Modeling of Infiltration Process – A Review

KEYWORDS: Infiltration, Modeling, Computing Software, Mathcad

Capt (Dr) N P Sonaje

Deputy Registrar,Shivaji University, Kolhapur Maharashtra

ABSTRACT *Hydrological studies aim at modeling of fundamental transport processes in hydrologic cycle. Moreover, hydrological study is a useful tool to decide adequate availability of water, its storage and release which are regarded as the complex issues in water management. More so, in the process of infiltration, hydrological system assumes a critical dimension for simulation of water movement. Many researchers have conceptualized numerous methods / models for performing simulations related to the movement of water and unanimously concluded that the rate of infiltration of water is most vital parameter required for modeling. Interestingly, the process of water movement in subsoil is very dynamic, changing dramatically in temporal and spatial paradigms. Hydrological application aspects are evident more specifically for flood protection projects, rehabilitation of aging dams, floodplain management, water-quality evaluation, and water-supply forecasting. Present communication reviews state-of-art models used for infiltration studies.*

1.0 INTRODUCTION

Infiltration is process whereby water enters the soil, generally through soil-atmosphere boundary. It produces a downward flux that changes water content and pore-water pressure gradients with depth. Horton (1940), showed that during a period of constant precipitation, the rate of infiltration decreases with time. When there is plenty of water available, infiltration rates follow the limiting function, until a constant rate is reached.

Literature review reveals numerous methods for estimation of water infiltration rate such as,

i. In situ measurement techniques
ii. Empirical models,
iii. Green-Ampt models,
iv. Richard's equation models and
v. Information systems for infiltration.

A review of these methods is taken in this paper.

1.2 IN SITU MEASUREMENT TECHNIQUES

Hydraulic properties may be measured or estimated either by measurement on undisturbed samples in laboratory or in situ measurements. Jejurkar (2005) has reported the comparison of various infiltration equations with experimental results and found best fit equation for specific land cover and land use. The sub-methods of situ measurements are as follows:

1.2.1 One-dimensional ponded infiltration measurement techniques – the double ring infiltrometer (DRI)

The DRIs are usually thin-walled, generally pushed or driven to a short distance into the soil. Area inside the concentric cylinders is filled with water and rate of water loss from ring is taken as an estimate of one-dimensional infiltration rate. The DRIs are used for measuring cumulative infiltration, infiltration rate and field-saturated hydraulic conductivity.

1.2.2 Constant head model for double-ring infiltrometer

Unlike the need of refilling in order to maintain water level in DRI, this model maintains a constant water depth. The rate of water loss from the inner ring is taken as an estimate of the 1-D infiltration rate of soil. Theoretical basis of this method is 'quasi-steady-state infiltration rate' that estimates 'quasi-steady flow' (K_{fs}) in the near-surface soil under measuring cylinder. Time required to reach quasi-steady-state flow depends on various soil attributes like texture and structure.

However, for a deep soil profile, a unit hydraulic gradient is commonly assumed. Steady state infiltration rate (q_∞) in the above case is considered equals to K_{fs} and mathematically expressed as,

$q_\infty = K_{fs}$, (1)

K_{fs} can be obtained as,

$$K_{fs} = \frac{q_s}{\left(\frac{H}{(C_1 d + C_2 a)}\right) + \left(\frac{1}{(\alpha(C_1 d + C_2 a))}\right) + 1} \quad (2)$$

where,

q_s = quasi-steady-state infiltration rate, and

$C_1 = 0.316\,\pi$; $C_2 = 0.184\,\pi$

a, α = Soil parameters selected from the soil texture and structure

1.2.3 Three-dimensional (unconfined) tension infiltrometer

After discussing the one-dimensional methods, it is worthwhile to explore the three dimensional infiltrometer that yields rich information owing to exploration in multiple dimensions. The bubble tower in the tension infiltrometer contains a moveable air-entry tube (Perroux and White, 1988). The air-entry tube is used to impose the desired negative water pressure at base of disc by varying the distance between air-entry point and water level. Measurements are conducted for each imposed negative water pressure. Subsequently, volume of water infiltrating into the soil is measured by recording the change in height of water in reservoir, manually or automated reservoir level.

1.2.4 Limitations of in situ measurement techniques

- Rate of infiltration decreases with increase in the depth and / or diameter of infiltrometer.
- Rate of infiltration increases with increase in head of water.
- Boundary condition of infiltrometer affect on rate of infiltration.
- The driving of tube or rings disturbs the soil structure; and raindrop-impact is not simulated.
- Complex and less accurate.

Volume : 3 | Issue : 9 | Sept 2013 | ISSN - 2249-555X

1.3 EMPIRICAL MODELS
Unlike the complexity associated with in situ methods, empirical models are usually in simple form of equations. They provide estimates of cumulative infiltration and infiltration rates quite accurately. However, they are unable to provide information regarding water content distribution. Few of them are,

1.3.1 Kostiakov's Equation
The empirical basis of one of the leading methods proposed by Kostiakov's (Sonaje and Waikar 2009) is,

$i(t) = \alpha \bullet t^{-\beta}$ (3)

where,

i = infiltration rate at a time t.;
α and β = empirical constants.

It describes the infiltration quite well at small time but less accurate at large times. It was further modified as below.

1.3.2 Modified Kostiakov's Equation:
$I = k \bullet t^a + f_0 \bullet t$ (4)

where,

I = cumulative infiltration; t = opportunity time, and

k; a and f_0 = empirical coefficients.

This equation is most commonly used in surface irrigation applications.

1.3.3 Horton's Equation
Yet another theory proposed by Horton (1940) describes the basic behavior of infiltration as shown by equation. However, the physical interpretation of exponential constant is poorly defined. Mathematical equation for determining the rate curve of infiltration capacity is given by,

$i(t) = i_f + (i_o - i_f) \bullet e\ \gamma^{\bullet t}$ (5)

where,

i_o, i_f = initial and final infiltration rate,

t = time since start of rainfall; γ = empirical constant.

The striking drawback of this equation is inadequacy to represent the rapid decrease of 'i' from very high values at 't' as shown by Philip [5].

1.3.4 Philip's Equation
The equation put forth by Philips (1974) is valid in the limiting condition of 't' not being too large,

$q(t)= 0.5 \bullet S \bullet t^{(-½)} + A$ (6)

$S = I / t^{½}$ (7)

where,

q(t) = rate of infiltration;
t = time for infiltration;

S = sorptivity of soil;
A = constant.

Further, Philip (1974) revealed the reasonable values for 'A' are, $K_s/2$, $2K_s/3$, and $0.38K_s$. With this work the failure of the model for very large values of times was justified.

1.3.5 Mezencev's Equation
The limitations of Kostiakov's equation for large times, were modified by Mezencev as,

$i(t) = i_f + \alpha \bullet t^{-\beta}$ (8)

where,

i_f = final infiltration rate at time t;
α and β = empirical constants.

1.3.6 NRCS (SCS) Equation
Yet another work in this field is reported by the USDA Soil Conservation Service (USDA-SCS, 1972), popularly referred to as SCS equation and renamed as 'Curve Number Method' and 'Natural Resources Conservation Service (NRCS) method' subsequently. The equation assumes that for a single storm, ratio of actual soil retention to potential maximum retention is equal to the ratio of direct runoff to available rainfall. In case of lack of soil moisture data or insufficient definition of boundary conditions, the NRCS model is suitable semi-empirical model. The basic mathematical function of this model is given by equations 9 to 11.

$$R = \frac{(P - I_a)^2}{(P + I_a)} \quad \text{When } P > Ia \qquad (9)$$

Otherwise,

R = 0 When P < Ia (10)

$I_a = \lambda \bullet S$ (11)

where,

P = daily precipitation;
I_a = initial abstraction,

λ = initial abstraction ratio, and
S = potential maximum retention.

The lacuna in this equation was consideration of fixed level of λ (i.e. 0.2) that however, either underestimate or overestimate the runoff (Chunale et.al., 2001). Therefore, it was suggested to make λ as a variable.

The potential maximum retention was given as,

$$S = \frac{25400}{CN} - 254 \qquad (12)$$

where,

CN = Curve number for concerned Antecedent Moisture Condition (AMC).

Infiltration (I) is calculated as the excess of rainfall (P) over runoff (R), i.e.,

I = P – R (13)

1.3.7 Holtan's Equation
This model specifically takes into account the effects of vegetation and soil water condition in the form of available pore space for moisture storage (Holtan, 1961).

$i(t) = i_f + a \bullet b \bullet (\omega - I)^{1.4}$ (14)

where,
a = constant (0.25 to 0.8); b = scaling factor,

ω = initial moisture deficit and I = cumulative infiltration at time t.

1.3.8 Advantages of Empirical Models
- They are based on widely-accepted concepts of soil

Volume : 3 | Issue : 9 | Sept 2013 | ISSN - 2249-555X

physics and easy to use.

- Hydraulic parameters can be easily obtained from literature and electronic databases.
- Site-specific measurements of all parameters are not necessary for obtaining preliminary estimates of water flux.
- Spatial variability of soil parameters can be more easily incorporated into the mathematical models.

1.4 GREEN AMPT (GA) MODELS

After taking in-depth account of in situ and empirical models, the Green-Ampt models that addresses surface ponding and movement of wetting front is dealt here. The core importance in these methods is evaluation of soil moisture-pressure profile.

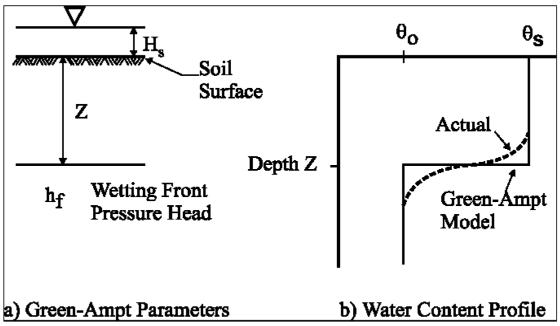

Figure 1. Illustration of Green-Ampt parameters.

The GA theory considers water to move downwards as piston flow with a well-defined wetting front as shown in figure 1. It uses Darcy's equation and is based on infiltration into deep homogeneous reservoirs with a homogeneous initial water content distribution.

$$q = \frac{dI}{dt} = -K_s\left(\frac{h_f-(h_s+Z)}{Z}\right) \qquad (15)$$

where,

q = rate of infiltration; K_s = hydraulic conductivity to the surface water content,

$I(t)$ = cumulative infiltration at time t ; h_f = soil-water pressure head at the wetting front,

h_s = soil-water pressure at surface and

Z = penetration of the infiltrating water front.

$I(t) = Z \bullet (\theta_s - \theta_o)$ (16)

The statement of the GA model was derived by integrating equation (16) as given below,

$$I = K_s t-(h_f-h_s)(\theta_s-\theta_0)\log_e\left[1-\frac{I}{(h_f-h_s)(\theta_s-\theta_0)}\right] \qquad (17)$$

1.4.1 GA Model for Layered Systems (GALAYER)

GA model is widely worked out modification for calculation of infiltration into non-uniform soils. Flerchinger et.al. (1988), developed a model (GALAYER), for calculating infiltration over time in vertically heterogeneous soils. Mathematical formulations is,

$f = f^* \bullet K_n$ (18)

$f^* = (F^* + 1) / (F^* + z^*)$ (19)

$F^* = 0.5\,[t^* - 2z^* + \{(t^* - 2z^*)^2 + 8\,t^*\}^{½}]$ (20)

where,

f = infiltration rate;
f* = dimensionless infiltration,

K_n = hydraulic conductivity of layer n containing wetting front,

F* = dimensionless accumulated infiltration in layer n,

z* = dimensionless depth accounting for thickness and conductivity of layers behind the wetting front and

t = time

1.4.2 Explicit GA Model (GAEXP)

More general kind of (GAEXP) model for q(t) and I(t), facilitated a straightforward and accurate estimation of infiltration for any given time. Mathematical formulations is,

$$q(t) = \left[\left(\frac{\sqrt{2}}{2}\right)\cdot\tau(t)^{\left(-\frac{1}{2}\right)}+\left(\frac{2}{3}\right)-\left(\frac{\sqrt{2}}{6}\right)\cdot\tau(t)^{\frac{1}{2}}+\left(\frac{1-\sqrt{2}}{3}\right)\cdot\tau(t)\right]\cdot K_s \qquad (21)$$

$$\tau(t) = \frac{t}{(t+\chi)} \qquad (22)$$

$$\chi = \frac{(h_s - h_f)\cdot(\theta_s-\theta_0)}{K_s} \qquad (23)$$

where,

q(t) = infiltration rate at time (t); K_s = saturated hydraulic conductivity,

h_s = ponding depth; h = capillary pressure head at wetting front,

θ_s = saturated volumetric water content and

θ_o = initial volumetric water content.

1.4.3 Constant Flux GA Model (GACONST)

The specific case derived for infiltration into a sandy loam soil under non ponding conditions is 'GACONST Model (Singh and Woolhiser, 2002). Underlying mathematical formulations for varying boundary conditions is,

For $r > K_s$ and $t > 0$,

$q = r$ (24)

For $r > K_s$ and $t < t_o$,

$q = r$ (25)

For $r > K_s$ and $t > t_o$,

$$q = K_s\left[1-(\theta_s-\theta_0)\cdot\frac{h_f}{I}\right] \qquad (26)$$

$$t_0 = \frac{-K_s\cdot h_f(\theta_s-\theta_0)}{r\cdot(r-K_s)} \qquad (27)$$

where,

q = surface infiltration rate; r = constant water application rate at the surface,

Volume : 3 | Issue : 9 | Sept 2013 | ISSN - 2249-555X

t = time; K = saturated hydraulic conductivity,

θ_s, θ_o = saturated and initial volumetric water content, and

h_f = capillary pressure head (< 0) at the wetting front.

1.4.4 Advantages of Green-Ampt Models
- Simplicity to use.
- Adaptability to varying scenarios
- Easily-measurable variables.

1.5 RICHARD'S EQUATION MODELS
Here the problem is essentially treated as one-dimensional. The foremost condition for analysis is vertical soil water movement. Depending on the simplicity (or complexity) of input parameters, Richard's equation has been solved exactly or partially. However, this infiltration equation only relates to the cumulative infiltration to time; they do not provide information on moisture profile or water flux distribution (Philip, 1991). Basis of this model is Darcy-Buckingham law, for soil water flux,

$q = - K(\theta)\nabla\psi(\theta)$ (28)

where,
q = water flux;
θ = volumetric water content,

K = unsaturated hydraulic conductivity of soil,
ψ = total soil-water head, and
z = the vertical coordinate.

The Darcy-Buckingham law has been combined with continuity equation to obtain a general form of Richard's equation as stated below,

$$\frac{\partial\theta}{\partial t} = \nabla\left(K(\theta)\nabla h(\theta)\right) - \frac{\partial K(\theta)}{\partial z} \qquad (29)$$

1.5.1 Limitations of Richard Equation
- Colloidal swelling and shrinking of soils may demand that the water movement be considered relative to the movement of soil particles; this phenomenon may also cause significant changes in soil permeability.
- Two-phase flow involving air movement may be important when air pressures differ significantly from atmospheric pressure.
- Thermal effects may be important, especially for evaporation during redistribution of infiltrated water, in which case the simultaneous transfer of both heat and moisture needs to be considered.
- Depending on the simplicity (or complexity) of these input parameters, the Richards equation can be solved exactly or numerically.

1.6 Information Systems for infiltration
Author (Sonaje, 2011) have conceptualized and explored the usefulness of information systems applied at different scales, from field to basin to continent. This poses a unique advantage of universality of a single model that can analyze the infiltration irrespective of system dimensions. Available computing tools are divided into two broad categories such as,

i. Numerical Computing Software tools and
ii. Hydrological Software Tools.

1.6.1 Numerical Computing Software tools Vs. Hydrological Software Tools
Numerical Computing Software tools such as Mathcad, Mathematica, MathView, MATLAB, and Maple offer powerful computational paradigm. Most of these tools empower the analyst with following unique capabilities:

- Numeric and symbolic computations
- Algebraic, trigonometric, and matrix functions
- Graphics and Visualization
- Conditional programming
- Flexible, easy to use interface.

Some of leading hydrological software tools includes HYMOS, SLURP, SWAT, USGS PRMS, MIKE SHE, WEHY, SWAP Model, etc. They are generally classified as black-box packages with the intended functionality.

On other hand, a mathematical software package facilitates a great deal of computing power, flexibility and customization in comparison to their hydrological counterparts. Regarded as white-box packages, this offers the user immediate visualization of end effects with respect to data. Moreover, their reliability is unquestionable.

1.6.2 Numerical Computing Software tools
These tools have become the backbone of studies for planning, design, operation and management of projects, to conserve water and soil resources and to protect their quality.

1.6.2.1 MATLAB
MATrix LABoratory (Gilat Amos, 2004) (MATLAB) was invented in late 1970s since then it is well adapted to numerical experiments owing to constantly evolving algorithms, built-in functions and m-files those are based on standard libraries such as LINPACK and EISPACK.

1.6.2.2 Mathcad
Unlike MATLAB, the Mathcad (Sonaje, 2011) offers a rich problem-solving environment just like a pad of pencil and paper with wide choice of tools, supported by a variety of analysis and visualization techniques. It is combination of,

- a powerful technical computing environment centered on real math notation and
- a flexible, full-featured technical word processor.

1.6.2.3 Mathematica
Whenever working with symbolic complex formulae is concerned, there is nothing like Mathematica, high-level programming tool with graphics support (www.sims.berkeley.edu/~hal). It enables the followings,

a. to handle probabilistic design,
b. to handle laws of computing with random variables and graphic capabilities and
c. able to create an expression which can be tabulated, plotted, or used in subsequent computations.

1.6.2.4 Maple
One of the superior numerical tools used today is Maple that offers,

a. intuitive smart and self document environment with math equation editor,
b. task templates and interactive task assistants,
c. 2-D and 3-D plotting and animations.
d. code generation and
e. compatibility to Excel, MATLAB, C, Java, and FORTRAN.

1.6.3 Hydrological Software Tools
Several hydrological softwares are of complex and require large input details. However, prediction accuracy of these models is very low as compared to detailed inputs.

1.6.3.1 HYMOS
HYMOS is well known and widely used conceptual hydrological state of art information system for water resources management, hydrology, meteorology, water quality and environmental assessment. HYMOS adopts the Sacramento rainfall-runoff model.

1.6.3.2 SWAT (Soil and Water Assessment Tool)
SWAT predicts runoff, sediment, nutrients, bacteria and pes-

Volume : 3 | Issue : 9 | Sept 2013 | ISSN - 2249-555X

ticides from both rural and urban land uses [15].

Strengths,

a. Physically based with GIS interface,
b. Good land management modules and
c. Suitable for any sized watershed.

Limitations
a. Not for simulating single storm event,
b. Complex requires many inputs and
c. Difficult to manage and modify input files.

1.6.3.3 USGS PRMS (Precipitation Runoff Modeling System) Model
It is conceptual, distributed parameter model capable of continuous simulations and GIS interface (www.brc.tamus.edu/swat). The watershed is divided up into sub watersheds and hydrologic response units (HRUs).

System inputs are daily precipitation, daily maximum and minimum air temperature. Output includes simulated mean daily discharge, monthly and annual summaries of precipitation, interception, evapotranspiration, and inflow and outflow from the groundwater reservoirs.

1.6.3.5 MIKE-SHE Model
MIKE-SHE model is a combination of the SHE Model and MIKE-11. It is physics based model with lumped and distributed parameter capabilities. Its modular format allows simulation of any or all components of land phase of hydrologic cycle (www.dhisoftware.com).

1.6.3.6 WEHY (Watershed Environmental HYdrology)
WEHY model (Fukami and Matsuura, 2003) accounts for effect of heterogeneity within natural watersheds. Toward this purpose, point location scale conservation equations for various hydrologic processes have been up scaled, in order to obtain their ensemble averaged forms at the scale of computational grid areas. It has a confined groundwater aquifer component that represents a possible series of several confined, pressurized groundwater aquifers, underlying a watershed.

1.6.3.7 Soil – Water – Atmosphere - Plant (SWAP) Model
At smallest scale, the field, vertical water balance SWAP model has been used to show the relationship between water quantity and quality. At intermediate or irrigation-scheme level, this model has been used to represent types of crop, soil and irrigation. This model provides information on the effects of management changes in water distribution and allocation on productivity of irrigation schemes.

1.6.7 Advantages of information system
- Information system can be used to understand processes that are difficult to measure because of complexity or temporal and / or spatial scale.
- Information system can be used to study the effects of changes in land cover, water management or climate: the impacts of alternative scenarios.

1.7. Conclusion
The paper has reviewed various frameworks for measurement of infiltration in use for analytical and experimental comparison. With extensive coverage an analyst may go for a suitable and economical method and further tailor the same for obtaining numerical solutions for scientific estimations. Such infiltration studies are helpful for rainfall – runoff simulation. The paper has revealed that current infiltration models are comprehensive, and formed their basis on many physical parameters. In general, usage of numerical computing software tools is growing by leafs and bounds. With extensive survey these tools are found to be capable of simulating not only water quantity but also quality. The review empowers the researchers to choose appropriate model for water infiltration investigations.

REFERENCE
1. Chunale G L, Atre A A, Bangal G B and Kareppa S M. 2001, "On the initial abstraction of SCS curve number model for various storm sizes in sub humid region of Maharashtra –I," Indian Journal of Soil Conservation 29 (3), 248 – 255. | 2. Flerchinger, G.N., F.J. Waltts, and G.L. Bloomsburg. 1988. "Explicit solution to Green-Ampt equation for non uniform soils." J. Irrig. and Drain. Div., ASCE. | 3. Fukami K. and Matsuura T., 2003. "Watershed environmental hydrology (WEHY) model: Hydrologic module," ASCE J. of Hydrol. Engg., 2003. | 4. Gilat Amos, 2004, "MATLAB: An introduction with applications", John Wiley & Sons. ISBN 0-471-69420-7. | 5. Holtan, H.N. 1961. "A concept for infiltration estimates in watershed engineering." USDA-ARS 41-51. | 6. Horton, R.E. 1940. "An approach towards a Physical Interpretation of Infiltration Capacity." Soil Sci. Soc. Am. Proceedings 5: 399-417. | 7. Jejurkar, 2005, "Formulation of different infiltration equations on different land covers", Unpublished M.E. dissertation, SGGS College of Engineering and Technology, Nanded, Maharashtra. | 8. Perroux, K.M., and I. White. 1988. "Design for disc permeameters." Soil Sci. Soc. Am. J. 52:1205-1215. | 9. Philip, J.R. 1974. "Recent Progress in the Solution of Nonlinear Diffusion Equations." Soil Sci. 117(5): 257-264. | 10. Philip, J.R. 1991. "Horizontal Redistribution with Capillary Hysteresis," Water Resources Research 27(7): 1459-1469. | 11. References | 12. Singh Vijay P., and Woolhiser David A., 2002, "Mathematical modeling of watershed hydrology," Journal of Hydrologic Engineering, Vol. 7, No. 4, July 1, 2002. ASCE, 270–292. | 13. Sonaje N. P. and Waikar M. L., 2009, "Application of Mathcad for Hydrological Analysis Using Kostiakov's Method." Asian Journal of Environmental and Chemical Research, Vol. 2, pg., 57 - 63. | 14. Sonaje N. P., "Mathcad A Tool For Infiltration Modeling" Lambert Acedemic Publication, Germeny, 2011. | 15. USDA-SCS. 1972. "National Engineering Handbook, Hydrology," Section 4. USDA. | 16. www.brc.tamus.edu/swat as retrieved on Apr 24, 2013. | 17. www.buttecounty.net as retrieved on Apr 30, 2013. | 18. www.dhisoftware.com as retrieved on Apr 24, 2013. | 19. www.sims.berkeley.edu/~hal as retrieved on Mar 28, 2013. |