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(1,2)*- π gr-Closed Sets in Topological Spaces

KEYWORDS: (1,2)*- πgr-closed sets, (1,2)*- πgr-continuous and (1,2)*- πgr- irresolute maps, (1,2)*- πgr-T1/2- space

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ABSTRACT *The aim of this paper is to introduce a new class of sets called (1 ,2)*- π gr-closed sets in topological spaces and to study their properties. Further, we define and study (1,2)*- π gr -continuity, (1,2)*- π gr-irresoluteness .*

1.Introduction

The study of bitopological spaces was first initiated by J.C. Kelly[7]in the year 1963. Levine [9] introduced generalized closed sets and studied their properties. Later on N.Palaniappan[16,17] studied the concept of regular generalized closed set in topological space. In 1985, Fukutake [6], introduced the concepts of g-closed sets in bitopological spaces. Dontchev.J, Noiri.T [5]introduced and studied the concepts of πg- closed set in topological spaces.Ravi, Lellis Thivagar , Ekici and many others [18,19] have defined different weak forms of the topological notions namely , semi open, pre-open, regular open and α-open sets in bitopological spaces. Zaitsev [21]introduced the concept of π-closed sets in topological space. Andrijevic.D[1],Levine[9] ,Nagaveni[15],K.Balachandran and Arokiarani[4], Gnanambal [10],Mashour.A.S et al [14]and Maki et al [13]introduced the concepts of semi-open sets , weakly closed sets, generalized pre-closed sets,pre-regular closed sets,pre-closed sets and α-closed sets respectively.In 2012,Jeyanthi.V and Janaki.C [12] introduced the notion of πgr-closed sets in topological spaces .

In this paper, we introduce the notion of (1,2)*- πgr-closed sets and investigate their properties.Further,we study (1,2)*- πgr- continuous, (1,2)*- πgr-irresolute maps,(1,2)*- πgr-$T_{1/2}$-space and their properties.

2. Preliminaries

Throughout this paper, X and Y denote the bitopological spaces (X,τ_1,τ_2) and (Y,σ_1 , σ_2) respectively, on which no separation axioms are assumed.

Definition: 2.1

Let A be a subset of X. then A is called $\tau_{1,2}$-open [18,19,2] if A= $A_1 \cup B_1$, where A_1 Îτ_1 , B_1Î τ_2 The complement of $\tau_{1,2}$ -open set is $\tau_{1,2}$ -closed set.The family of all $\tau_{1,2}$ -open (resp. $\tau_{1,2}$-closed) sets of X is denoted by (1,2)* -O(X) and (resp. (1,2)* -C(X)).

Example :2.2

Let X= {a,b,c}, τ_1 = {ϕ,X,{a},{a,c}}, τ_2 = { ϕ,X,{c}}. Then $\tau_{1,2}$ - open sets ={ϕ,X,{a},{c},{a,c}} and $\tau_{1,2}$ -closed sets ={ϕ,X,{b,c},{a,b},{b}}

Definition :2.3

Let A be a subset of a bitopological space X. Then

(i) $\tau_{1,2}$ -closure of A [18,19,2] denoted by $\tau_{1,2}$ -cl(A) is defined by the intersection of all $\tau_{1,2}$ -closed sets containing A.
(ii) $\tau_{1,2}$ -interior of A [18,19,2] denoted by $\tau_{1,2}$ -int (A) is defined by the union of all open sets contained in A.

Remark : 2.4

Notice that $\tau_{1,2}$ -open subsets of X need not necessarily form a topology.

Now, we recall some definitions and results which are used in this paper.

Defintion: 2.5

A subset A of a bitopological space X is said to be

(i) (1,2)* -pre -open[18,19] if AÌ $\tau_{1,2}$ -int ($\tau_{1,2}$ -cl(A)).
(ii) (1,2)* -semi open [18,19] if AÌ τ_1,2-cl ($\tau_{1,2}$ -int(A)).
(iii) Regular (1,2)* -open [8] if A = $\tau_{1,2}$ -int ($\tau_{1,2}$ -cl(A)).
(iv) (1,2)* - α-open [18,19] if AÌ $\tau_{1,2}$ -int ($\tau_{1,2}$ -cl ($\tau_{1,2}$ -int (A))).
(v) (1,2)* -π- open[5,21] if A is the finite union of regular (1,2)* -open sets.

The complements of all the above mentioned open sets are called their respective closed sets.The family of all (1,2)* -open sets [(1,2)* -regular open, (1,2)* - π-open, (1,2)* -semi open) sets of X will be denoted by (1,2)* O(X)(resp. (1,2)* RO(X), (1,2)* - πO(X), (1,2)*-SO(X)].

Definition: 2.6

$\tau_{1,2}$-r-closure of a set A is defined as the intersection of all (1,2)*-regular closed sets containing the set and $\tau_{1,2}$-r-interior of a set A is the union of (1,2)*-regular open set contained in the set.The above are denoted by (1,2)*-rcl(A) and(1,2)*- r int(A)

Definition :2.7

A subset A of bitopological space X is said to be

a (1,2)* - generalized closed set [18,19]((1,2)* - g closed set) if $\tau_{1,2}$ -cl (A) Ì U whenever AÌ U and U Î (1,2)*- O(X).
a (1,2)* - generalized pre closed set[8] (briefly (1,2)* -gpr-closed set) if (1,2)* - pcl (A) ÌU whenever A Ì U and U Î (1,2)* - RO(X) .
a (1,2)* - π- generalized closed [5](briefly (1,2)*- πg -closed set) $\tau_{1,2}$ - cl (A) Ì U whenever A Ì U and U Î (1,2)* -πO(X).
a (1,2) * - πgα - closed set [3]if $\tau_{1,2}$ - αcl (A) Ì U whenever A Ì U and U Î (1,2)* -πO(X).
a (1,2)*- $T_{1/2}$-space [18,19] if every (1,2)*- g-closed set in X is $\tau_{1,2}$ -closed in X.

Definition 2.8

A map f: X→Y is said to be

(1,2)*- continuous[18,19] if $f^{-1}(V)$ is $\tau_{1,2}$ -closed in X for every $\sigma_{1,2}$ -closed set V in Y.

(1,2)*- regular- continuous [17]if $f^{-1}(V)$ is (1,2)*-regular closed in X for every $\sigma_{1,2}$- closed set V in Y.

a (1,2) * - πgb - continuous [20]if $f^{-1}(V)$ is (1,2)*-πgb-closed in X for every $\sigma_{1,2}$- closed set V in Y. (1,2)*- πg-continuous[5]

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if $f^{-1}(V)$ is (1,2)*-πg closed in X for every $\sigma_{1,2}$-closed set V in Y. (1,2)*- gpr-continuous [8]if f^{-1} (V) is (1,2)*- gpr closed in X for every $\sigma_{1,2}$ - closed set V in Y.

a (1,2) * - πgα - continuous[3] if $f^{-1}(V)$ is (1,2)*-πgα- closed in X for every $\sigma_{1,2}$-closed set V in Y.

a (1,2) * - πwg - continuous[11] if $f^{-1}(V)$ is (1,2)*-πwg-closed in X for every $\sigma_{1,2}$-closed set V in Y.

3. (1,2) * - πgr - closed sets in bitopological spaces.
Defintion 3.1

A subset A of X is called (1,2) *-πgr- closed set in X if $\tau_{1,2}$-rcl(A)) Ì U whenever AÌ U and U Ì(1,2)* -πO(X). The complement of (1,2)* - πgr-closed set is (1,2)*-πgr-open set.We denote the family of all (1,2)*- πgr- closed (resp. πgr- open)sets in X by (1,2)*- πGRC(X)(resp. (1,2)*- πGRO(X)).

Theorem 3.2

1. Every $\tau_{1,2}$-regular-closed set is (1,2) *-πgr- closed set .
2. Every (1,2)* - πgr -closed set is (1,2)* - πg -closed set.
3. Every (1,2)* - πgr -closed set is (1,2)* - πwg -closed set.
4. Every (1,2)* - πgr -closed set is (1,2)* - πgα -closed set.
5. Every (1,2)* - πgr -closed set is (1,2)* - πgb -closed set.
6. Every (1,2)* - πgr -closed set is (1,2)* - gpr -closed set.

Proof : Straight forward.

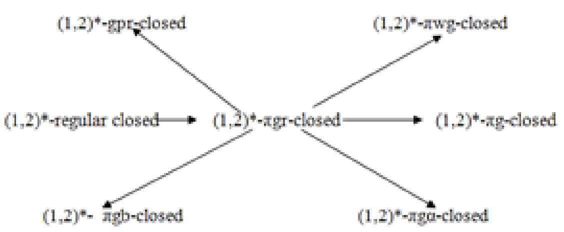
Remark :3.3
The converse of the above results need not be true as seen in the following examples.

Example: 3.4
Let X= {a,b,c,d}, τ_1 = {ϕ, X, {b},{b,c}}, τ_2 = {ϕ, X, {c},{b,c,d}}.Then $\tau_{1,2}$ -open = {ϕ, X,{b},{c},{b,c},{b,c,d}} and $\tau_{1,2}$ –closed = {ϕ, X,{({a,c,d},{a,b,d}, {a,d},{a}}.Here all (1,2)*-regular closed sets are (1,2)*-πgr-closed, but {b}, {d}, {a,b},{c,d},{b,c},{b,d},{c,d},{a, b,c},{a,c,d} are not (1,2)*-regular closed .

Example : 3.5
Let X= {a,b,c,d}, τ_1 = {ϕ, X, {b},{b,c}}, τ_2 = {ϕ, X, {c},{b,c,d}}. Then $\tau_{1,2}$ -open = {ϕ, X, {b},{c},{b,c},{b,c,d}} and $\tau_{1,2}$ -closed = {ϕ,X,{a,c,d},{a,b,d},{a,d},{a}} The set {a,c} is (1,2)*-gpr-closed , but not (1,2)*-πgr-closed.

Example 3.6
Let X= {a,b,c,d}, τ_1 = {ϕ, X, {c},{c,d},{a,c,d}}, τ_2 = {ϕ, X, {d}, {b,d} ,{b,c,d}} . Then $\tau_{1,2}$ -open = {ϕ, X,{c},{d},{c,d},{b,d},{a,c,d},{b, c,d}} and $\tau_{1,2}$ -closed = {ϕ, X,{a,b,d},{a,b,c}, {a,b},{a,c},{b},{a}}. Here the set {b} is (1,2)*-πg-closed,(1,2)*-πwg-closed and (1,2)*-πgα-closed and(1,2)*- πgb-closed but not (1,2)*-πgr-closed.

Note: 3.7
The above discussions are summarized in the following diagram.



Remark:3.8
Finite union of (1,2)* - πgr-closed sets need not be (1,2)* - πgr- closed set and is shown in the following example.

Example :3.9
Let X={a,b,c,d}, τ_1 = {ϕ, X, {a},{b,c,d}}, τ_2 = {ϕ, X, {c},{a,b,d}}. Then $\tau_{1,2}$ -open = {ϕ, X, {a},{c},{a,c},{b,c,d},{a,b,d}} and $\tau_{1,2}$ -closed = {ϕ, X, {b,c,d},{a,b,d},{b,d},{a},{c}} .Here A = {a} and B= {c} are (1,2)* -πgr- closed sets , but A ∪ B = {a,c} is not (1,2)*- πgr- closed.

Remark :3.10
Finite intersection of two (1,2)* - πgr- closed sets need not be (1,2)* - πgr- closed set.

Example: 3.11
Let X={a,b,c,d}, τ_1 = {ϕ, X, {a},{b,c,d}}, τ_2 = {ϕ, X, {c},{a,b,d}}. Then $\tau_{1,2}$ -open= {ϕ,X,{a},{c},{a,c},{b,c,d},{a,b,d}} and $\tau_{1,2}$ -closed ={ϕ,X,{b,c,d},{a,b,d},{b,d} ,{a},{c}} .Here A={a,b,c} and B={a,c,d} are (1,2)*- πgr-closed sets but A∩B={a,c} is not (1,2)*- πgr-closed.

Theorem: 3.12
If a subset A of X is both are (1,2)*-π-open and are (1,2)*-πgr-closed , then it is $\tau_{1,2}$-closed .

Proof: Let A be a subset of X which is both (1,2)*-π-open and (1,2)*-πgr-closed. Then $\tau_{1,2}$-rclAÌ U whenever AÌ U and U is (1,2)*-π-open. Also, AÌ $\tau_{1,2}$-rclA, which implies $\tau_{1,2}$-rclA=A. Hence A is (1,2)*-regular closed.Hence A is $\tau_{1,2}$-closed.

Theorem: 3.13
If A is (1,2)*-πgr- closed and AÌ BÌ $\tau_{1,2}$-rcl (A) , then B is also (1,2)*- πgr -closed subset of X.

Proof: Let A be (1,2)*-πgr -closed set in X and B Ì U , where U is (1,2)*-π- open.Since A Ì B,A Ì U. Since A is (1,2)*-πgr-closed, $\tau_{1,2}$-rcl(A) Ì U. Given B Ì $\tau_{1,2}$-rcl(A). Then $\tau_{1,2}$-rcl(B) Ì $\tau_{1,2}$-rcl(A) ÌU. Hence $\tau_{1,2}$-rcl(B) ÌU and hence B is (1,2)*-πgr - closed.

Theorem 3.14
If A is (1,2)*-πgr -closed,then $\tau_{1,2}$-rcl(A) – A does not contain any non-empty (1,2)*-π -closed set.

Proof: Let F be a non-empty (1,2)*- π-closed set such that F Ì (1,2)*-rcl(A) – A

Þ F ÌX-A .The above implies A ÌX-F.Since A is (1,2)*-πgr-closed, X–F is (1,2)*-π-open.Since $\tau_{1,2}$-rcl(A)Ì X–F, FÌX- $\tau_{1,2}$-rcl(A).Thus, FÌ $\tau_{1,2}$-rcl(A)∩(X– $\tau_{1,2}$-rcl(A)).Thus,F Ì ϕ, which is a contradiction.Hence F= ϕ , whence $\tau_{1,2}$-rcl(A)-A does not contain non-empty (1,2)*-π- closed set.

Corollary 3.15
Let A be (1,2)*-πgr-closed set in X. Then A is (1,2)*-regular closed iff $\tau_{1,2}$-rcl(A) – A is (1,2)*-π-closed.

Proof: First,let A be (1,2)*-regular closed. Then $\tau_{1,2}$-rcl(A) =A and so $\tau_{1,2}$-rcl(A) – A = ϕ, which is (1,2)8- π-closed.On the otherhand,let us suppose the $\tau_{1,2}$-rcl(A) – A is (1,2)*-π -closed. Then $\tau_{1,2}$-rcl(A)– A = ϕ, since A is (1,2)*-πgr-closed.(i.e) $\tau_{1,2}$-rcl(A) =A.Hence A is (1,2)*- regular closed.

4.(1,2)*-πgr-Open Sets
Definition 4.1

A set AÌ X is called (1,2)*-πgr-open set iff its complement is (1,2)*-πgr-closed and the collection of all (1,2)*-πgr-open sets is denoted by πGRO(X).

Remark 4.2
For a subset A of X, $\tau_{1,2}$-rcl(X–A) =X– $\tau_{1,2}$-r-int(A).

Theroem 4.3
Let AÌ X is (1,2)*-πgr-open iff F Ì $\tau_{1,2}$-r-int (A) whenever A is (1,2)*-π-closed and FÌA.

Proof: Let A be (1,2)*-πgr-open . Let F be (1,2)*-π-closed set and F ÌA. Then X–A Ì X–F, where X–F is (1,2)-π-open.Since A is (1,2)*-πgr-open, X–A is (1,2)*-πgr-closed .Then $\tau_{1,2}$- rcl(X–A) Ì X–F. Since $\tau_{1,2}$-rcl(X–A) = X- $\tau_{1,2}$-r-int(A).The above implies X– $\tau_{1,2}$-r -int(A) Ì X–F.Hence FÌ $\tau_{1,2}$-r- int(A).

Conversely, let F is (1,2)*-π-closed and FÌ A implies FÌ $\tau_{1,2}$-r-int(A). Let X–A ÌU, where U is (1,2)*-π-open. Then X–UÌA, where X–U is (1,2)*-π-closed. By hypothesis, X–U Ì(1,2)*- r-int(A).Hence X– $\tau_{1,2}$-r int(A) ÌU.Since $\tau_{1,2}$-rcl(X–A) =X–rint(A). The above implies $\tau_{1,2}$-rcl(X–A) ÌU, whenever X–A is (1,2)*-π-open.Hence X–A is (1,2)*-πgr-closed and hence A is (1,2)*-πgr-open.

Theorem 4.4
If $\tau_{1,2}$-r-int (A) ÌBÌA and A is (1,2)*-πgr-open, then B is (1,2)*-πgr-open.

Proof: Given $\tau_{1,2}$-r-int (A) ÌBÌA. The X –A ÌX –BÌ $\tau_{1,2}$-rcl(X–A). Since A is (1,2)*-πgr-open, X–A is (1,2)*-πgr-closed.Then X –B is also (1,2)*-πgr-closed. Hence B is (1,2)*-πgr-open.

Remark: 4.5
For any set AÌX, $\tau_{1,2}$-r- int($\tau_{1,2}$-rcl(A) –A)) = ϕ.

Theorem: 4.6
If AÌX is πgr-closed, the rcl(A)-A is πgr-open.

Proof: Let A be (1,2)*-πgr-closed.Let F be a (1,2)*-π-closed set such that FÌ $\tau_{1,2}$-rcl(A)-A. Then F= ϕ.

So, FÌ $\tau_{1,2}$-r int($\tau_{1,2}$-rcl(A)-A). The above implies $\tau_{1,2}$-rcl(A)-A is (1,2)*-πgr-open.

Theorem :4.7
Suppose that A and B are (1,2)*-π-separated subsets of X. (i.e) A∩ $\tau_{1,2}$-πcl(B) = B∩ $\tau_{1,2}$-πcl(A)=ϕ. If A and B are (1,2)*-πgr-open, then A ∪ B is also(1,2)*- πgr-open.

Proof: Let F be a (1,2)*-π-closed set such that FÌ A ∪ B .

Then F∩ $\tau_{1,2}$-πcl(A) Ì (A ∪ B) ∩ $\tau_{1,2}$-πcl(A)

Ì (A∩ $\tau_{1,2}$-πcl(A)) ∪ (B∩ $\tau_{1,2}$-πcl(A))

ÌA∩ $\tau_{1,2}$-πcl(A) ∪ ϕ

Hence F∩ $\tau_{1,2}$-πcl(A) ÌA.Similarly , we prove F∩ $\tau_{1,2}$-πcl(B) ÌB.. Since F∩ $\tau_{1,2}$-πcl(B) and F∩ $\tau_{1,2}$-πcl(A) are (1,2)*- π-closed. Then by theorem 4.3 , F∩ $\tau_{1,2}$-πcl(A) Ì $\tau_{1,2}$-r-int(A) and F∩ $\tau_{1,2}$-πcl(B)Ì $\tau_{1,2}$r-int(B).

Hence F= F∩πcl(A ∪ B) Ì (F∩πcl(A)) ∪ (F∩πcl(B)) Ì r-int(A) ∪ r-int(B) Ì r-int(A ∪ B). Hence by theorem 4.3, A ∪ B is πgr-open.

5.(1,2)*- πgr-$T_{1/2}$ -Space.
Definition:5.1
A space X is called a πgr-$T_{1/2}$-Space if every (1,2)*-πgr-closed set is(1,2)*- regular closed.

Theorem: 5.2
For a bitopological space the following conditions are equivalent.

X is (1,2)*-πgr-$T_{1/2}$ -Space.

Every singleton of X either (1,2)*-π-closed or (1.2)*-regular open.

Proof:(i)Þ(ii): Let x∈X and assume that {x} is not (1,2)*-π-closed. Then clearly X-{x} is not (1,2)*-π-open and X-{x} is trivially (1,2)*-πgr-closed.Since X is (1,2)*-πgr-$T_{1/2}$-space, every (1,2)*-πgr-closed set is (1,2)*-regular closed.ÞX-{x} is (1,2)*-regular closed and hence {x} is (1,2)*-regular open.

(ii) Þ(i) :Assume every singleton of X is either (1,2)*-π-closed or (1,2)*-regular open.Let AÌX be (1,2)*-πgr-closed and obviously, AÌ $\tau_{1,2}$-rcl(A) and let x∈ $\tau_{1,2}$-rcl(A) . To prove $\tau_{1,2}$-rcl(A) ÌA.

Case(i)
Let {x} be (1,2)*-π-closed. Suppose {x} does not belong to A.Then {x} Ì $\tau_{1,2}$-rcl(A)–A .Since {x}∈A. Hence $\tau_{1,2}$-rcl(A) ÌA. The above implies $\tau_{1,2}$-rcl(A)=A.Hence A is (1,2)*-regular closed. Thus every (1,2)*-πgr-closed set is (1,2)*-regular closed and hence X is (1,2)*- πgr-$T_{1/2}$- space.

Case(ii)
Let {x} be (1,2)*-regular open. Since {x}∈ $\tau_{1,2}$-rcl(A), we have {x}∩A≠ϕ. Hence {x}∈A. Therefore, A is (1,2)*-regular closed and hence every (1,2)*-πgr-closed set is (1,2)*- regular closed.

Theorem :5.3
(i) (1,2)*-RO(X) Ì(1,2)*-πGRO(X),

(ii) A space X is (1,2)*-πgr-$T_{1/2}$-space iff (1,2)*-RO(X)=(1,2)*-πGRO(X)

Proof:(i)Let A be (1,2)*- regular open. Then X–A is (1,2)*-regular closed and so(1,2)*- πgr-closed.Hence A is (1,2)*-πgr-open. Hence (1,2)*-RO(X) Ì(1,2)*-πGRO(X)

(ii)Necessity: Let X be (1,2)*-πgr-$T_{1/2}$-space. Let A ∈ (1,2)*-πGRO(X). Then X–A is (1,2)*-πgr-closed. Since the space X is (1,2)*-πgr-$T_{1/2}$-space , X–A is (1,2)*-regular closed. The above implies A is(1,2)*- regular open in X.

Hence (1,2)*-RO(X)=(1,2)*-πGRO(X)

Sufficiency: Let (1,2)*-RO(X)=(1,2)*-πGRO(X).

Let A be (1,2)*-πgr-closed. Then X–A is (1,2)*-πgr-open and X–A∈(1,2)*-RO(X) Hence A is (1,2)*-regular closed and hence A is (1,2)*- πgr-$T_{1/2}$-space.

6. (1,2)*-πgr-Continuous and (1,2)*- πgr-Irresolute functions
Definition:6.1

A function f: X →Y is called an (1,2)*-R-map[15] if f^{-1}(V) is (1,2)*-regular closed in X for every (1,2)*-regular closed set V in Y.

Definition :6.2
A function f: X →Y is called (1,2)*-πgr- continuous if every f^{-1}(V) is (1,2)*-πgr- closed in X for every $\sigma_{1,2}$- closed set V of Y.

A function f: X →Y is called (1,2)*-πgr- irresolute if every f^{-1}(V) is (1,2)*-πgr- closed in X for every (1,2)*-πgr -closed set V of Y.

Remark :6.3
Every regular (1,2)*-continuous is (1,2)*-πgr-continuous but not conversely.

Proof: Straight forward

Example: 6.4
Let X= {a,b,c,d}, τ_1={ ϕ,X,{d},{c,d},{b,d},{a,c,d},{b,c,d}, τ_2={ϕ,X,{c}},$\tau_{1,2}$-open ={ϕ,X,{c},{d},{c,d},{b,d},{a,c,d},{b,c,d}}.Let Y= {a,b,c,d} , S_1={ ϕ, Y }, S_2={ ϕ, Y ,{b}},$\sigma_{1,2}$-open= {ϕ, Y,{b}}} , $\sigma_{1,2}$-closed= {ϕ, Y, {a,c,d}}.Define f: X →Y by f(a)=a ,f(b)=b, f(c)=c,f(d)=d. Here the inverse image of the closed set {a,c,d } in Y is πgr-closed in X but not regular closed in X.Hence (1,2)*-πgr-continuity need not be (1,2)*-regular continuous in X.

Remark: 6.5
(1,2)*-continuity and (1,2)*- πgr-continuity are independent concepts.

Example: 6.6
Let X= {a,b,c,d}, τ_1={ ϕ,X,{d},{c,d},{b,d},{a,c,d},{b,c,d}, τ_2={ ϕ,X,{c}},$\tau_{1,2}$ -open = {ϕ, X, {c} ,{d},{c,d},{b,d},{a,c,d},{b,c,d}}, $\tau_{1,2}$-closed = {ϕ, X, {a },{b },{a, c},{a,b },{a,b,c },{a,b,d } } . Let Y= {a,b,c,d} , σ_1= {ϕ,Y,{c,d}}, σ_2 = {ϕ, Y, {a,c,d}}, $\sigma_{1,2}$ -open= {ϕ, Y, {c,d},{a,c,d}} , $\sigma_{1,2}$-closed= {ϕ,Y,{a,b},{b}}. Define f: X →Y by f(a)=a,f(b)=b,f(c)=c,f(d)=d. Here the inverse image of the

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closed set {b} in Y is not (1,2)*-πgr-closed in X but $\tau_{1,2}$-closed .Also , the (1,2)*-πgr-closed in X are not $\tau_{1,2}$-closed in X.

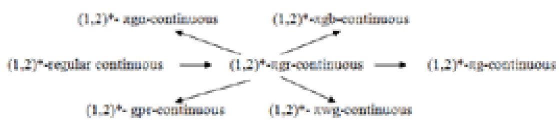
Remark :6.7
Every πgr-continuous function is (1,2)*-πwg-continuous, (1,2)*-πg-continuous ,(1,2)*- πgb-continuous, (1,2)*- πgα-continuous,(1,2)*-gpr-continuous .

Proof: Straight forward.

Remark:6.8
The converse of the above need not be true as shown in the following examples.

Example:6.9
a)Let X= {a,b,c,d}, τ_1={ ϕ,X,{d},{c,d},{b,d},{a,c,d},{b,c,d}, τ_2={ ϕ,X,{c}},$\tau_{1,2}$ -open = {ϕ, X, {c},{d},{c,d},{b,d},{a,c,d},{b,c,d}}, $\tau_{1,2}$-closed = {ϕ, X, {a}, {b}, {a,c},{a,b},{a,b,c},{a,b,d}} , σ_1 = {ϕ,Y,{c,d}}, σ_2 = {ϕ, Y, {a,c,d}}, $\sigma_{1,2}$-open= {ϕ, Y, {c,d},{a,c,d}},$\sigma_{1,2}$-closed= {ϕ, Y, {a,b},{b}}.Define f: X →Y by f(a)=a,f(b)=b,f(c)=c,f(d)=d.Here the inverse image of the closed set {b} in Y is not (1,2)*-πgr-closed in X but (1,2)*-πwg-closed, (1,2)*-πg-closed,(1,2)*-πgα-closed,(1,2)*-πgb-closed and (1,2)*- gpr-closed in X.

Remark: 6.10
The above discussions are summarized in the following diagrammatic representation.



Remark :6.11
The composition of two (1,2)*-πgr-continuous functions need not be (1,2)*-πgr-continuous is shown in the following example.

Example :6.12
Let X= {a,b,c,d}=Y=Z, τ_1 ={ ϕ, X, {c},{c,d},{a,c,d}},t_2={ ϕ, X, {d} {b,d}, {b,c,d}},$\tau_{1,2}$ open = {ϕ, X, {c},{d},{c,d},{b,d},{a,c,d},{b,c,d}}, σ_1= {ϕ, Y, {a},{a,d},{a,b,d}}, σ_2= {ϕ, Y,{d},{b,d},{a,b,d}} ,$\sigma_{1,2}$-open= {ϕ, Y, {a} , {d}, {a,d},{b,d},{a,b,d}} and η_1 = {ϕ, Z,{a},{a,b}}, η_2= {ϕ, Z,{b}}, $\eta_{1,2}$-open= {ϕ, Z,{a},{b},{a,b}}. Let f: (X,τ) →(Y, σ) and g: (Y, σ) →(Z, η) are identity mappings.Here f and g are (1,2)*-πgr-continuous functions. But the $(gof)^{-1}$ of the closed sets in Z are not (1,2)*-πgr-closed in X.Hence their composition (gof) is not (1,2)*-πgr-continuous.

Theorem: 6.13
Every (1,2)*-πgr-irresolute function is (1,2)*-πgr -continuous but not conversely.

Proof: Follows from the definitions.

Example:6.14
Let X= {a,b,c}=Y, t_1={ ϕ, X, {a}},t_2={ ϕ, X,{b},{a,b}},$\tau_{1,2}$-open = {ϕ, X, {a}, {b},{a,b}}, $\tau_{1,2}$-closed = {ϕ, X, {b,c},{a,c},{c}}, s_1={ ϕ, Y, {a}},s_2={ ϕ, Y, {a,b}}, $\sigma_{1,2}$-open={ ϕ,Y,{a},{a,b}}, $\sigma_{1,2}$-closed={ ϕ,Y,{b,c},{c}}. Let f: X →Y by f(a)=b,f(b)=a,f(c)=c. Here the inverse image of the closed sets in Y are (1,2)*-πgr-closed in X ,but the inverse image of the (1,2)*-πgr-closed sets in Y are not (1,2)*-πgr-closed in X.Hence (1,2)*-πgr-continuity need not be (1,2)*- πgr-irresolute.

Theorem: 6.15
The composition of two (1,2)*-πgr-irresolute functions is (1,2)*-πgr-irresolute .

Proof: Straight forward.

Theorem :6.16
Let f: X →Y and g:Y →Z be any two functions. Then

(i) (gof) is (1,2)*-πgr-continuous if g is (1,2)*-regular continuous and f is (,12)*-πgr-continuous
(ii) (gof) is (1,2)*-πgr-irresolute if g is (1,2)*-πgr-continuous and f is (1,2)*- πgr-irresolute.
(iii) (gof) is (1,2)*-regular-continuous if f is (1,2)*-πgr-irresolute and g is (1,2)*-πgr-continuous and X is (1,2)*-πgr-$T_{1/2}$-space.

Proof: (i)Let V be $\eta_{1,2}$-closed set in Z. Then $g^{-1}(V)$ is (1,2)*-regular closed in Y.Then $g^{-1}(V)$ $s_{1,2}$-closed in Y.Hence (1,2)*-πgr-continuity of f implies $f^{-1}(g^{-1}(V))$ is (1,2)*-πgr- closed in X.$(gof)^{-1}$ (V) is (1,2)*-πgr-closed in X. Hence (gof) is (1,2)*-πgr-continuous.

(ii)Let V be a $\eta_{1,2}$-closed set in Z. Since g is (1,2)*-πgr-continuous, $g^{-1}(V)$ is (1,2)*-πgr-closed in Y. As f is (1,2)*-πgr-irresolute,$f^{-1}(g^{-1}(V))$ =$(gof)^{-1}(V)$ is (1,2)*-πgr-closed in X. Hence (gof) is (1,2)*-πgr-irresolute.

(iii)Let V be a $\eta_{1,2}$- closed set in Z.Since g is (1,2)*-πgr-continuous, $g^{-1}(V)$ is (1,2)*-πgr-closed in Y. (1,2)*-pgr-irresoluteness of f implies that $f^{-1}[g^{-1}(V)]$= $(gof)^{-1}(V)$ is (1,2)*-pgr-closed in X.Since X is a (1,2)*-pgr-$T_{1/2}$-space, Hence $(gof)^{-1}(V)$ is (1,2)*-regular closed in X and hence (gof) is (1,2)*-regular continuous.

Theorem :6.17
Let f: X →Y be a function, then the following are equivalent.

a) f is (1,2)*-pgr-continuous.
b) The inverse image of every open set in Y is (1,2)*-pgr-open in X.

Proof: Follows from the definitions.

Theorem:6.18
If f: X →Y is (1,2)*-πgr-continuous, then f((1,2)*-πgr-cl(A)) Ì$t_{1,2}$-cl(f(A)) for every subset A of X.

Proof: Let AÌX. Since f is (1,2)*-πgr-continuous and A Ì$f^{-1}(t_{1,2}$-cl(f(A))), we obtain (1,2)*-πgr-cl(A)Ì$f^{-1}(t_{1,2}$-cl(f(A)) and then f((1,2)*-πgr-cl(A)) Ì$t_{1,2}$-cl(f(A))

Proposition:6.19
Let f: X →Y be a (1,2)*-πgr-continuous function and H be (1,2)*-π-open, (1,2)*-πgr-closed subset of X. Assume that (1,2)*-πgrC(X, τ) closed under finite intersections. Then the restriction be f/H: (H, $\tau_{1,2}$/H) →(Y, $\sigma_{1,2}$) is πgr-continuous.

Proof: Let F be any (1,2)*-regular closed subset in Y. By hypothesis and our assumption $f^{-1}(F)\cap H_1$, it is (1,2)*-πgr-closed in X.Since $(f/H)^{-1}(F)=H_1$,it is sufficient to show that H_1 is (1,2)*-πgr-closed in H.Let H_1 Ì G_1, where G_1 is any (1,2)*-π-open set in H.We know that a subset A of X is $t_{1,2}$-open , then (1,2)*-πO(A, $\tau_{1,2}$/A)= {V∩A: V∈ (1,2)*-πO(X,τ)}-------(1)

By (1), G_1= G∩H for some (1,2)*-π-open set G in X. Then H_1 ÌG_1ÌG and H_1 is (1,2)*-πgr-closed in X implies $rcl_x(H_1)$ = $rcl_x(H_1)$ ∩H

ÌG∩H= G_1 and so H_1 is (1,2)*-πgr-closed in H. Therefore, f/H is (1,2)*-πgr-continuous.

Generalization of Pasting Lemma for (1,2)*-πgr-continuous maps.
Theorem:6.20

Let X = G ∪ H be a topological space with topology $\tau_{1,2}$, Y be a topological space with topology $\sigma_{1,2}$ and let us assume that X is closed under finite union.Let f: (G, $\tau_{1,2}$/G) →(Y, $\sigma_{1,2}$) and g: (H, $\tau_{1,2}$/H) →(Y, σ) be (1,2)*-πgr-continuous functions such that f(x)=g(x) for every x ∈ G∩H. Suppose that both G and H are (1,2)*-π-open and (1,2)*- πgr-closed in X. Then their combination (f∇g): (X, τ_1,t_2) →(Y, σ_1,s_2) defined by (f∇g)(x)=f(x) if x∈G and (f∇g)(x) =g(x) if x∈H is (1,2)*-πgr-continuous.

Proof: Let F be any closed set in Y.Clearly $(f\nabla g)^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$. Since $f^{-1}(F)$ is (1,2)*-πgr-closed in G and G is (1,2)*-π-open in X and (1,2)*-πgr-closed in X, $f^{-1}(F)$ is (1,2)*-πgr-closed in X.Similarly, $g^{-1}(F)$ is (1,2)*-πgr-closed in X. Therefore, $(f\nabla g)$ is (1,2)*-πgr-continuous.

Proposition :6.21

If a function f: X →Y is πgr-irresolute, then

f((1,2)*-πgr-cl(A)) Ì(1,2)*-πgr-cl (f(A)) fr every subset A of X.

(1,2)*-πgr-cl(f^{-1}(B)) Ìf^{-1}((1,2)*-πgr-cl (B))for every subset B of Y.

Proof : (i)For every AÌX,(1,2)*- πgr-cl(f(A)) is (1,2)*-πgr-closed in Y.By hypothesis, f^{-1}((1,2)*-πgr-cl(f(A)) is (1,2)*-πgr-closed in X. Also, AÌf^{-1}(f(A)) Ìf^{-1}((1,2)*-πgr-cl(A)). By the definition of (1,2)*-πgr-closure , we have (1,2)*-πgr-cl(A) Ìf^{-1}((1,2)*-πgr-cl(A)).Hence , we get f((1,2)*-πgr-cl(A)) Ì (1,2)*-πgr-cl(f(A)).

(ii)(1,2)*-πgr-cl(B) is (1,2)*-πgr-closed in Y and so by hypothesis , f^{-1}((1,2)*-πgr-cl(B)) is (1,2)*-πgr-closed in X. Since f^{-1}(B) Ìf^{-1}((1,2)*-πgr-cl(B)),it follows that (1,2)*-πgr-cl(f^{-1}(B)) Ìf^{-1}((1,2)*-πgr-cl(B)).

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