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Eoq Model With Partial Backlogging, Linear Demand Under Inflation, Permissible Delay in Payments and Cash Discounts

KEYWORDS deterioration, inventory, linear demand, partial backlogging, inflation, permissible delay in payment, cash discount

Raman Patel	Shital S. Patel
Department of Statistics, Veer Narmad South Gujarat University, Surat-395007, INDIA	Department of Statistics, Veer Narmad South Gujarat University, Surat-395007, INDIA

ABSTRACT *An EOQ model for deteriorating items with linear demand under inflation, permissible delay in payments and cash discount is considered. Holding cost is linear function of time. Shortages are allowed and are partially backlogged. Numerical example is taken and sensitivity is also carried out to support the model.*

INTRODUCTION:

In the traditional inventory models it is assumed that items can be stored indefinitely to meet the future demand. This assumption is not true for all items. For example, the commonly used goods items like fruits, vegetables, electric components, etc. where deterioration is usually observed during their normal storage period. Therefore controlling and maintaining the inventory of deteriorating items becomes problem for decision makers. Inventory models for deteriorating items were widely studied in past. Ghare and Schrader [8] first developed an EOQ model with constant rate of deterioration. Covert and Philip [7] extended this model by considering variable rate of deterioration. Shah [19] further extended the model by considering shortages. The related work are found in (Nahmias [17], Raffat [18], Goyal and Giri [10], Wu et al. [24], Mishra et al. [15]).

Goyal [9] first considered the economic order quantity model under the condition of permissible delay in payments. Aggarwal and Jaggi [1] extended Goyal's [9] model to consider the deteriorating items. Aggarwal and Jaggi's [1] model was further extended by Jamal et al. [12] to consider shortages. The related work are found in (Chung and Dye [5], Chung et al. [6], Chang et al. [3]).

Buzacott [2] developed the first economic order quantity model by considering inflationary effects into account. Su et al. [22] developed model under inflation for stock dependent consumption rate and exponential decay. Moon et al. [16] developed models for ameliorating/ deteriorating items with time varying demand pattern over a finite planning horizon taking into account the effects of inflation and time value of money. Mishra et al. [14] considered a model for deterministic perishable items that follows variable type demand rate with infinite time horizon, constant deterioration. Hou [11] developed an inflation model for deteriorating items with stock dependent consumption rate and shortages by assuming a constant length of replenishment cycles and a constant fraction of the shortage length with respect to the cycle length. Chern et al. [4] developed an EOQ lot size model for deteriorating items with partial backlogging and inflation. An inventory model under inflation for deteriorating items with stock dependent consumption rate and partial backlogging is given by Yang et al. [23]. Singh et al. [21] developed an inventory model for decaying items with selling price dependent demand and partial backlogging under inflation. An inventory model for stock dependent consumption and permissible delay in payment under inflationary conditions was developed by Liao et al. [13]. Singh [20] developed an EOQ model with linear demand and permissible delay in payments. The effect of inflation and time value of money were also taken into account.

In this paper we have developed EOQ model when demand rate is linear function of time, deterioration is two parameter Weibull distribution and inventory holding cost is linear function of time under inflationary conditions with permissible delay in payments and cash discount. Shortages are allowed and partially backlogged. Numerical example is taken and sensitivity analysis is also carried out.

NOTATIONS AND ASSUMPTIONS:

The following notations and assumptions are used here:

NOTATIONS:

$D(t)$: $a + bt$, Demand is linear function of time, where $a > 0$, $0<b<1$

A : Ordering cost per order

c : Unit purchasing cost per item

p : Unit selling price of the item ($p>c$)

$h(t)$: $x+yt$, Inventory variable holding cost per unit excluding interest charges

c_1 : Shortage cost per unit

c_2 : Cost of lost sales per unit

Q_1 : Inventory level initially

Q_2 : Shortage of inventory

Q : Order quantity

$I(t)$: Inventory level at any instant of time t, $0 \le t \le T$

r : Cash discount rate

δ : Lost sales

I_e : Interest earned per year

I_p : Interest paid per year

R : Inflation rate

M : Permissible period of delay in setting the accounts with the supplier

T : Time interval between two successive orders

α : Scale parameter ($0 < \alpha < 1$)

β : Shape parameter ($\beta > 0$)

$\alpha\beta t^{\beta-1}$: the two parameter Weibull deterioration rate.

ASSUMPTIONS:

The following assumptions are used in the development of the model:

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and are partially backlogged.

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- The deteriorated units can neither be repaired nor replaced during the cycle time.
- During the time, the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

THE MATHEMATICAL MODEL AND ANALYSIS:

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

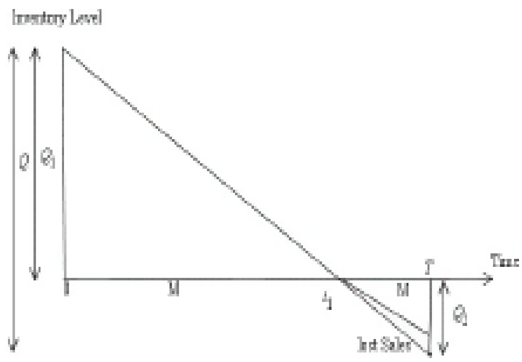

Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -(a+bt), \qquad 0 \le t \le t_1, \quad (1)$$

$$\frac{dI(t)}{dt} = -e^{-\delta(T-t)}(a+bt), \qquad t_1 \le t \le T. \quad (2)$$

with the boundary conditions at $I(0) = Q_1$, $I(t_1) = 0$ and $I(T) = -Q_2$.

The solution of equation (1) and (2) using boundary conditions are:

$$I(t) = a\left[(t_1 - t) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right) - \alpha t^{\beta}(t_1 - t)\right] + b\left[\frac{1}{2}(t_1^2 - t^2) + \frac{\alpha\beta}{\beta+2}\left(t_1^{\beta+2} - t^{\beta+2}\right) - \frac{\alpha t^{\beta}}{2}(t_1^2 - t^2)\right]$$

$$0 \le t \le t_1, \quad (3)$$

$$I(t) = \begin{bmatrix} -\frac{1}{3}b\delta T^3 - \left[\frac{1}{2}a\delta + b(1-\delta T)\right]T^2 - a(1-\delta T)T \\ +\frac{1}{3}b\delta t^3 + \left[\frac{1}{2}a\delta + b(1-\delta T)\right]t^2 + a(1-\delta T)t \end{bmatrix},$$

$$t_1 \le t \le T. \quad (4)$$

(by neglecting higher powers of α and δ)

The initial order quantity at t=0 is obtained by putting t=0 in equation (3)

$$Q_1 = \left[a t_1 + \frac{b t_1^2}{2} + \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha\beta t_1^{\beta+2}}{2(\beta+2)}\right]. \quad (5)$$

For t = T, I(T) = $-Q_2$. So from equation (4), we have

$$Q_2 = -\begin{bmatrix} -\frac{1}{3}b\delta T^3 - \left[\frac{1}{2}a\delta + b(1-\delta T)\right]T^2 - a(1-\delta T)T \\ +\frac{1}{3}b\delta t_1^3 + \left[\frac{1}{2}a\delta + b(1-\delta T)\right]t_1^2 + a(1-\delta T)t_1 \end{bmatrix}. \quad (6)$$

The associated costs are:

(i) Ordering cost (OC) = A (7)

(ii) Holding cost:

$$HC = \int_0^{t_1} h(t) I(t) e^{-Rt} dt$$

$$= \int_0^{t_1} (x+yt)\begin{bmatrix} a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{a\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right) \\ + \frac{b\alpha\beta}{\beta+2}\left(t_1^{\beta+2} - t^{\beta+2}\right) \\ - a\alpha t^{\beta}(t_1 - t) - \frac{b\alpha}{2}t^{\beta}(t_1^2 - t^2) \end{bmatrix} e^{-Rt} dt$$

$$= \frac{1}{24(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}$$

$$\left[t_1^2 \left(-24 \left(\begin{array}{l} (3 - 3Rt_1 + t_1^2R^2)(yt_1 + x)(bt_1 + a) \ldots \\ + \left(\begin{array}{l} [illegible] R^2 t_1^4 + \frac{27}{2}((xb + ya)R - 2yb)Rt_1^3 \\ + (15R^2 xa + (-30ya - 30xb)R + 60yb)t_1^2 \\ + (30xb + 30ya - 30xRa)t_1 + 30xa \end{array} \right) \beta \\ + \left(\begin{array}{l} [illegible] R^2 t_1^4 + \left(\left(\frac{127}{2}ya + \frac{[illegible]}{2}xb\right)R^2 - 127yRb\right)t_1^3 \\ + (78R^2 xa + (-156ya - 154xb)R + [illegible]yb)t_1^2 \\ + (150ya - 150xRa + 150xb)t_1 + 234xa \end{array} \right) \beta^2 \\ + \left(\begin{array}{l} [illegible] R^2 t_1^4 + ((120ya + 108xb)R^2 - 216yRb)t_1^3 \\ + (164R^2 xa + (-300xb - 328ya)R + 300yb)t_1^2 \\ + (402xb + 402ya - 402xRa)t_1 + 604xa \end{array} \right) \beta^3 \\ + 120\left(\frac{3}{4}R^2 t_1^2 y + R(-2y + Rx)t_1^2 + (3y - 3Rx)t_1 + 4x\right) \\ \left(\frac{1}{2}bt_1 + a\right) \end{array} \right) \alpha t_1^{\beta} \right. \right.$$

$$\left. \left. + (\beta+3)(\beta+4)(\beta+5)(\beta+6) \left(\begin{array}{l} 4((bt_1\beta + b)t_1 + 2a\beta + a)\left(\frac{3}{4}R^2 t_1^2 y + R(-2y + Rx)t_1^2 + (3y - 3Rx)t_1 + 6x\right)t_1^2 \\ + (\beta+1)(\beta+2)\left(\begin{array}{l} \frac{1}{2}yR^2 b t_1^4 + \left(\left(\frac{3}{5}ya + \frac{4}{5}xb\right)R^2 - \frac{8}{5}yRb\right)t_1^3 \\ + [illegible] \\ + (-4xRa + 4ya + 4xb)t_1 + 12xa \end{array} \right) \end{array} \right) \right) \right]$$

(8)

(by neglecting third and higher powers of R)

(iii) Shortage cost:

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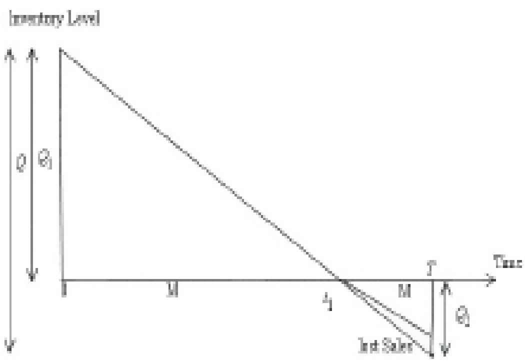

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$$0 \le t \le t_1, \quad (3)$$

$$I(t) = \begin{bmatrix} -\frac{1}{3}b\delta t^3 - \frac{1}{2}\left(a\delta + b(1-\delta T)\right)t^2 - a(1-\delta T)t \\ + \frac{1}{3}b\delta t_1^3 + \frac{1}{2}\left(a\delta + b(1-\delta T)\right)t_1^2 + a(1-\delta T)t_1 \end{bmatrix},$$

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$$= \frac{1}{24(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}$$

$$\left[t_1^2 \left(\begin{aligned} &(\beta+3)(\beta+4)(\beta+5)(\beta+6)\Big(4\big((bt_1+3a)x + 2at_1 y\big) - R\big(\tfrac{3}{4}R t_1^2 y + R(-2y+bx)t_1^2 + (3y-3Rx)t_1 + 6x\big)t_1\Big) \\ &+ \ldots \Big(\tfrac{1}{2}\ldots\Big) \end{aligned} \right) \right] \quad (8)$$

[illegible]

(by neglecting third and higher powers of R)

(iii) Shortage cost:

$$SC = -c_2\int_{t_1}^{T} I(t)e^{-Rt}dt$$

$$= -c_2\int_{t_1}^{T}\begin{bmatrix} -\frac{1}{3}b\ {}^3\ \frac{1}{2}[a\delta + b(1-\delta T)]T\ {}^2 \\ -a(\delta T)T+\ \frac{1}{3}b\delta t\ \ {}^3_1 \qquad e \\ +\frac{1}{2}[a+b(1-\delta T)]t\ \ {}^2_1 a(1-\delta T)t\ \ {}_1 \end{bmatrix} dt^{-Rt}$$

$$= -\frac{1}{45}(T-t_1)^2 c_2$$

$$\begin{bmatrix} -\frac{5}{4}t_1 R^2 b\ Rb\delta T + \quad a\delta\left(\frac{3}{2}\ \ b\ \frac{3}{2}R-\right)\ b\delta\frac{9}{2}Rt\Big)\ {}^3_1 \\ +\begin{pmatrix} \frac{3}{4}T^4R^2b \quad \frac{9}{8}\left(\left(\frac{8}{3}a\delta\ R-\right)b\delta\ RT\right) \\ -\frac{15}{8}aR^2 + \left(b\frac{45}{8}Rb + \frac{45}{8}\delta a\right)\quad \frac{45}{4} \end{pmatrix} t_1^2 \end{bmatrix}$$

$$-\frac{1}{45}(T-t_1)^2 c_2$$

$$\begin{bmatrix} +\begin{pmatrix} 2T^2bR\ {}^3- \frac{3}{4}R\ ((6b+a\delta)R+7b\delta)T\ {}^2 \\ +\left(-\frac{15}{4}aR^2 + \left(b\frac{15}{4}R+ \quad \frac{45}{4}b\delta\right)T\ \ t\ \frac{15}{2}\right) \\ -15a\delta+\frac{15}{2}aR-15b \end{pmatrix}{}_1 \\ +b\delta R^2T^4 + \frac{27}{8}\left(\left(-\frac{2}{3}b+a\delta\right)R-\frac{7}{9}b\delta\right)RT^3 \\ +\left(-\frac{45}{8}aR^2\ \delta\left(\frac{45}{8}b-\frac{75}{8}a\right)R+\frac{15}{4}\delta bT\right)^2 \\ +\left(15aR-\frac{15}{2}b+\delta 15b-\right)\quad a\frac{45}{2} \end{bmatrix} \quad (9)$$

(iv) Cost due to lost sales:

$$LS = c_3\int_{t_1}^{T}(a+bt)[1-(R(T-t)]e\quad dt$$

$$= c_3\begin{bmatrix} -\frac{1}{10}bRb\quad T^5\left(-t^5\ \ {}^5_1\right) \\ +\frac{1}{4}\left(\frac{1}{2}R^2(b\delta T\ R\ +)b\delta R\quad T\ -\right)\left(\ {}^4\quad {}^4_1\right) \\ \delta\frac{1}{3}\delta R\left(\frac{1}{2}a-a\delta\ {}^2(\delta T\ R-b\delta)\ T-t\right)\left(\ {}^3\quad {}^3_1\right) \\ +\frac{1}{2}(\delta R-a\delta+b\delta T\quad 1)\left(-t^2\ \ {}^2_1\right) \\ +a\delta T(T-t_1) \end{bmatrix} \quad (10)$$

(v) Deterioration cost:

$$DC = c\left[Q_1 - \int_0^{t_1}(a+bt)e^{-Rt}dt\right]$$

$$= c\begin{bmatrix} \frac{1}{2}\frac{bt_1^2}{\beta+1} + \frac{a\alpha t_1^{\beta+1}}{\beta+2} + \frac{b\alpha t_1^{\beta+2}}{8} - \frac{1}{}bR^2t_1^4 \\ -\frac{1}{3}\left(\frac{1}{2}aR^2-bR\right)t_1^3 - \frac{1}{2}(-aR+b)t_1^2 \end{bmatrix} \quad (11)$$

(vi) Cash discount:

$$CD = rcQ$$

$$= rc\begin{bmatrix} 2at_1 + bt_1^2 + \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} \\ -aT\ \delta T\frac{1}{6}+\ {}^3\ a\delta\frac{1}{2}T - \ {}^2b\frac{1}{2}T\quad {}^2 \\ -\frac{1}{2}bRt\ \ {}^2_1\ \ \frac{1}{3}b\delta t\ +{}^3_1\ \ \frac{1}{2}a\delta t\ \ {}_1 a\delta Tt\ {}_1 \end{bmatrix} \quad (12)$$

To determine the interest payable and interest earned, there will be two cases i.e. case I: $(0 \le M \le t_1)$ and case II: $(0 \le t_1 \le M)$.

Case I: $(0 \le M \le t_1)$: In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to t_1.

(vii) Interest earned per cycle:

$$IE_1 = pI_e\int_0^{M}(a+bt)te^{-Rt}dt$$

$$= pI_e\begin{bmatrix} \frac{1}{10}bR^2M^5 + \frac{1}{4}\left(\frac{1}{2}aR^2-bR\right)M^4 \\ +\frac{1}{3}(-aR+b)M^3 + \frac{1}{2}aM^2 \end{bmatrix} \quad (13)$$

(viii) Interest payable per cycle for the inventory not sold after the due period M is

$$IP_1 = cI_p\int_M^{t_1} I(t)e^{-Rt}dt$$

$$= cI_p\int_M^{t_1}\begin{bmatrix} a(t_1-t) + \frac{1}{2}\frac{b}{\beta+1}(t_1^2 \ldots) \ldots (t_1\ \ -t\ \ ) \\ +\frac{b\alpha}{\beta+2}(t_1^{\beta+2}-t^{\beta+2}) - a\ \ t(t_1\ \ ) \\ -\frac{b\alpha}{2}t^{\beta}(t_1^2-t^2) \end{bmatrix} e$$

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$$-\frac{1}{20(\theta+1)(\theta+2)(\theta+3)(\theta+4)(\theta+5)}\left[\ldots\right]$$

[illegible]

$$+\frac{1}{20(\theta+1)(\theta+2)(\theta+3)(\theta+4)(\theta+5)}\left[\ldots\right]$$

[illegible]

(14)

The total cost per unit during a cycle $C_1(t_1,T)$ is consisted of the following:

$$C_1(t_1,T)=\frac{1}{T}\begin{bmatrix} OC+HC+DC+SC \\ +LS+IP_1-IE_1-CD \end{bmatrix} \tag{15}$$

Putting values from equations (7) to (14) in equation (15) we get the total cost for case I.

Differentiating equation (15) with respect to t_1 and T and equate it to zero, we have

$$\frac{\partial C_1(t_1,T)}{\partial T}=0,\ \frac{\partial C_1(t_1,T)}{\partial t_1}=0. \tag{16}$$

By solving equation (16) for t_1 and T, we obtain the optimal cycle length $t_1=t_1^*$ and $T=T^*$ provided it satisfies equation

$$\frac{\partial^2 C_1(t_1,T)}{\partial T^2}>0,\ \frac{\partial^2 C_1(t_1,T)}{\partial t_1^2}>0 \text{ and}$$

$$\text{and}\left[\frac{\partial^2 C_1(t_1,T)}{\partial T^2}\right]\left[\frac{\partial^2 C_1(t_1,T)}{\partial t_1^2}\right]-\left[\frac{\partial^2 C_1(t_1,T)}{\partial T\partial t_1}\right]>0. \tag{17}$$

Case II: $(0\le t_1\le M)$:

In this case, the retailer earns interest on the sales revenue up to the permissible delay period and no interest is payable during this period. So

(ix) Interest earned up to the permissible delay period is:

$$IE_2=pI_e\left[\int_0^{t_1}(a+bt)te^{-Rt}dt+(a+bt_1)t_1(M-t_1)\right]$$

$$=pI_e\begin{bmatrix} \frac{1}{10}bR^2t_1^5+\frac{1}{4}\left(\frac{1}{2}aR^2-bR\right)t_1^4 \\ +\frac{1}{3}(-aR+b)t_1^3 \\ +\frac{1}{2}at_1^2+(a+bt_1)t_1(M-t_1) \end{bmatrix} \tag{18}$$

(x) $IP_2=0.$ (19)

The total cost per unit during a cycle $C_2(t_1,T)$ is consisted of the following.

$$C_2(t_1,T)=\frac{1}{T}\begin{bmatrix} OC+HC+DC+SC \\ +LS+IP_2-IE_2+CD \end{bmatrix} \tag{20}$$

Putting values from equations (7) to (12) and (18), (19) in equation (20) we get the total cost for case II.

Differentiating equation (20) with respect to t_1 and T and equate it to zero, we have

$$\frac{\partial C_2(T)}{\partial T}=0,\ \frac{\partial C_2(T)}{\partial t_1}=0. \tag{21}$$

By solving equation (21) for t_1 and T, we obtain the optimal cycle length $t_1=t_1^*$ and $T=T^*$ provided it satisfies equation

$$\frac{\partial^2 C_2(T)}{\partial T^2}>0,\ \frac{\partial^2 C_2(T)}{\partial t_1^2}>0$$

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$$\text{and} \left[\frac{\partial^2 C_2(T)}{\partial T^2}\right]\left[\frac{\partial^2 C_2(T)}{\partial t_1^2}\right] - \left[\frac{\partial^2 C_2(T)}{\partial T \partial t_1}\right] > 0. \qquad (22)$$

NUMERICAL EXAMPLES:

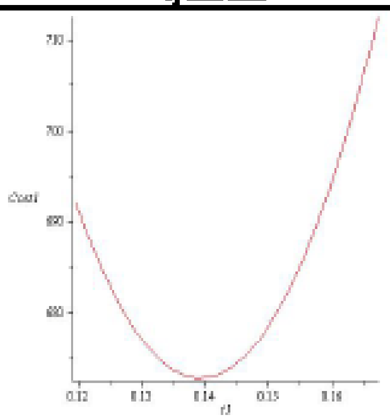
Case I: Considering A= Rs 100, c = Rs. 25, p = Rs 40, I_p = Rs 0.15, I_e=0.12, M=0.08 years, α = 0.04, β=2, a=1000, b=0.05, x=5, y=0.05, c_2 = Rs. 8, c_3 = Rs 2, r=0.012, R = 0.01, δ=0.8, t_d=0.05 in appropriate units. Then we obtained the optimal value of t_1^* = 0.1392, T*=0.1803 and the optimal total cost C_1^* = Rs. 672.7226 and the optimum order quantity Q*= 179.6610.

Case II: Considering A= Rs 100, c = Rs. 25, p = Rs 40, I_p = Rs 0.15, I_e=0.12, M=0.14 years, α = 0.04, β=2, a=1000, b=0.05, x=5, y=0.05, c_2 = Rs. 8, c_3 = Rs 2, r=0.01, R = 0.01, δ=0.8, t_d=0.05 in appropriate units. Then we obtained the optimal value of t_1^*= 0.1389, T*=0.1545 and the optimal total cost C_2^* = Rs. 445.5520 and the optimum order quantity Q*= 154.4389.

The second order conditions given in equations (17) and (22) are also satisfied. The graphical representation of the convexity of the cost function for the two cases is also given.

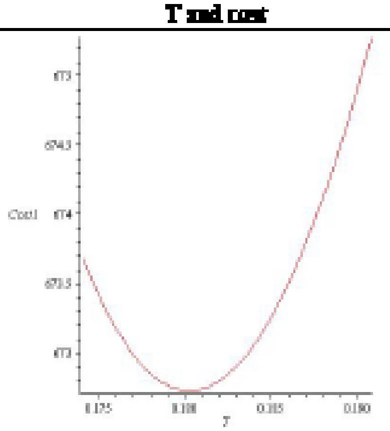
Case I

t_1 and cost



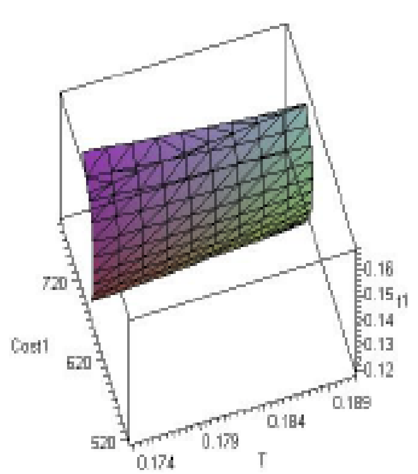
Graph 1

T and cost



Graph 2

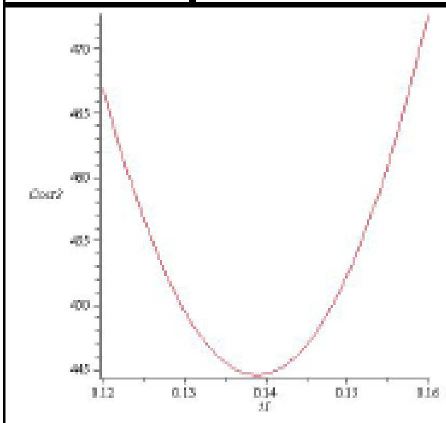
t_1, T and cost



Graph 3

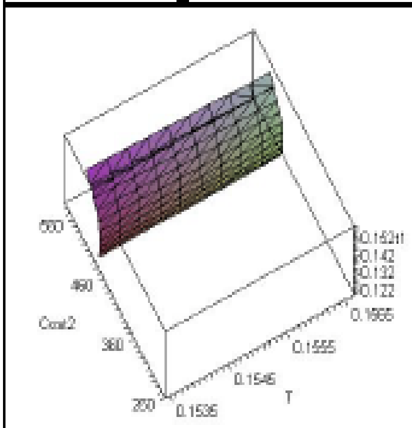
Case II

t_1 and cost



Graph 4

t_1, T and cost



Graph 5

SENSITIVITY ANALYSIS:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the

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following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis : Case I: ($0 \leq M \leq t_1$)

Parameter	%	t_1	T	Cost	Q
a	+50%	0.1168	0.1336	690.4914	202.2523
	+20%	0.1285	0.1586	689.2478	189.9226
	-20%	0.1531	0.2088	640.9628	166.0865
	-50%	0.1860	0.2786	554.5458	137.6294
α	+50%	0.1384	0.1798	675.2102	179.1682
	+20%	0.1389	0.1801	675.2144	179.4754
	-20%	0.1395	0.1805	671.7179	179.8573
	-50%	0.1400	0.1808	670.1995	180.1532
x	+50%	0.1196	0.1741	790.2640	172.9354
	+20%	0.1306	0.1775	723.5749	176.6506
	-20%	0.1491	0.1837	615.6666	183.2661
	-50%	0.1673	0.1908	518.2019	190.6424
M	+50%	0.1397	0.1644	527.0818	164.1929
	+20%	0.1397	0.1745	617.2542	174.0526
	-20%	0.1384	0.1854	724.8650	184.5525
	-50%	0.1365	0.1918	797.3814	190.6115
δ	+50%	0.1395	0.1784	675.3649	177.3289
	+20%	0.1393	0.1795	673.8080	178.5672
	-20%	0.1391	0.1811	671.5974	180.5722
	-50%	0.1389	0.1823	669.8309	181.9594
R	+50%	0.1381	0.1799	679.2971	179.2370
	+20%	0.1388	0.1801	672.7298	179.4541
	-20%	0.1397	0.1804	670.0654	179.7745
	-50%	0.1404	0.1806	666.0503	179.9912
r	+50%	0.1444	0.1576	570.4989	157.5710
	+20%	0.1418	0.1722	636.8525	171.8690
	-20%	0.1360	0.1871	702.9312	186.0899
	-50%	0.1302	0.1953	738.9135	193.6351

Table 2
Sensitivity Analysis: Case II: ($0 \leq t_1 \leq M$)

Parameter	%	t_1	T	Cost	Q
α	+50%	0.1382	0.1542	447.4319	154.1509
	+20%	0.1385	0.1544	445.7082	154.3433
	-20%	0.1391	0.1547	443.3909	154.6819
	-50%	0.1395	0.1549	441.6379	154.8238
x	+50%	0.1226	0.1535	582.7961	153.1432
	+20%	0.1319	0.1540	503.9225	153.8358
	-20%	0.1487	0.1552	378.7520	155.2137
	-50%	0.1608	0.1566	265.6057	156.8500
δ	+50%	0.1389	0.1536	445.0335	153.5066
	+20%	0.1389	0.1541	444.7515	154.0254
	-20%	0.1389	0.1549	444.3428	154.8544
	-50%	0.1389	0.1555	444.0090	155.5805

From the table we observe that with increase and decrease in parameters a, there is corresponding increase/ decrease in total cost for both case I and case II and there is decrease/ increase in total quantity for both case I and case II.

We observe that with increase and decrease in parameter x, there is corresponding increase/ decrease in total cost but there is very little decrease/ increase in quantity for both the cases.

Also we observe that with increase and decrease in parameter δ, there is corresponding increase/ decrease in total cost and decrease/ increase in total quantity for case I and case II.

Moreover, we observe that with increase and decrease in the value of a, there is corresponding increase/ decrease in total cost and quantity for case I.

Moreover, we observe that with increase and decrease in the value of M, there is corresponding decrease/ increase in total cost and quantity for case I.

Also we observe that with increase and decrease in parameter R, there is corresponding increase/ decrease in total cost and decrease/ increase in total quantity for case I.

Also, we observe that with increase and decrease in the value of r, there is corresponding decrease/ increase in total cost and quantity for case I.

CONCLUSION:

In this paper we have developed an EOQ model for deteriorating items with linear demand, time varying holding cost and partial backordering under inflationary conditions and permissible delay in payments. Sensitivity with respect to parameters has been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of cost.

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