 - \alpha_0{}^2 - \beta_0{}^2) - \frac{\sigma}{Da}(D^2 - \alpha_0{}^2 - \beta_0{}^2)\right] V_1(y)$$

$$+\left[\sigma\{-4(\alpha_0\alpha_1 + \beta_0\beta_1)(D^2 - \alpha_0{}^2 - \beta_0{}^2)\} - i\{2c_0(\alpha_0\alpha_1 + \beta_0\beta_1) + [-c_1 + h(y)\alpha_0(R_{T0} - R_{S0})](D^2 - \alpha_0{}^2 - \beta_0{}^2) - \alpha_0(R_{T0} - R_{S0})D^2h\} + \frac{\sigma}{Da}2(\alpha_0\alpha_1 + \beta_0\beta_1)\right]V_0(y) =$$

$$i\sigma\alpha_0(R_{T0}D\theta_0 - R_{S0}D\phi_0) + \sigma cos\varphi\{(\alpha_0{}^2 + \beta_0{}^2)(R_{T0}\theta_1 + R_{T1}\theta_0 - R_{S0}\phi_1 - R_{S1}\phi_0) + 2(\alpha_0\alpha_1 + \beta_0\beta_1)(R_{T0}\theta_0 - R_{S0}\phi_0)\}$$ (36)

$[(D^2 - \alpha_0{}^2 - \beta_0{}^2) + ic_0]\theta_1(y) + 2V_1(y) = \{2(\alpha_0\alpha_1 + \beta_0\beta_1) - ic_1 + i\alpha_0(R_{T0} - R_{S0})h\}\theta_0$ (37)

$[\gamma(D^2 - \alpha_0{}^2 - \beta_0{}^2) + ic_0]\phi_1(y) + 2V_1(y) = \{2\gamma(\alpha_0\alpha_1 + \beta_0\beta_1) - ic_1 + i\alpha_0(R_{T0} - R_{S0})h\}\phi_0$(38)

On using $c_0 = 0$, and $\alpha_0 + \beta_0 = \frac{\pi^2}{2}$ we get

$$\left[\sigma\left(D^2 - \frac{\pi^2}{2}\right)^2 - \frac{\sigma}{Da}\left(D^2 - \frac{\pi^2}{2}\right)\right] V_1(y) - \sigma\left\{\frac{\pi^2}{2}(R_{T0}\theta_1 - R_{S0}\phi_1)\right\}$$

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$$= \sigma\left\{4(\alpha_0\alpha_1 + \beta_0\beta_1)\left(D^2 - \frac{\pi^2}{2}\right)V_0(y)\right\} - i\alpha_0(R_{T0} - R_{S0})V_0D^2h$$
$$+i\alpha_0(R_{T0} - R_{S0})h\left(D^2 - \frac{\pi^2}{2}\right)V_0 - ic_1\left(D^2 - \frac{\pi^2}{2}\right)V_0 + i\alpha_0\sigma(R_{T0}D\theta_0 - R_{S0}D\phi_0)$$
$$+\sigma\frac{\pi^2}{2}(R_{T1}\theta_0 - R_{S1}\phi_0) + 2\sigma(\alpha_0\alpha_1 + \beta_0\beta_1)(R_{T0}\theta_0 - R_{S0}\phi_0) \quad (39)$$

$$(D^2 - \alpha_0{}^2 - \beta_0{}^2)\theta_1(y) + 2V_1(y) = i\alpha_0(R_{T0} - R_{S0})h\theta_0 - ic_1\theta_0 + 2(\alpha_0\alpha_1 + \beta_0\beta_1)\theta_0 \quad (40)$$

$$\gamma(D^2 - \alpha_0{}^2 - \beta_0{}^2)\phi_1(y) + 2V_1(y) = i\alpha_0(R_{T0} - R_{S0})h\phi_0 - ic_1\phi_0 + 2\gamma(\alpha_0\alpha_1 + \beta_0\beta_1)\phi_0 \quad (41)$$

Where $h(y) = \frac{U(y)}{(R_T - R_S)sin\varphi} = Da\left[1 - 2y + 2\sqrt{Da}\left\{\frac{sin\,h\left(\frac{y}{\sqrt{Da}}\right) - sin\,h\left(\frac{y-1}{\sqrt{Da}}\right)}{1 + cos\,h\left(\frac{1}{\sqrt{Da}}\right)}\right\}\right] \quad (42)$

$$\sigma\left(D^2 - \frac{\pi^2}{2}\right)^2 V_1(y) - \sigma R_{T0}\frac{\pi^2}{2}\theta_1 + \sigma R_{S0}\frac{\pi^2}{2}\phi_1$$
$$= 3\sigma\pi^2(\alpha_0\alpha_1 + \beta_0\beta_1)sin\pi y - i\alpha_0(R_{T0} - R_{S0})sin\pi y D^2h$$
$$-i\alpha_0\frac{3\pi^2}{2}h(R_{T0} - R_{S0})sin\pi y - ic_1\frac{3\pi^2}{2}sin\pi y + i\alpha_0\sigma\frac{9\pi^3}{2}cos\pi y$$
$$+\sigma\frac{2}{3}\left(R_{T1} - \frac{1}{\gamma}R_{S1}\right)sin\pi y \quad (43)$$

$$\left(D^2 - \frac{\pi^2}{2}\right)\theta_1(y) + 2V_1(y) = i\alpha_0\frac{4}{3\pi^2}h(R_{T0} - R_{S0})sin\pi y - ic_1\frac{4}{3\pi^2}sin\pi y$$
$$+\frac{8}{3\pi^2}(\alpha_0\alpha_1 + \beta_0\beta_1)sin\pi y \quad (44)$$

$$\gamma\left(D^2 - \frac{\pi^2}{2}\right)\phi_1(y) + 2V_1(y) = i\alpha_0\frac{4}{3\pi^2\gamma}h(R_{T0} - R_{S0})sin\pi y - ic_1\frac{4}{3\pi^2\gamma}sin\pi y + \frac{8}{3\pi^2\gamma}(\alpha_0\alpha_1 + \beta_0\beta_1)sin\pi y \quad (45)$$

Since the inhomogeneous part of equation must be orthogonal to the homogeneous adjoint solution, the eigen value R_{T1} , can be computed as follows: Multiplying the equation (43) by V^* , (44) by θ^* and equation (45) by ϕ^*, summing and then integrating from y = 0 to y= 1 , yields

$$R_{T1} - \frac{1}{\gamma}R_{S1} = (\alpha_0\alpha_1 + \beta_0\beta_1)\left[\frac{4}{3\pi^2}\left(R_{T0} - \frac{1}{\gamma^2}R_{S0}\right) - \frac{9\pi^2}{4} - \frac{6}{Da}\right] - ic_1\left\{\frac{9\pi^2}{4}\frac{1}{\sigma} + \frac{2}{3\pi^2}\left(\frac{1}{\gamma}R_{T0} - \frac{1}{\gamma^2}R_{S0}\right)\right\} \quad (46)$$

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$$R_{T1}-\frac{1}{\gamma}R_{S1}=(\alpha_0\alpha_1+\beta_0\beta_1)\left[\frac{4}{3\pi^2\gamma}\left(1-\frac{1}{\gamma}\right)R_{S0}-\frac{3}{Da}\right]-ic_1\left\{\frac{9\pi^2}{4}\left(1+\frac{1}{\sigma}\right)+\frac{2}{3\pi^2\gamma}\left(1-\frac{1}{\gamma}\right)R_{S0}+\frac{3}{2Da}\right\} \quad (47)$$

Since $R_{T1}\&R_{S1}$ are real, C_1 must be imaginary. Thus to this order there is no oscillatory motion and the neutral state C_1 and hence

$$R_{T1}-\frac{1}{\gamma}R_{S1}=(\alpha_0\alpha_1+\beta_0\beta_1)\left[\frac{4}{3\pi^2\gamma}\left(1-\frac{1}{\gamma}\right)R_{S0}-\frac{3}{Da}\right] \quad (48)$$

In view of equation (34) and the fact that R_{T1} , R_{S1} and C_1 , equations (44) to (46) becomes

$$\left[\sigma\left(D^2-\frac{\pi^2}{2}\right)^2-\frac{\sigma}{Da}\left(D^2-\frac{\pi^2}{2}\right)\right]V_1(y)-\sigma R_{T0}\frac{\pi^2}{2}\theta_1+\sigma R_{S0}\frac{\pi^2}{2}\phi_1$$
$$=i\alpha_0\left(\frac{9\pi^3}{2}+\frac{3\pi}{Da}\right)\sigma cos\pi y$$
$$-i\alpha_0\left(R_{T0}-R_{S0}\right)\left[\frac{2\sqrt{Da}}{1+cosh\left(\frac{1}{\sqrt{Da}}\right)}\left(1+\frac{3\pi^2}{2}\right)\left\{\sinh\left(\frac{y}{\sqrt{Da}}\right)+\sinh\left(\frac{y-1}{\sqrt{Da}}\right)\right\}+\frac{3\pi^2}{2}Da(1-2y)\right]sin\pi y$$
$$+\sigma(\alpha_0\alpha_1+\beta_0\beta_1)\left[3\pi^2+\frac{8}{9\pi^2\gamma}\left(1-\frac{1}{\gamma}\right)R_{S0}-\frac{1}{Da}\right]sin\pi y \quad (49)$$

$$\left(D^2-\frac{\pi^2}{2}\right)\theta_1(y)+2V_1(y)$$
$$=i\alpha_0\frac{4Da}{3\pi^2}(R_{T0}-R_{S0})\left\{1-2y+\frac{2\sqrt{Da}}{1+cosh\left(\frac{1}{\sqrt{Da}}\right)}\left\{\sinh\left(\frac{y}{\sqrt{Da}}\right)+\sinh\left(\frac{y-1}{\sqrt{Da}}\right)\right\}\right\}sin\pi y$$
$$+\frac{8}{3\pi^2}(\alpha_0\alpha_1+\beta_0\beta_1)sin\pi y \quad (50)$$

$$\gamma\left(D^2-\frac{\pi^2}{2}\right)\phi_1(y)+2V_1(y)$$
$$=i\alpha_0\frac{4Da}{3\pi^2\gamma}(R_{T0}-R_{S0})\left\{1-2y+\frac{2\sqrt{Da}}{1+\cosh\left(\frac{1}{\sqrt{Da}}\right)}\left\{\sinh\left(\frac{y}{\sqrt{Da}}\right)+\sinh\left(\frac{y-1}{\sqrt{Da}}\right)\right\}\right\}sin\pi y$$
$$+\frac{8}{3\pi^2\gamma}(\alpha_0\alpha_1+\beta_0\beta_1)sin\pi y \quad (51)$$

Further elimination of θ_1 and ϕ_1 yields

$$\sigma\left[\left(D^2-\frac{\pi^2}{2}\right)^3-\frac{1}{Da}\left(D^2-\frac{\pi^2}{2}\right)^2+\pi^2\left(R_{T0}-\frac{1}{\gamma}R_{S0}\right)\right]V_1(y)$$
$$=\sigma\,\frac{4}{3}\left(R_{T0}-\frac{1}{\gamma^2}R_{S0}\right)(\alpha_0\alpha_1+\beta_0\beta_1)sin\pi y$$

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$$+i\,\alpha_0\frac{2}{3}\sigma\,Da\left(R_{T0}-\frac{1}{\gamma^2}R_{S0}\right)(R_{T0}-R_{S0})\Bigg[1-2y+\frac{2\sqrt{Da}}{1+cosh\left(\frac{1}{\sqrt{Da}}\right)}\left\{\sinh\left(\frac{y}{\sqrt{Da}}\right)+\sinh\left(\frac{y-1}{\sqrt{Da}}\right)\right\}\Bigg]sin\pi y$$

$$-i\alpha_0\left(\frac{27\pi^5}{4}+\frac{9\pi^3}{2}\frac{1}{Da}\right)\sigma cos\pi y-i\,\alpha_0(R_{T0}-R_{S0})\Bigg[\frac{2\sqrt{Da}}{1+cosh\left(\frac{1}{\sqrt{Da}}\right)}\left(1+\frac{3\,\pi^2}{2}\right)\left\{\sinh\left(\frac{y}{\sqrt{Da}}\right)+\sinh\left(\frac{y-1}{\sqrt{Da}}\right)\right\}\left\{\left(\frac{1}{Da}-\frac{3\pi^2}{2}\right)sin\pi y+\frac{2\,\pi}{\sqrt{Da}}\,cos\pi y\right\}-6\,\pi^3\,Da\,cos\pi y-\frac{9\,\pi^4}{4}\,Da\,(1-2y)sin\pi y\Bigg]$$

$$-\sigma(\alpha_0\alpha_1+\beta_0\beta_1)\left[3\pi^2+\frac{8}{9\,\pi^2\gamma}\left(1-\frac{1}{\gamma}\right)R_{S0}-\frac{1}{Da}\right]\frac{3\,\pi^2}{2}sin\pi y \qquad (52)$$

where$R_{T0}=\frac{1}{\gamma}R_{S0}+\frac{27\,\pi^4}{8}+\frac{1}{Da}\frac{9\pi^2}{4}$

$$\sigma\left[\left(D^2-\frac{\pi^2}{2}\right)^3-\frac{1}{Da}\left(D^2-\frac{\pi^2}{2}\right)^2+\left(\frac{27\pi^6}{8}+\frac{1}{Da}\frac{9\,\pi^4}{4}\right)\right]V_1(y)=\sigma\;(\alpha_0\alpha_1+\beta_0\beta_1)\left[\frac{9\,\pi^2}{2}\frac{1}{Da}-\frac{8}{\gamma}\left(1-\frac{1}{\gamma}\right)R_{s0}\right]sin\pi y \qquad (53)$$

By substituting

$$V_1(y)=i\alpha_0\overline{V_1}(y)+2(\alpha_0\alpha_1+\beta_0\beta_1)\tilde{V}_1(y) \qquad (54)$$

in equation (52) we get

$$\left[\left(D^2-\frac{\pi^2}{2}\right)^3-\frac{1}{Da}\left(D^2-\frac{\pi^2}{2}\right)^2+\left(\frac{27\pi^6}{8}+\frac{1}{Da}\frac{9\,\pi^4}{4}\right)\right]\bar{V}_1(y)$$

$$= \frac{2}{3}\, Da\, \left(R_{T0} - \frac{1}{\gamma^2} R_{S0}\right)(R_{T0} - R_{S0}) \left[1 - 2y + \frac{2\sqrt{Da}}{1 + cosh\left(\frac{1}{\sqrt{Da}}\right)} \left\{\sinh\left(\frac{y}{\sqrt{Da}}\right) + \sinh\left(\frac{y-1}{\sqrt{Da}}\right)\right\}\right] sin\pi y$$

$$-\left(\frac{27\pi^5}{4} + \frac{9\pi^3}{2}\frac{1}{Da}\right)\sigma cos\pi y$$

$$-(R_{T0} - R_{S0})\left[\frac{2\sqrt{Da}}{1+cosh\left(\frac{1}{\sqrt{Da}}\right)}\left(1 + \frac{3\,\pi^2}{2}\right)\left\{\sinh\left(\frac{y}{\sqrt{Da}}\right) + \sinh\left(\frac{y-1}{\sqrt{Da}}\right)\right\}\left\{\left(\frac{1}{Da} - \frac{3\pi^2}{2}\right) sin\pi y + \frac{2\,\pi}{\sqrt{Da}}\, cos\pi y\right\} - 6\,\pi^3\, Da\, cos\pi y - \frac{9\,\pi^4}{4}\, Da\,(1-2y) sin\pi y\right] \quad (55)$$

$$\sigma\left[\left(D^2 - \frac{\pi^2}{2}\right)^3 - \frac{1}{Da}\left(D^2 - \frac{\pi^2}{2}\right)^2 + \left(\frac{27\pi^6}{8} + \frac{1}{Da}\frac{9\,\pi^4}{4}\right)\right]\tilde{V}_1(y) = \frac{1}{2}\left[\frac{9\,\pi^2}{2}\frac{1}{Da} - \frac{8}{\gamma}\left(1 - \frac{1}{\gamma}\right) R_{s0}\right] sin\pi y \quad (56)$$

With boundary conditions

$$\overline{V_1}(y) = D^2\overline{V_1} = 0 \text{ at } y = 0, 1$$

Equation (49) and (50) can be simplified by substituting

$$\theta_1(y) = i\alpha_0\, \overline{\theta_1}(y) + 2(\alpha_1\alpha_0 + \beta_1\beta_0)\widetilde{\theta_1}(y) \quad (57)$$

$$\phi_1(y) = i\alpha_0\overline{\phi_1}(y) + 2(\alpha_1\alpha_0 + \beta_1\beta_0)\widetilde{\phi_1}(y) \quad (58)$$

Then

$$\left(D^2 - \frac{\pi^2}{2}\right)\overline{\theta_1}(y) + 2\bar{V}_1(y) = \frac{4Da}{3\pi^2}(R_{T0} - R_{S0})\left(1 - 2y + \frac{2\sqrt{Da}}{1+cosh\left(\frac{1}{\sqrt{Da}}\right)}\left\{\sinh\left(\frac{y}{\sqrt{Da}}\right) + \sinh\left(\frac{y-1}{\sqrt{Da}}\right)\right\}\right) sin\pi y \quad (59)$$

$$\left(D^2 - \frac{\pi^2}{2}\right)\widetilde{\theta_1}(y) + 2\widetilde{V_1}(y) = \frac{4}{3\pi^2} sin\pi y \quad (60)$$

$$\gamma\left(D^2 - \frac{\pi^2}{2}\right)\bar{\phi}_1(y) + 2\bar{V}_1(y) = \frac{4\,Da}{3\pi^2\gamma}(R_{T0} - R_{S0})\left[1 - 2y + \frac{2\sqrt{Da}}{1+cosh\left(\frac{1}{\sqrt{Da}}\right)}\left\{\sinh\left(\frac{y}{\sqrt{Da}}\right) + \sinh\left(\frac{y-1}{\sqrt{Da}}\right)\right\}\right] sin\pi y \quad (61)$$

$$\gamma\left(D^2 - \frac{\pi^2}{2}\right)\widetilde{\phi_1}(y) + 2\widetilde{V_1}(y) = \frac{4}{3\pi^2\gamma} sin\pi y \quad (62)$$

As for $\overline{V_1}(y)$, $\bar{\theta}_1(y)$ and $\overline{\phi}_1(y)$, these had to be obtained via a numerical solution of equations (51), (55) and (58) are shown in figures . As required by their governing equations and the associated boundary conditions, both functions are anti-symmetric conditions with respect to mid-point y = 0.5. Also, it is apparent from figure that, for Prandtl number higher than 1.0.

Considering the second order terms in $sin^2\varphi$ we get ,

$$\left[\sigma\left(D^2 - \frac{\pi^2}{2}\right)^2 - \frac{\sigma}{Da}\left(D^2 - \frac{\pi^2}{2}\right)\right] V_2\,(y) - \sigma\frac{\pi^2}{2} R_{T0}\theta_2 + \sigma\frac{\pi^2}{2} R_{S0}\phi_2$$

$$= \alpha_0^2 \Big[(R_{T0} - R_{S0}) \bar{V}_1 D^2 h + (R_{T0} - R_{S0}) h \Big(D^2 - \frac{\pi^2}{2} \Big) \bar{V}_1 - R_{T0} \sigma D \bar{\theta}_1 + R_{S0} \sigma D \bar{\phi}_1 \Big]$$

$$+ i\alpha_0 \xi \Big[2\sigma \Big(D^2 - \frac{\pi^2}{2} \Big) \bar{V}_1 + h(R_{T0} - R_{S0}) \Big(D^2 - \frac{\pi^2}{2} \Big) \tilde{V}_1 - (R_{T0} - R_{S0}) \tilde{V}_1 D^2 h - \frac{\sigma}{Da} \bar{V}_1 - h(R_{T0} - R_{S0}) V_0 + R_{T0} \sigma D \tilde{\theta}_1 - R_{S0} \sigma D \tilde{\phi}_1 + \sigma R_{T0} \bar{\theta}_1 - \sigma R_{S0} \bar{\phi}_1 \Big]$$

$$+ \sigma \xi^2 \Big[2 \Big(D^2 - \frac{\pi^2}{2} \Big) \tilde{V}_1 - \frac{1}{Da} \tilde{V}_1 - V_0 + R_{T0} \tilde{\theta}_1 - \sigma R_{S0} \tilde{\phi}_1 \Big]$$

$$+ \sigma \xi \Big[\frac{\pi^2}{2} R_{T1} \tilde{\theta}_1 - \frac{\pi^2}{2} R_{S1} \tilde{\phi}_1 + R_{T1} \theta_0 - R_{S1} \phi_0 \Big]$$

$$+ i\alpha_1 \Big[(R_{T0} - R_{S0}) h \Big(D^2 - \frac{\pi^2}{2} \Big) V_0 - (R_{T0} - R_{S0}) V_0 D^2 h + \sigma R_{T0} D \theta_0 - \sigma R_{S0} D \phi_0 \Big]$$

$$+ \sigma \zeta \Big[2 \Big(D^2 - \frac{\pi^2}{2} \Big) V_0 - \frac{1}{Da} V_0 + R_{T0} \theta_0 - R_{S0} \phi_0 \Big]$$

$$+ i\alpha_0 \Big[(R_{T1} - R_{S1}) h \Big(D^2 - \frac{\pi^2}{2} \Big) V_0 - (R_{T1} - R_{S1}) V_0 D^2 h + \sigma R_{T1} D \theta_0 - \sigma R_{S1} D \phi_0 + \sigma \frac{\pi^2}{2} R_{T1} \bar{\theta}_1 - \sigma \frac{\pi^2}{2} R_{S1} \bar{\phi}_1 \Big]$$

$$+ \sigma \frac{\pi^2}{2} R_{T2} \theta_0 - \sigma \frac{\pi^2}{2} R_{S2} \phi_0 - i c_2 \Big(D^2 - \frac{\pi^2}{2} \Big) V_0 \qquad (63)$$

$$\Big(D^2 - \frac{\pi^2}{2} \Big) \theta_2 + 2V_2 = - i c_2 \theta_0 + \xi^2 \tilde{\theta}_1 + \zeta \theta_0 + i\alpha_0 \xi (R_{T0} - R_{S0}) h \tilde{\theta}_1 - i\alpha_0 \xi \bar{\theta}_1 - \alpha_0^2 (R_{T0} - R_{S0}) h \bar{\theta}_1 + i\alpha_1 (R_{T0} - R_{S0}) h \theta_0 + i\alpha_0 (R_{T1} - R_{S1}) h \theta_0 \qquad (64)$$

$$\gamma \Big(D^2 - \frac{\pi^2}{2} \Big) \phi_2 + 2V_2 = - i c_2 \phi_0 + \gamma \xi^2 \tilde{\phi}_1 + \gamma \zeta \phi_0 + i\alpha_0 \xi (R_{T0} - R_{S0}) h \tilde{\phi}_1 + i\alpha_0 \gamma \xi \bar{\phi}_1 - \alpha_0^2 (R_{T0} - R_{S0}) h \bar{\phi}_1 + i\alpha_1 (R_{T0} - R_{S0}) h \phi_0 + i\alpha_0 (R_{T1} - R_{S1}) h \phi_0 \qquad (65)$$

Where $\xi = 2\alpha_1 \alpha_0 + 2\beta_1 \beta_0$,

$\zeta = 2\alpha_2 \alpha_0 + \alpha_1^2 + 2\beta_2 \beta_0 + \beta \beta_1^2$

Again, multiplying the equation (62) by V^*, (63) by θ^* and the (64) by ϕ^*, summing and integrating, yields we get

$$R_{T2} - \frac{1}{\gamma} R_{S2} = -\alpha_0^2 R_{T0} k_1(\sigma) + \alpha_0^2 R_{S0} k_2(\sigma) - i\alpha_0 \xi k_3(\sigma) - \xi^2 k_4(\sigma) - \xi \frac{3\pi^2}{2} k_5 - i\alpha_0 \frac{3\pi^2}{2} k_5 + \frac{3}{2} \xi^2 - \frac{2\xi}{\pi^2} \Big(R_{T1} - \frac{1}{\gamma} R_{S1} \Big) - \frac{3\zeta}{2Da} - i c_2 \Big[\frac{9\pi^2}{4} \frac{1}{\sigma} - \frac{2}{3\pi^2} \Big(R_{T0} - \frac{1}{\gamma^2} R_{S0} \Big) \Big] \qquad (66)$$

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$$k_1(\sigma) = \frac{3}{\sigma}\int_0^1 \left[\left\{\bar{V}_1 D^2 h - h\left(D^2 - \frac{\pi^2}{2}\right)\bar{V}_1 - \sigma D\bar{\theta}_1\right\}V^* - h\bar{\theta}_1\theta^* - h\bar{\Phi}_1\Phi^*\right]dy$$

$$k_2(\sigma) = \frac{3}{\sigma}\int_0^1 \left[\left\{\bar{V}_1 D^2 h - h\left(D^2 - \frac{\pi^2}{2}\right)\bar{V}_1 - \sigma D\bar{\Phi}_1\right\}V^* - h\bar{\theta}_1\theta^* - h\bar{\Phi}_1\Phi^*\right]dy$$

$$k_3(\sigma) = \frac{3}{\sigma}\int_0^1 \left[\left\{2\sigma\left(D^2 - \frac{\pi^2}{2}\right)\bar{V}_1 + h(R_{T0} - R_{S0})\left(D^2 - \frac{\pi^2}{2}\right)\tilde{V}_1 - (R_{T0} - R_{S0})\tilde{V}_1 D^2 h - \frac{\sigma}{Da}\bar{V}_1 + \sigma R_{T0} D\tilde{\theta}_1 - \sigma R_{S0} D\tilde{\phi}_1 + \sigma R_{T0}\,\bar{\theta}_1 - \sigma R_{S0}\bar{\Phi}_1\right\}V^* + \{(R_{T0} - R_{S0})h\tilde{\theta}_1 + \bar{\theta}_1\}\theta^* + \{(R_{T0} - R_{S0})h\tilde{\phi}_1 + \gamma\bar{\phi}_1\}\Phi^*\right]dy$$

$$k_4(\sigma) = \int_0^1 \left[3\left\{2\left(D^2 - \frac{\pi^2}{2}\right)\tilde{V}_1 - \frac{1}{Da}\tilde{V}_1 + R_{T0}\tilde{\theta}_1 - R_{S0}\tilde{\phi}_1\right\}V^* + \frac{3}{\sigma}\{\tilde{\theta}_1\theta^* + \gamma\tilde{\phi}_1\phi^*\}\right]dy$$

$$k_5 = R_{T1}\int_0^1 \tilde{\theta}_1\, V^*\, dy - R_{S1}\int_0^1 \tilde{\phi}_1\, V^*\, dy$$

At the neutral state, c_2 must be real, since $R_{T2} - \frac{1}{\gamma}R_{S2}$ is real, equating imaginary and real parts of equations becomes

$$R_{T2} - \frac{1}{\gamma}R_{S2} = -\alpha_0^2 R_{T0} k_1(\sigma) + \alpha_0^2 R_{S0} k_2(\sigma) - \xi^2 k_4(\sigma) + \frac{3}{2}\xi^2 - \frac{2\xi}{\pi^2}\left(R_{T1} - \frac{1}{\gamma}R_{S1}\right) - \frac{3\zeta}{2Da}$$

and

$$c_2\left[\frac{9\pi^2}{4}\frac{1}{\sigma} + \frac{2}{3\pi^2}\left(R_{T0} - \frac{1}{\gamma^2}R_{S0}\right)\right] = \alpha_0\xi k_3(\sigma) + \alpha_0\frac{3\pi^2}{2}k_5$$

$$c_2 = -\frac{\left[\alpha_0\xi k_3(\sigma) + \alpha_0\frac{3\pi^2}{2}k_5\right]}{\left[\frac{9\pi^2}{4}\frac{1}{\sigma} + \frac{2}{3\pi^2}\left(R_{T0} - \frac{1}{\gamma^2}R_{S0}\right)\right]} = 0$$

Results and Discussion

This paper is an attempt to bring in the effects of porous media on the double diffusive instability in an inclined porous slot, when the infinite surfaces bounding the fluid saturated by porous media are tilted at a small angle of inclination from the horizontal direction. The expression for the critical Rayleigh number is obtained by using the linear stability analysis. Solution obtained in terms of perturbation expansion with angle of inclination as small perturbation parameter. A second order perturbation method is employed to determine the correction to critical Rayleigh number for horizontal slot on the lines of weakly nonlinear theory. The critical Rayleigh number expression given by the equation (35) and which excludes the oscillatory motions. The linear stability theory when applied to the system depicted in figure (1) leads to very interesting prediction that when the critical Rayleigh number is increased past the critical point, the ensuing motion for a small values of φ will consists of steady parallel rolls having definite wavelength and with their axis in the x-direction. The rolls are very much modified by the presence of heating and salinity gradients with the increasing Darcy numbers. These theoretical considerations are rather similar to those obtained by Liang and Acrivos (1970) for double diffusive convection, Paliwal and Chen (1980),Chikwall (1982) and others. It has been observed that the critical Rayleigh numbers are smallest for longitudinal disturbances having their axis aligned in the direction of mean flow(for $a_x = 0$). For other disturbances ($a_x \neq 0$) which generally lead to oscillatory instability and the critical Rayleigh number increase sharply with increasing Prandtl number. In the present problem the neutral state remains stationary for all disturbance wave numbers and in addition the absolute values of $K_1(\sigma)$ and $K_2(\sigma)$ approaches constant values as the Prandtl number is increased and are dependent on Darcy number and solutal Rayleigh number as well. These parameters $K_1(\sigma)$, $K_2(\sigma)$ which also depend on the other parameters approach constant values as the Prandtl numbers, Darcy numbers are increased. Thus the critical Rayleigh numbers increase to an asymptotic value independent of σ and diffusivity ratio. The problem also bearing closest resemblance to that of stability natural convection in a vertical slot and is characterised by Grashoff number rather than the Rayleigh number as in the case of transverse rolls i.e. rolls having their axes normal to the direction of the mean flow. These results are confirmed experimentally by Vest and Arpaci (1969). Thus it would appear that, as φ is increased from 0 to 90°, a transition from stationary longitudinal to stationary transverse rolls might result and it is interesting to see such transition experimentally also.

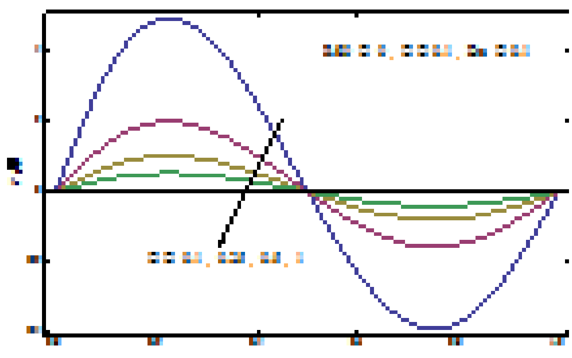

Figure (2) Variation of $K_2(y)$ with y for different values of σ

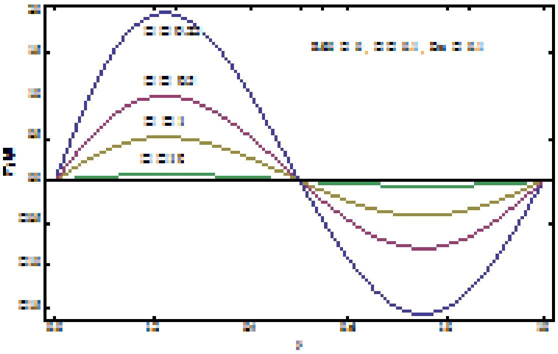

Figure (3) Variation of $\bar{V}_1(y)$ with y for different values of σ

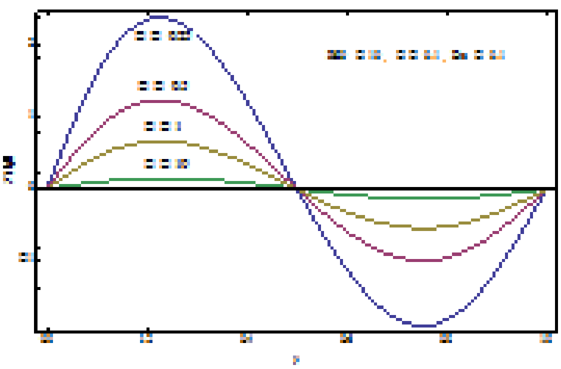

Figure (4) Variation of $\bar{V}_1(y)$ with y for different values of σ

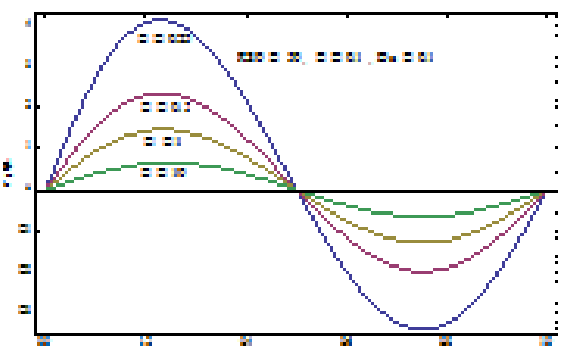

Figure (5) Variation of $\bar{V}_1(y)$ with y for different values of σ

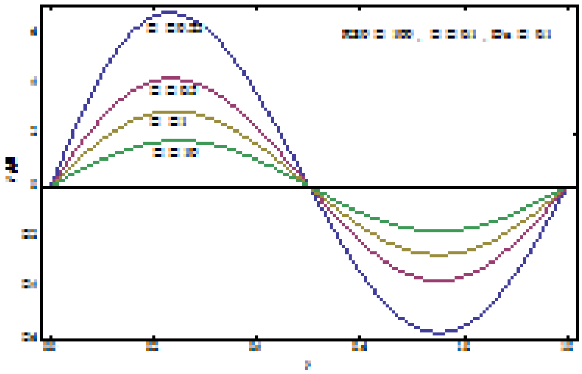

Figure (6) Variation of $\bar{V}_1(y)$ with y for different values of σ

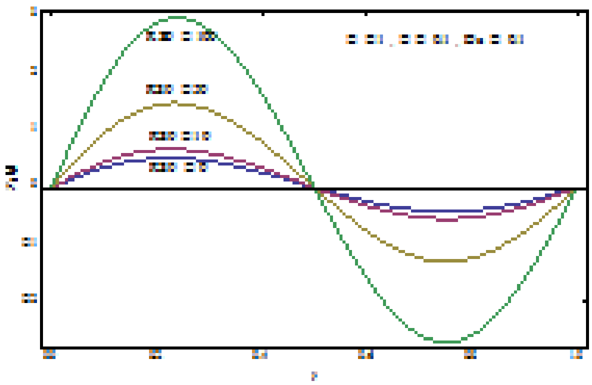

Figure (7) Variation of $\bar{V}_1(y)$ with y for different values of RS0

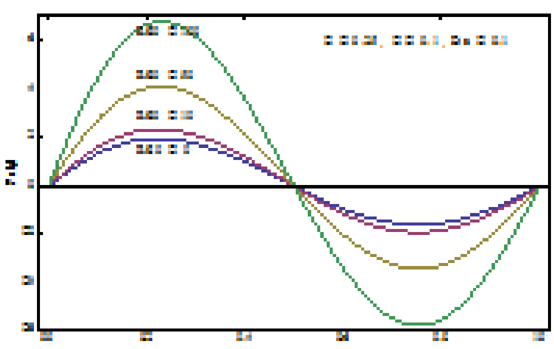

Figure (8) Variation of $\bar{V}_1(y)$ with y for different values of RS0

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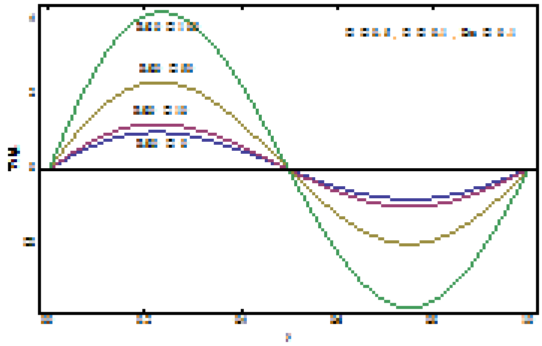

Figure (9) Variation of $\bar{V}_1(y)$ with y for different values of RS0

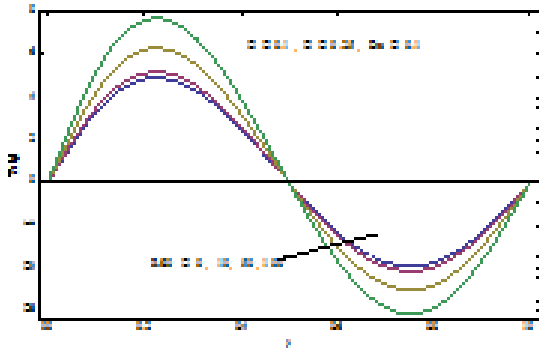

Figure (10) Variation of $\bar{V}_1(y)$ with y for different values of RS0

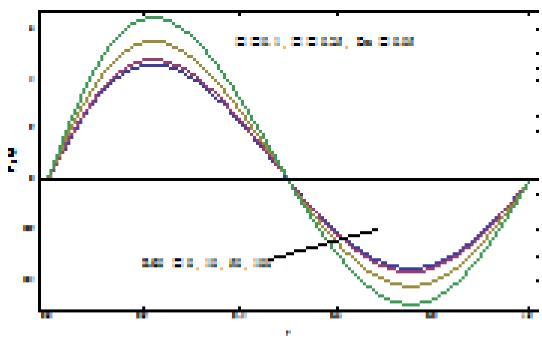

Figure (11) Variation of $\bar{V}_1(y)$ with y for different values of RS0

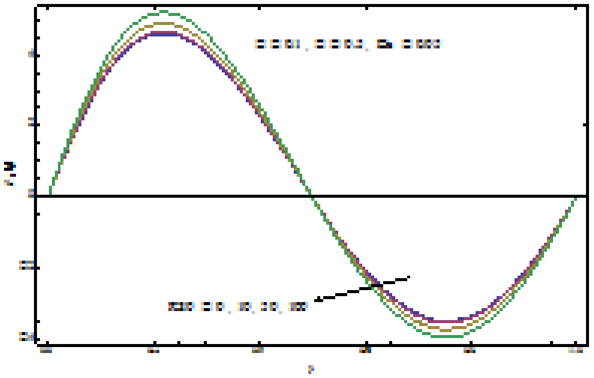

Figure (12) Variation of $\bar{V}_1(y)$ with y for different values of RS0

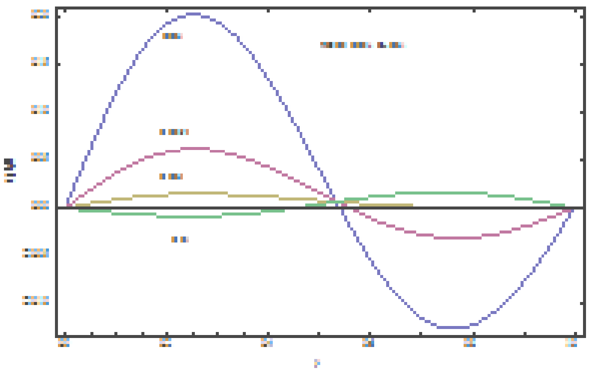

Figure (13) Variation of $\bar{\theta}_1(y)$ with y for different values of RS0

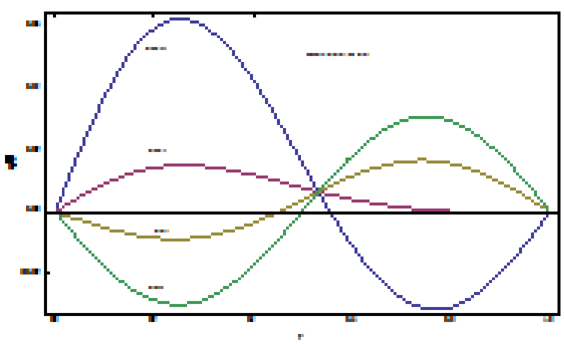

Figure (14) Variation of $\bar{\theta}_1(y)$ with y for different values of σ

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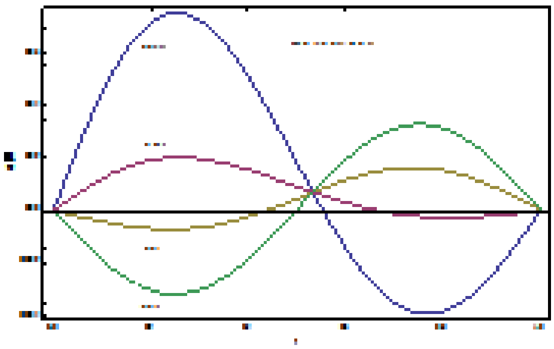

Figure (15) Variation of $\bar{\theta}_1(y')$ with y for different values of σ

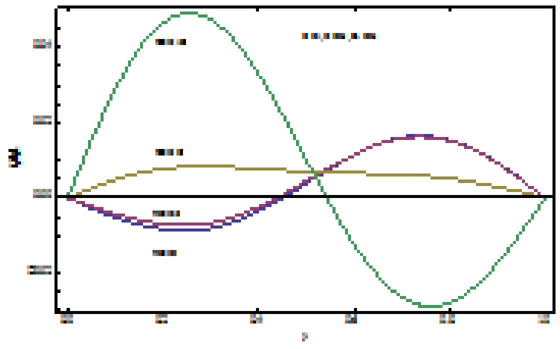

Figure (18) Variation of $\bar{\theta}_1(y')$ with y for different values of RS0

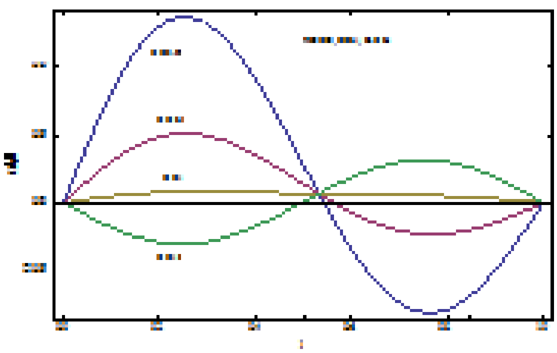

Figure (16) Variation of $\bar{\theta}_1(y')$ with y for different values of σ

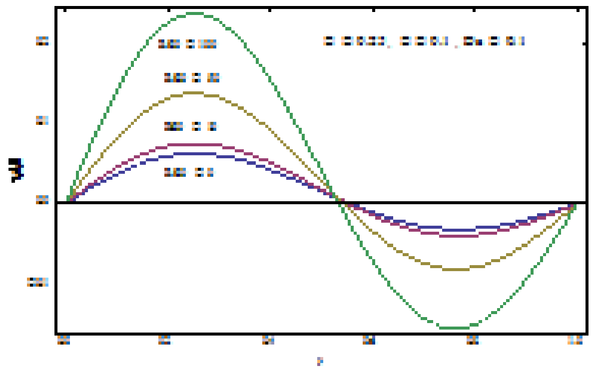

Figure (19) Variation of $\bar{\theta}_1(y')$ with y for different values of RS0

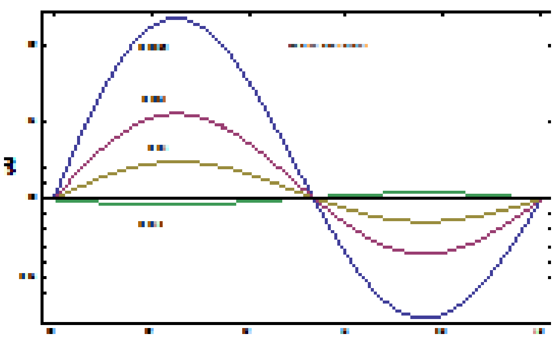

Figure (17) Variation of $\bar{\theta}_1(y')$ with y for different values of σ

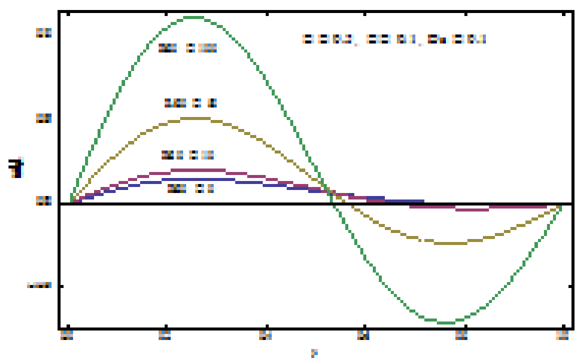

Figure (20) Variation of $\bar{\theta}_1(y')$ with y for different values of RS0

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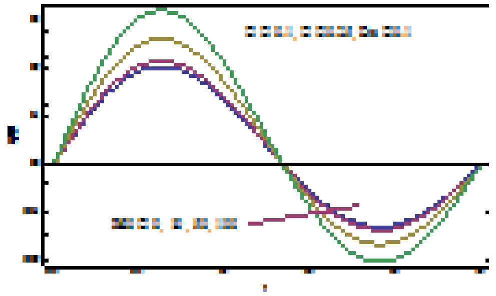

Figure (21) Variation of $\bar{\sigma}_1(y)$ with y for different values of RSD

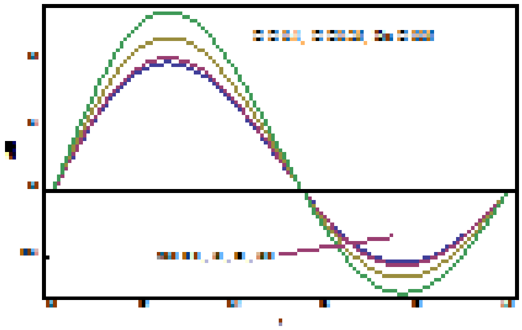

Figure (22) Variation of $\bar{\sigma}_1(y)$ with y for different values of RSD

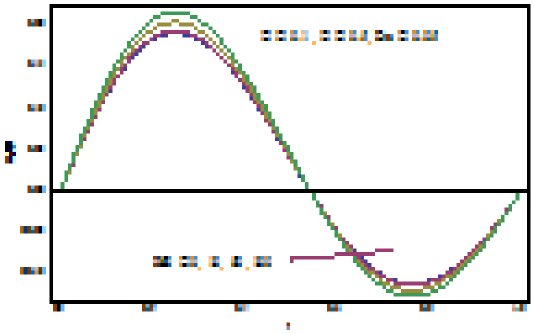

Figure (23) Variation of $\bar{\sigma}_1(y)$ with y for different values of RSD

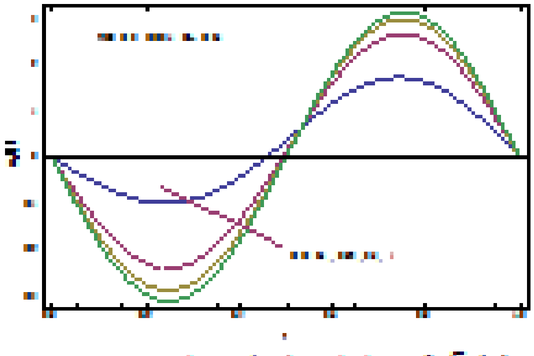

Figure (24) Variation of $\bar{\sigma}_1(y)$ with y for different values of a

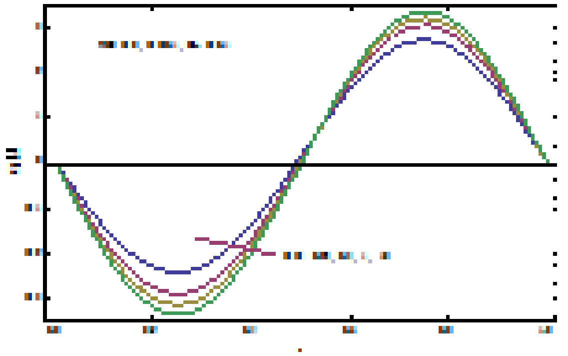

Figure (25) Variation of $\bar{\sigma}_1(y)$ with y for different values of a

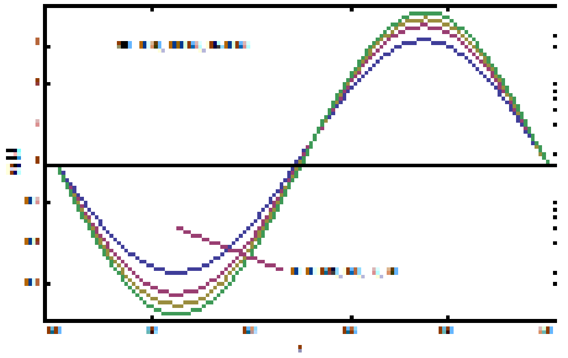

Figure (26) Variation of $\bar{\sigma}_1(y)$ with y for different values of a

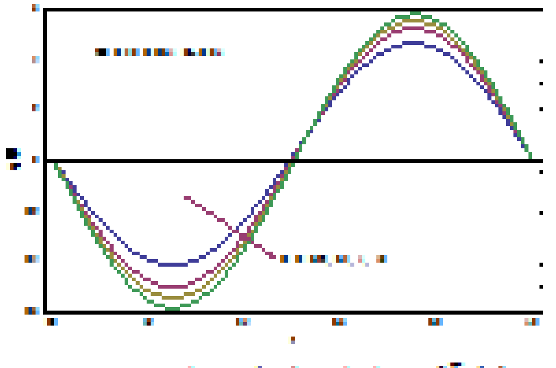

Figure (27) Variation of $\bar{\sigma}_1(y)$ with y for different values of a

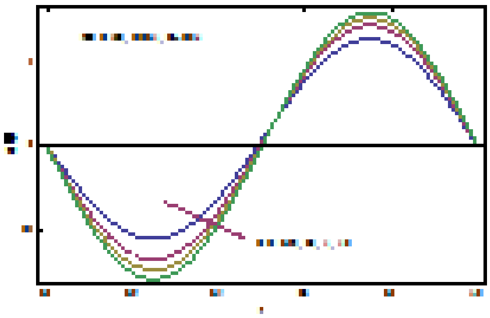

Figure (28) Variation of $\bar{\phi}_1(y)$ with y for different values of α

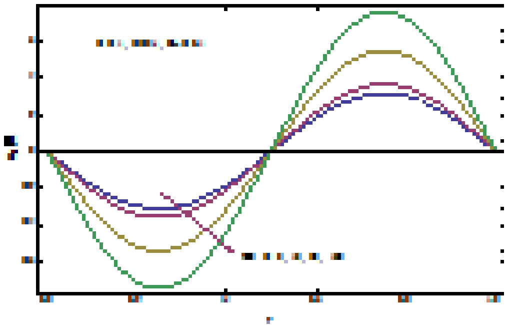

Figure (29) Variation of $\bar{\phi}_1(y)$ with y for different values of R50

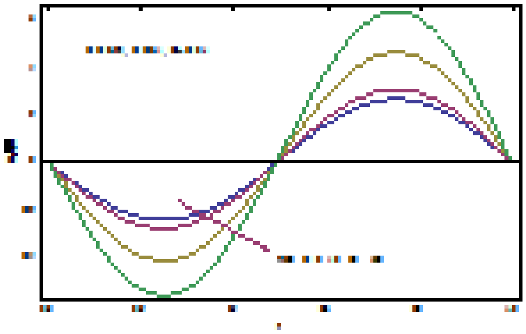

Figure (30) Variation of $\bar{\phi}_1(y)$ with y for different values of R50

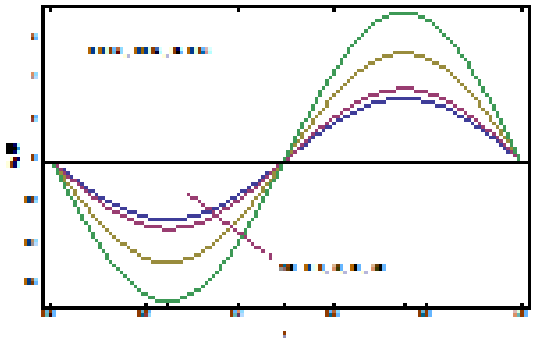

Figure (31) Variation of $\bar{\phi}_1(y)$ with y for different values of R50

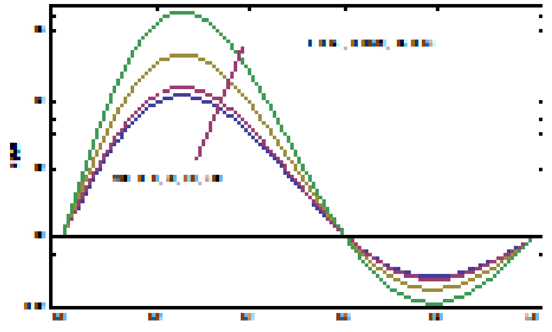

Figure (32) Variation of $\bar{\phi}_1(y)$ with y for different values of R50

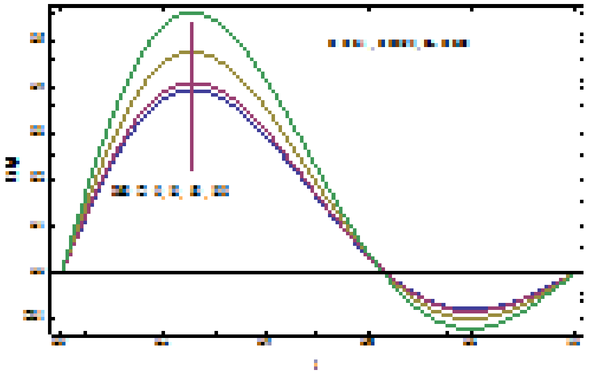

Figure (33) Variation of $\bar{\phi}_1(y)$with y for different values of R50

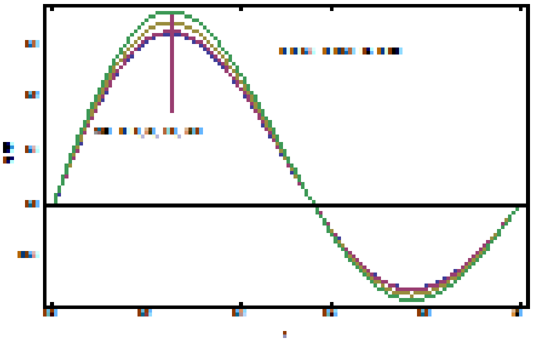

Figure (54) Variation of $\bar{\sigma}_1(y)$ with y for different values of R30

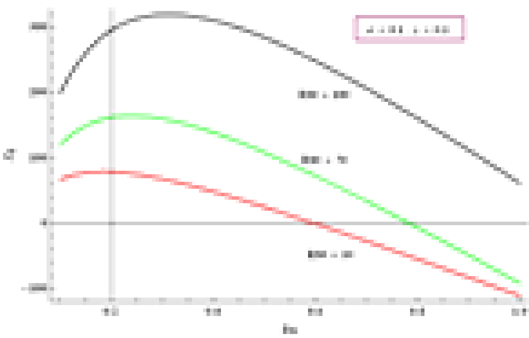

Figure (55) Variation of K1 for different values of Da

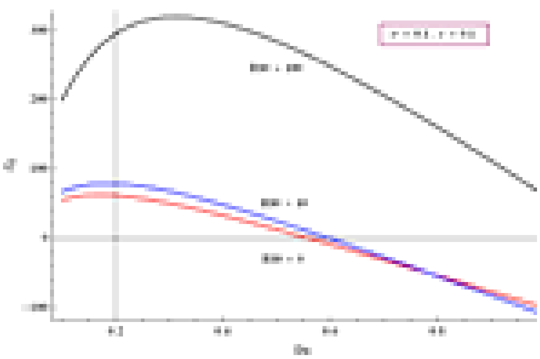

Figure (56) Variation of K1 for different values of Da

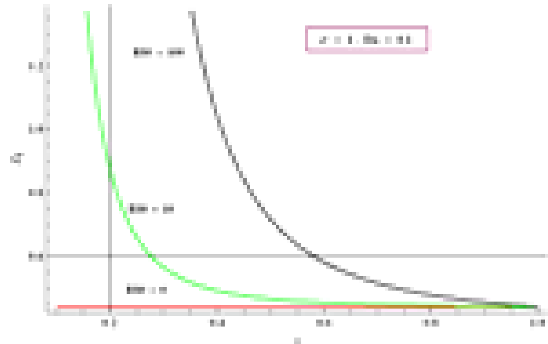

Figure (57) Variation of K1 for different values of R30

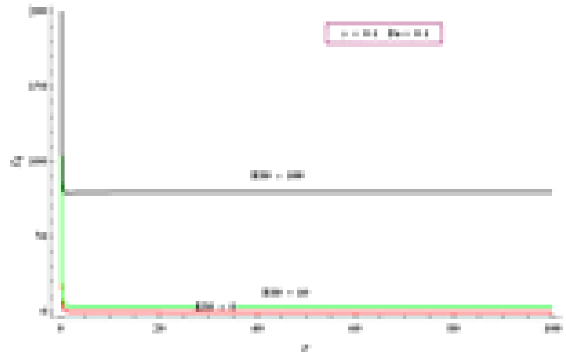

Figure (58) Variation of K1 for different values of Da

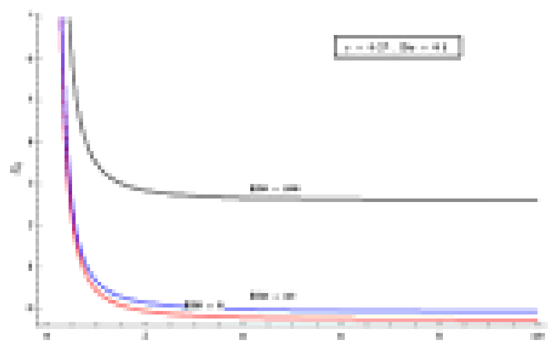

Figure (59) Variation of K1 for different values of R30

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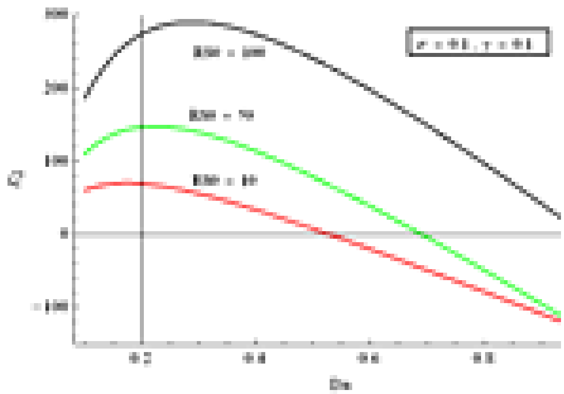

Figure (39) Variation of K2 for different values of RSD

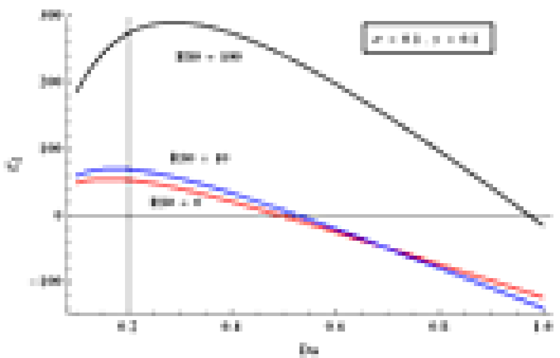

Figure (40) Variation of K2 for different values of RSD

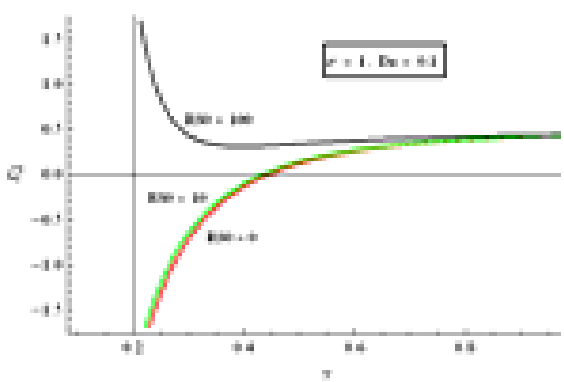

Figure (41) Variation of K2 for different values of RSD

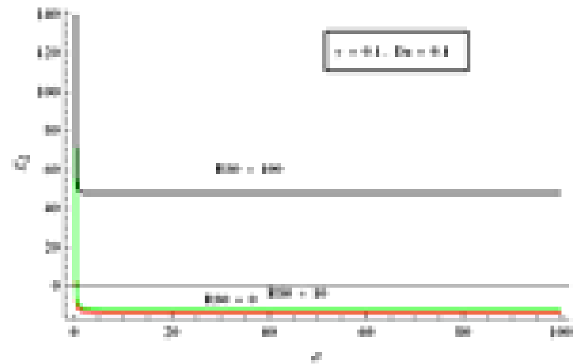

Figure (42) Variation of K2 for different values of RSD

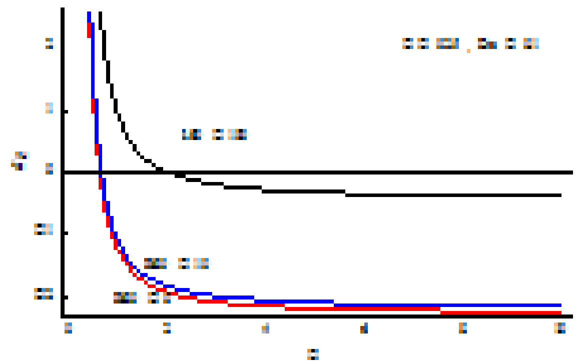

Figure (43) Variation of K2 for different values of RSD

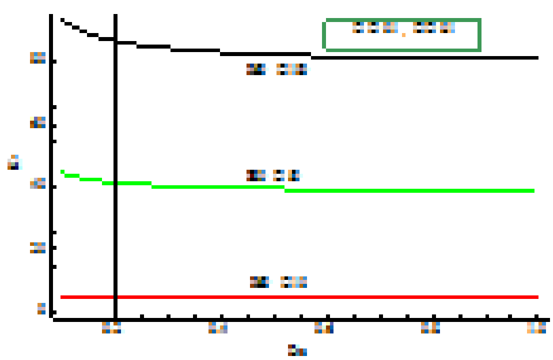

Figure (44) Variation of K4 for different values of RSD

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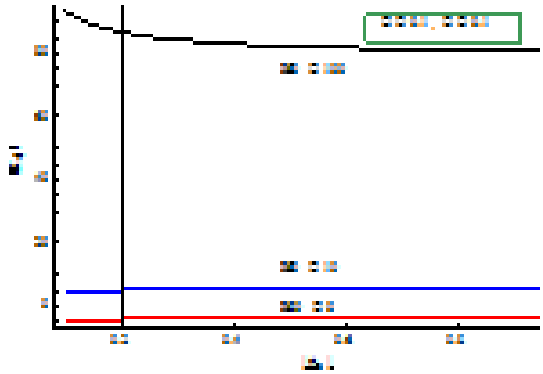

Figure (45) Variation of K4 for different values of R30

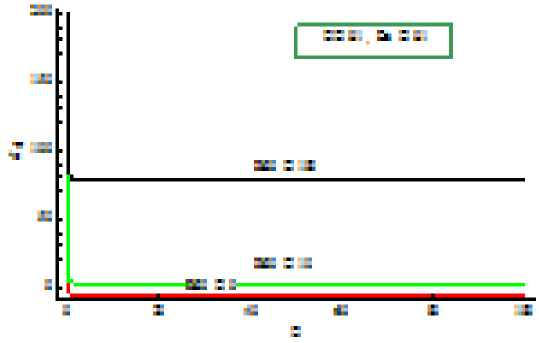

Figure (47) Variation of K4 for different values of R30

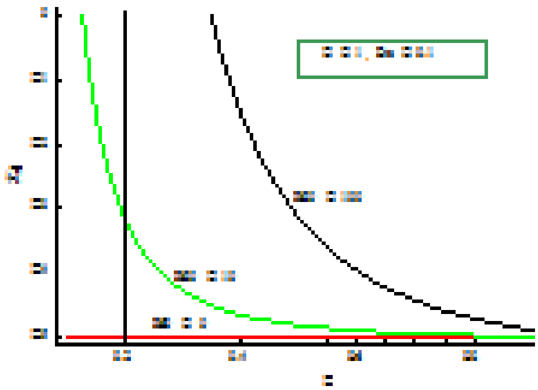

Figure (46) Variation of K4 for different values of R30

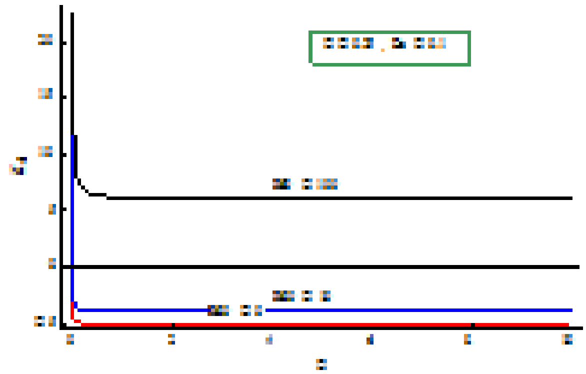

Figure (48) Variation of K4 for different values of R30

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