



# Theoretical and Experimental Vibration Study of Continuous Beam With Crack Depth and Location Effect

## KEYWORDS

Vibration beam, crack beam, health monitoring, experimental vibration crack beam, frequency of beam with crack effect.

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**ABSTRACT** *In this research the natural frequency of a cracked beam with different simply supported beam, is investigated analytically and experimental with different crack depth and location effect and the results are compared. The analytical results of the effect of a crack in a continuous beam are calculated the stiffness, EI, for a rectangular beam to involve an exponential function. A comparison made between analytical results from theoretical solution of general equation of motion of beam with crack effect with experimental results, where the biggest error percentage is about (9.4 %). Also it is found that the frequency of beam when the crack is in the middle position is less than the frequency with crack near the end position and the natural frequency of beam decreasing with increasing of crack depth due to decreasing of beam stiffness at any location of crack in beam.*

## INTRODUCTION

Many engineering components used in the aeronautical, aerospace and naval construction industries are considered by designers as vibrating structures, operating under a large number of random cyclic stresses. Consequently, it is natural to expect that fatigue crack initiation and propagation in critically stressed zones of such structures, in particular when local or general resonance occurs, [3]. The importance of the beam and its engineering applications is obvious, and it undergoes many different of loading. Many types of loading may cause cracks in the beam. These cracks and their locations effect on the shapes and values of the beam frequency. Recently these topics are so prevailing in the industry of spacecraft, airplanes, wind turbines, turbines, robot arm and many other applications. Many studies were performed to examine the vibration and dynamic of cracked beams, as,

**M. I. Friswell and J. E. T. Penny [1]**, presented a compares the different approaches to crack modeling, and demonstrates that for structural health monitoring using low frequency vibration, simple models of crack flexibility based on beam elements are adequate. This paper also addresses the effect of the excitation for breathing cracks, where the beam stiffness is bilinear, depending on whether the crack is open or closed. Most structural health monitoring methods assume that the structure is behaving linearly, whereas in practice the response will be nonlinear to an extent that varies with the form of the excitation. This paper will demonstrate these effects for a simple beam structure.

**K. El Bikri et. al. [3]**, presented a theoretical investigation of the geometrically non-linear free vibrations of a clamped-clamped beam containing an open crack. The approach uses a semi-analytical model based on an extension of the Rayleigh-Ritz method to non-linear vibrations, which is mainly influenced by the choice of the admissible functions. The general formulation is established using new admissible functions, called "cracked beam functions", and denoted as "CBF", which satisfy the natural and geometrical end conditions, as well as the inner boundary conditions at the crack location. The work is restricted to the fundamental mode in order to concentrate on the study of the influence of the crack on the non-linear dynamic response near to the fundamental resonance.

**H. R. Oz and M. T. Das [4]**, presented the in plane vibrations of cracked circular curved beams is investigated. The beam is an Euler-Bernoulli beam. Only bending and extension effects are included. The curvature is in a single plane. In plane

vibrations is analyzed using FEM. In the analysis, elongation, bending and rotary inertia effects are included. Four degrees of freedom for in-plane vibrations is assumed. Natural frequencies of the beam with a crack in different locations and depths are calculated using FEM. Comparisons are made for different angles.

**Liao-Liang Ke et. al. [5]**, presented the post-buckling response of beams made of functionally graded materials (FGMs) containing an open edge crack is studied based on Timoshenko beam theory and von Kármán nonlinear kinematics. Ritz method is employed to derive the nonlinear governing equations, which are then solved by using Newton-Raphson method to obtain the post-buckling load-end shortening curves and post-buckling deflection-end shortening curves. Unlike isotropic homogeneous beams, bifurcation buckling does not occur for both intact and cracked FGM beams due to the presence of bending-extension coupling effect.

The objective of this paper is to study the effect of crack depth and position on the natural frequency of the beam by using of analytical solution of general equation of motion of beam with crack effect and compared with experimental results evaluated by test of beam samples with crack effect of beam.

To achieve the above objectives, analytical solution is developed for dynamic analysis of beam with and without crack effect to evaluate the fundamental natural frequency of beam by using the analytical solution of general equation of motion of beam with crack effect, by building a computer program for analytical solution using Fortran power station 4.0 program. In addition to, evaluated the natural frequency of beam with crack effect by experimental work with different crack location and depth with different supported of beam.

## EXPERIMENTAL WORK

The experimental work include vibration test to calculate the fundamental natural frequency of beam with and without crack effect for different beam material, and boundary conditions of beam with dimensions (-length of beam, -width of the beam, -depth of the beam, -depth of crack in beam, and -location of the crack in the beam) and with different crack size and location effect.

The beam sample using in experimental study with crack effect shown in the Fig. 1. And The (TM16 universal vibration apparatus) from TQ company is employed at this study

shown in Fig. 2.

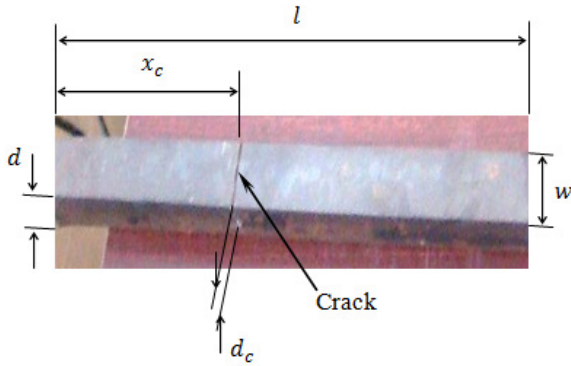


Figure 1. Sample Beam with Crack Used in Experimental Work.

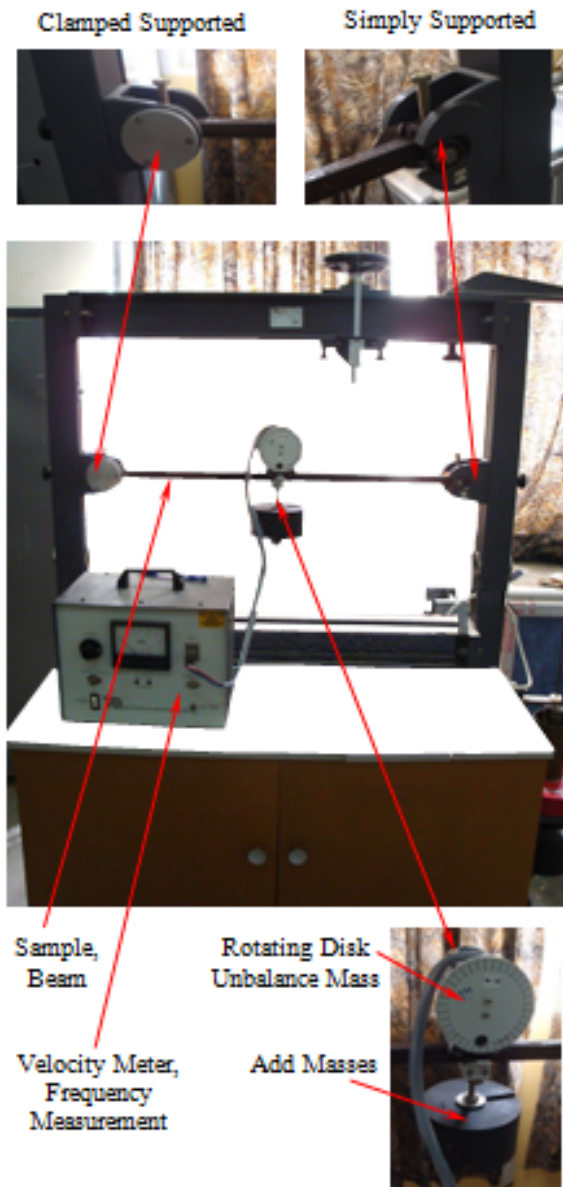


Figure 2. The Universal Vibration Apparatus

**THEORETICAL STUDY**

Consider the beam shown in Fig. 1 or 3, having the following geometrical and material characteristics ( $l$ , where;  $E$ -modulus

of elasticity; and  $\rho$ -density of beam and other notations as shown in the figure. The beam is supposed to be loaded with a bending moment and to have a uniform transverse surface crack of depth  $a$  located at a given position  $x_c$  from the left edge of the beam.

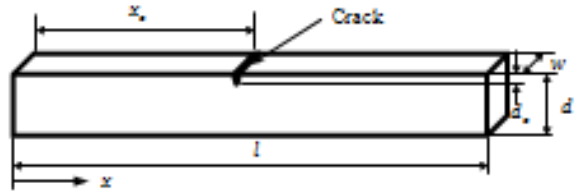


Figure 3. Dimensions of Beam with crack

The general equation of beam vibration can be written as,[2],

$$\frac{\partial^2}{\partial x^2} \left[ EI(x) \frac{\partial^2 w}{\partial x^2}(x, t) \right] + \rho A(x) \frac{\partial^2 w}{\partial t^2}(x, t) = 0 \quad (1)$$

The effect of a crack in a continuous beam and calculated the stiffness,  $EI$ , for a rectangular beam to involve an exponential function given by, [1]:

$$EI(x) = \frac{EI_0}{1 + C \exp(-2\alpha|x-x_c|/d)} \quad (2)$$

Where,

$$C = \frac{(I_0 - I_c)}{I_c}, \text{ for, } I_0 = \frac{wd^3}{12} \text{ and } I_c = \frac{w(d-d_c)^3}{12};$$

$w$  and  $d$  are the width and depth of the beam,  $d_c$  is the crack depth,  $x_c$  is the position along the beam, and  $x$  is the position of the crack.

$\alpha$  is a constant equal to (0.667), [1].

The mass for the beam can be calculated by,

$$\rho A(x) = \rho * w * d = \rho A \quad (3)$$

For vibration analysis of the beam having a crack with a finite length, relation Eq. (2) can be expanded as a sum of sine and cosine functions in the domain  $0 \leq x \leq L$  by Fourier series, as,

$$EI = A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x}{L} \quad (4)$$

Where,  $A_0$ ,  $A_n$ , and  $B_n$  are Fourier series constant can be evaluated as, [18],

$$A_0 = \frac{1}{L} \int_0^L EI(x) dx = \frac{1}{L} \int_0^L \frac{EI_0}{1 + C \exp(-2\alpha|x-x_c|/d)} dx$$

$$A_n = \frac{2}{L} \int_0^L EI(x) \cos \frac{2n \pi x}{L} dx = \frac{2}{L} \int_0^L \frac{EI_0}{1 + C \exp(-2\alpha|x-x_c|/d)} \cos \frac{2n \pi x}{L} dx$$

$$B_n = \frac{2}{L} \int_0^L EI(x) \sin \frac{2n \pi x}{L} dx = \frac{2}{L} \int_0^L \frac{EI_0}{1 + C \exp(-2\alpha|x-x_c|/d)} \sin \frac{2n \pi x}{L} dx \quad (5)$$

By integral equation (5) by  $x$ , using Simpson's rule integration method [19], get the Fourier series constant, as,

$$\int_{x_i}^{x_f} f(x) dx = \frac{1}{3} \left( \frac{x_f - x_i}{m_d} \right) \left[ f(x_i) + 4 \sum_{s=1,3,5,\dots}^{m_d-1} f(x_s) + 2 \sum_{s=2,4,6,\dots}^{m_d-2} f(x_s) + f(x_f) \right] \quad (6)$$

Where,  $x_i=0$  and  $x_f=L$ , and  $m_d$  is the subdivisions of interval , usually even number.

And,

$$x_s = x_i + \left(\frac{x_f-x_i}{m_d}\right) s$$

Then,

$$A_0 =$$

$$\frac{1}{3L} \left(\frac{x_f-x_i}{m_d}\right) \left( \frac{E I_0}{1+C \exp(-2\alpha|x_i-x_c|/d)} + \frac{4 \sum_{s=1,3,5,\dots}^{m_d-1} \frac{E I_0}{1+C \exp(-2\alpha|x_s-x_c|/d)}}{2 \sum_{s=2,4,6,\dots}^{m_d-2} \frac{E I_0}{1+C \exp(-2\alpha|x_s-x_c|/d)}} + \frac{E I_0}{1+C \exp(-2\alpha|x_f-x_c|/d)} \right)$$

$$A_n =$$

$$\frac{2}{3L} \left(\frac{x_f-x_i}{m_d}\right) \left( \frac{E I_0}{1+C \exp(-2\alpha|x_i-x_c|/d)} \cos \frac{2n \pi x_i}{L} + \frac{4 \sum_{s=1,3,5,\dots}^{m_d-1} \frac{E I_0}{1+C \exp(-2\alpha|x_s-x_c|/d)} \cos \frac{2n \pi x_s}{L}}{2 \sum_{s=2,4,6,\dots}^{m_d-2} \frac{E I_0}{1+C \exp(-2\alpha|x_s-x_c|/d)} \cos \frac{2n \pi x_s}{L}} + \frac{E I_0}{1+C \exp(-2\alpha|x_f-x_c|/d)} \cos \frac{2n \pi x_f}{L} \right)$$

$$B_n =$$

$$\frac{2}{3L} \left(\frac{x_f-x_i}{m_d}\right) \left( \frac{E I_0}{1+C \exp(-2\alpha|x_i-x_c|/d)} \sin \frac{2n \pi x_i}{L} + \frac{4 \sum_{s=1,3,5,\dots}^{m_d-1} \frac{E I_0}{1+C \exp(-2\alpha|x_s-x_c|/d)} \sin \frac{2n \pi x_s}{L}}{2 \sum_{s=2,4,6,\dots}^{m_d-2} \frac{E I_0}{1+C \exp(-2\alpha|x_s-x_c|/d)} \sin \frac{2n \pi x_s}{L}} + \frac{E I_0}{1+C \exp(-2\alpha|x_f-x_c|/d)} \sin \frac{2n \pi x_f}{L} \right) \tag{7}$$

Then, substitution eqs. 3 and 4 in to eq. 1, get,

$$\frac{\partial^2}{\partial x^2} \left[ \left( A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x}{L} \right) \frac{\partial^2 w}{\partial x^2} (x, t) \right] + \rho A \frac{\partial^2 w}{\partial t^2} (x, t) = 0 \tag{8}$$

Then, by differential eq. 8, get,

$$\frac{\partial^4 w}{\partial x^4} (x, t) \left( A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x}{L} \right) - \frac{\partial^3 w}{\partial x^3} (x, t) \left( \frac{4n \pi}{L} \right) \left( \sum_{n=1}^{\infty} A_n \sin \frac{2n \pi x}{L} - \sum_{n=1}^{\infty} B_n \cos \frac{2n \pi x}{L} \right) - \frac{\partial^2 w}{\partial x^2} (x, t) \left( \frac{2n \pi}{L} \right)^2 \left( \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x}{L} \right) + \rho A \frac{\partial^2 w}{\partial t^2} (x, t) = 0 \tag{9}$$

Assuming the effect of crack small on the deflection of beam, then can be assuming the behaviors of beam with crack same the behaviors of beam without crack, as, [2], for simply supported beam,

$$w(x) = \bar{A}_n \sin(\beta_n x) \tag{10}$$

For,  $\beta_1 l = \pi, \beta_2 l = 2\pi, \beta_3 l = 3\pi, \beta_4 l = 4\pi, \dots$

And, the general behavior of beam as a faction of x and time, as,

$$w(x, t) = \bar{A}_n \sin(\beta_n x) \sin(\omega t) \tag{11}$$

Then, by substitution eq. (11) in to (9), get the general equation of motion for beam with crack effect as,

$$\begin{aligned} & (\beta_n)^4 \sin(\beta_n x) \left( A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x}{L} \right) + \\ & (\beta_n)^3 \cos(\beta_n x) \left( \frac{4n \pi}{L} \right) \left( \sum_{n=1}^{\infty} A_n \sin \frac{2n \pi x}{L} - \sum_{n=1}^{\infty} B_n \cos \frac{2n \pi x}{L} \right) + \\ & (\beta_n)^2 \sin(\beta_n x) \left( \frac{2n \pi}{L} \right)^2 \left( \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x}{L} \right) - \\ & \omega^2 \sin(\beta_n x) \rho A = 0 \end{aligned} \tag{12}$$

By pre multiplying eq. (12) by  $\sin(\beta_n x)$  and integral with x for,  $0 \leq x \leq l$ , get, the natural frequency of beam with crack effect,

$$\omega^2 = \frac{\int_0^l \left( (\beta_n)^4 (\sin(\beta_n x))^2 \left( A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x}{L} \right) + (\beta_n)^3 \sin(2 \beta_n x) \left( \frac{2n \pi}{L} \right) \left( \sum_{n=1}^{\infty} A_n \sin \frac{2n \pi x}{L} - \sum_{n=1}^{\infty} B_n \cos \frac{2n \pi x}{L} \right) + (\beta_n)^2 (\sin(\beta_n x))^2 \left( \frac{2n \pi}{L} \right)^2 \left( \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x}{L} \right) \right) dx}{\int_0^l (\sin(\beta_n x))^2 \rho A dx} \tag{13}$$

By integral equation (13) by x, using Simpson's rule integration method [19], get the natural frequency of beam with crack effect, as,

$$\int_{x_i}^{x_f} f(x) dx = \frac{1}{3} \left( \frac{x_f-x_i}{m_d} \right) \left[ f(x_i) + 4 \sum_{s=1,3,5,\dots}^{m_d-1} f(x_s) + 2 \sum_{s=2,4,6,\dots}^{m_d-2} f(x_s) + f(x_f) \right] \tag{14}$$

Where,  $x_i=0$  and  $x_f=L$ , and  $m_d$  is the subdivisions of interval, usually even number.

And,

$$x_s = x_i + \left(\frac{x_f-x_i}{m_d}\right) s$$

Then,

$$\omega^2 = \frac{\left[ \begin{aligned} & \left( (\beta_n)^4 (\sin(\beta_n x_i))^2 \left( A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x_i}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x_i}{L} \right) + \right. \\ & \left. (\beta_n)^3 \sin(2 \beta_n x_i) \left( \frac{2n \pi}{L} \right) \left( \sum_{n=1}^{\infty} A_n \sin \frac{2n \pi x_i}{L} - \sum_{n=1}^{\infty} B_n \cos \frac{2n \pi x_i}{L} \right) + \right. \\ & \left. (\beta_n)^2 (\sin(\beta_n x_i))^2 \left( \frac{2n \pi}{L} \right)^2 \left( \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x_i}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x_i}{L} \right) \right]}{4 \sum_{s=1,3,5,\dots}^{m_d-1} \left[ \begin{aligned} & \left( (\beta_n)^4 (\sin(\beta_n x_s))^2 \left( A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x_s}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x_s}{L} \right) + \right. \\ & \left. (\beta_n)^3 \sin(2 \beta_n x_s) \left( \frac{2n \pi}{L} \right) \left( \sum_{n=1}^{\infty} A_n \sin \frac{2n \pi x_s}{L} - \sum_{n=1}^{\infty} B_n \cos \frac{2n \pi x_s}{L} \right) + \right. \\ & \left. (\beta_n)^2 (\sin(\beta_n x_s))^2 \left( \frac{2n \pi}{L} \right)^2 \left( \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x_s}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x_s}{L} \right) \right]} + 2 \sum_{s=2,4,6,\dots}^{m_d-2} \left[ \begin{aligned} & \left( (\beta_n)^4 (\sin(\beta_n x_s))^2 \left( A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x_s}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x_s}{L} \right) + \right. \\ & \left. (\beta_n)^3 \sin(2 \beta_n x_s) \left( \frac{2n \pi}{L} \right) \left( \sum_{n=1}^{\infty} A_n \sin \frac{2n \pi x_s}{L} - \sum_{n=1}^{\infty} B_n \cos \frac{2n \pi x_s}{L} \right) + \right. \\ & \left. (\beta_n)^2 (\sin(\beta_n x_s))^2 \left( \frac{2n \pi}{L} \right)^2 \left( \sum_{n=1}^{\infty} A_n \cos \frac{2n \pi x_s}{L} + \sum_{n=1}^{\infty} B_n \sin \frac{2n \pi x_s}{L} \right) \right]} \right] \tag{15}$$

By using of the building a computer program for analytical

solution using Fortran power station 4.0 program, can be results of above equations to evaluated natural frequency of beam.

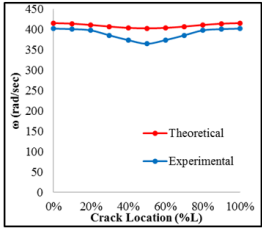


Fig. 4. Compare Natural Frequency Between Theoretical and experimental Study of simply supported Low Carbon Steel Beam with different crack location, from  $d_c = 0.25. d.$

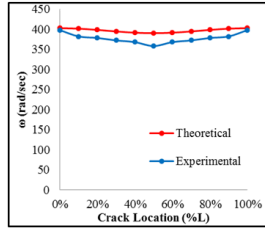


Fig. 7. Compare Natural Frequency Between Theoretical and experimental Study of simply supported alloys aluminum beam with different crack location, from  $d_c = 0.25. d.$

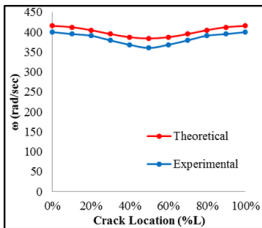


Fig. 5. Compare Natural Frequency Between Theoretical and experimental Study of simply supported Low Carbon Steel Beam with different crack location, from  $d_c = 0.5. d.$

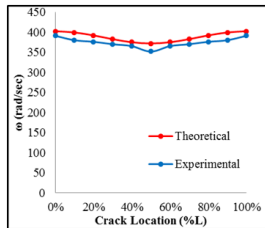


Fig. 8. Compare Natural Frequency Between Theoretical and experimental Study of simply supported alloys aluminum beam with different crack location, from  $d_c = 0.5. d.$

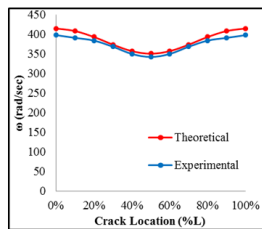


Fig. 6. Compare Natural Frequency Between Theoretical and experimental Study of simply supported Low Carbon Steel Beam with different crack location, from  $d_c = 0.75. d.$

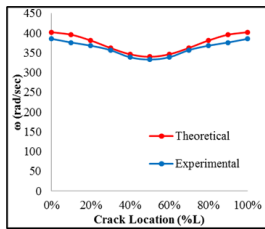


Fig. 9. Compare Natural Frequency Between Theoretical and experimental Study of simply supported alloys aluminum beam with different crack location, from  $d_c = 0.75. d.$

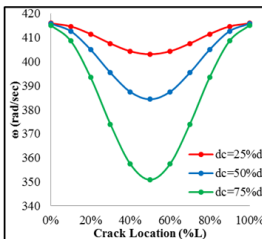


Fig. 10. Natural Frequency of simply supported Low Carbon Steel Beam with different crack depth and location.

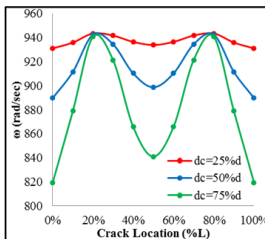


Fig. 13. Natural Frequency of clamped supported Low Carbon Steel Beam with different crack depth and location.

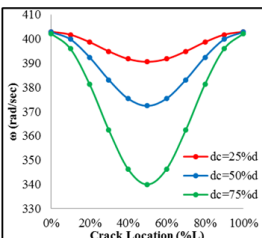


Fig. 11. Natural Frequency of simply supported alloys aluminum Beam with different crack depth and location.

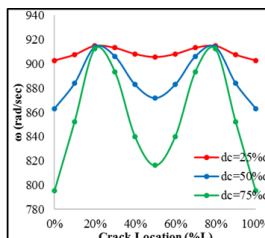


Fig. 14. Natural Frequency of clamped supported alloys aluminum Beam with different crack depth and location.

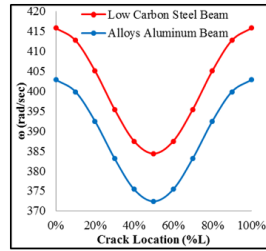


Fig. 12. Natural Frequency of simply supported different Beam materials with different crack location, for  $d_c = 0.5. d.$

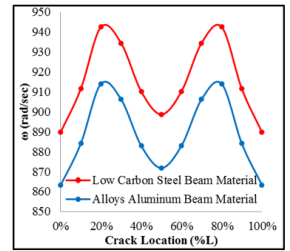


Fig. 15. Natural Frequency of clamped supported different Beam materials with different crack location, for  $d_c = 0.5. d.$

RESULTS AND DISCUSSION

The vibration results of beam with crack effect includes the evaluation of the natural frequency of different beam dimensions and materials, with crack effect of beam, included the effect of crack size, crack location, and boundary condition of beam.

Where The mechanical properties of beam studied, [20], are,

- Low Carbon Steel beam,  $E=207 \text{ Gpa}, G=80 \text{ Gpa}, \rho=7800 \text{ kg/m}^3, \nu=0.3$
- Alloys Aluminum beam,  $E=69 \text{ Gpa}, G=26 \text{ Gpa}, \rho=2770 \text{ kg/m}^3, \nu=0.33$

And dimensions beam,

- Width of beam,  $w=0.020 \text{ m}$
- Depth of beam,  $d=0.020 \text{ m}$
- Length of beam,  $l=0.84 \text{ m}$

The method studied to evaluated the natural frequency of beam with crack effect are, experimental study and theoretical study, by solving of general equation of motion of beam with crack effect.

The experimental work includes the study of the crack effect of beam with different crack size (depth of crack), different crack position, and clamped and simply supported boundary conditions of beam. And the theoretical work includes the study of the crack effect of beam with different crack size (depth of crack), different crack position, and simply supported beam.

The theoretical results are compared with those obtained experimental for each parameters effect studied as shown in Figs. 4 to 9. A comparison made between analytical results from solution of general equation of motion of beam, with crack effect, with experimental results, shows a good approximation where the biggest error percentage is about (9.4 %).

Figs. 10 to 12. Shown the natural frequency of simply supported beam with different crack location and depth effect for various materials beam. Form figure can be shown that the natural frequency decreases with increasing of the crack depth and the crack effect on the natural frequency more effect at crack near the middle of beam location from near the side of beam. And figs. 13 to 15. Shown the natural frequency of clamped beam with different beam types and depth and crack location effect of beam.

The effect of crack size as depth of crack for beam with different boundary conditions of different beam materials, with different location of crack are shown in figs. 10 to 15. From Figs. 10 to 15 shows the effect of the crack on the natural frequency of beam, can see that the natural frequency decreasing with increasing crack depth for different crack position, this is because the changing in stiffness beam.

CONCLUSIONS

The following concluding marks have been observed:

1. A comparison made between experimental results from vibration test of beam with crack effect with analytical results from solution of general equation of motion of beam with crack effect shows a good approximation.

2. The crack causes, as expected, a decrease in the natural frequencies of flexural vibrations of the beam.
3. The crack in the beam has an effect on the stiffness of the beam, this will affect the frequency of the beam. So, with increasing of the crack depth the stiffness of beam will decreased, this will cause a decreasing the natural frequency of the beam.
4. The position of crack in the beam near the middle of the beam has more effect on the stiffness and natural frequency of beam from the other positions (near to the ends of the beam), i.e. frequency of beam when the crack in the middle position it has a lower frequency of beam with respect to the cracks near to the end position.

## REFERENCE

- [1] M. I. Friswell and J. E. T. Penny 'Crack Modeling for Structural Health Monitoring' *Structural Health Monitoring*, Vol. 1, No. 2, pp. 139-148, 2002. | [2] Singiresu S. Rao 'Mechanical Vibration' Addison-Wesley Publishing Book Company, 2000. | [3] K. El Bikri, R. Benamar, and M.M. Bennouna 'Geometrically non-linear free vibrations of clamped-clamped beams with an edge crack' *Computers and Structures*, Vol. 84, pp. 485-502, 2006. | [4] H. R. Oz and M. T. Das 'In-Plane Vibrations of Circular Curved Beams With a Transverse Open Crack' *Mathematical and Computational Applications*, Vol. 11, No. 1, 2006. | [5] Liao-Liang Ke, Jie Yang, Sritawat Kitipornchai 'Postbuckling analysis of edge cracked functionally graded Timoshenko beams under end shortening' *Composite Structures*, Vol. 90, 2009. | [6] M. Al\_Waily " Investigation of Health Monitoring of Composite Plate Structures-Using Vibration Analysis" Ph.D. Thesis Submitted to the Mechanical Engineering Department, College of Engineering, Alnahrain University, 2012. | [7] D. Sreekanth Kumar, D. Roy Mahapatra, and S. Gopalakrishnan 'A spectral finite element for wave propagation and structural diagnostic analysis of composite beam with transverse crack' *Finite Elements in Analysis and Design*, Vol. 40, 2004. | [8] Ertugrul Cam, Sadettin Orhan, and Murat Luy 'An analysis of cracked beam structure using impact echo method' *NDT&E International*, Vol. 38, 2005. | [9] Luay S. Al-Ansari, Muhannad Al-Waily, and Ali M. H.Yusif Al-Hajjar 'Experimental and Numerical Study of Crack Effect on Frequency of Simple Supported Beam' *Al-Khwarizmi Engineering Journal*, Vol. 8, No. 2, 2012. | [10] Hiroyuki Okamura, H. W. Liu and Chong-Shin Chu 'A Cracked Column Under Compression' *Engineering Fracture Mechanics*, Vol. 1, 1969. | [11] M.G.D. Geers, R. de Borst, and R.H.J. Peerlings 'Damage and crack modeling in single-edge and double-edge notched concrete beams' *Engineering Fracture Mechanics*, Vol. 65, 2000. | [12] Mo-How H. Shen 'Natural Modes of Bernoulli-Euler Beams With A Single-Edge Crack' *American Institute of Aeronautics and Astronautics, AIAA*, 1990. | [13] Murat Kisa 'Free vibration analysis of a cantilever composite beam with multiple cracks' *Composites Science and Technology*, Vol. 64, 2004. | [14] Saber El Arem and Habibou Maitournam 'A cracked beam finite element for rotating shaft dynamics and stability analysis' *Journal of Mechanics of Materials and Structures*, Vol. 3, No. 5, 2008. | [15] T.G. Chondros, A.D. Dimarogonas, and J. Yao 'Longitudinal vibration of a bar with a breathing crack' *Engineering Fracture Mechanics*, Vol. 61, 1998. | [16] T.G. Chondros, A.D. Dimarogonas, and J. Yao 'Vibration of A Beam with a Breathing Crack' *Journal of Sound and Vibration*, Vol. 239, No. 1, 2001. | [17] T.T. Lu, H.Y. Hu, and Z.C. Li 'Highly accurate solutions of Motz's and the cracked beam problems' *Engineering Analysis with Boundary Elements*, Vol. 28, 2004. | [18] A. Jeffrey 'Advanced Engineering Mathematics' Harcourt / Academic Press, 2002. | [19] H. Anton, Irl Bivens, and Stephen Davis 'Calculus' Anton Textbooks, Inc, Seventh Edition, (2002). | [20] E. J. Hearn 'Mechanics of Materials-1 An Introduction to the Mechanics of Elastic and Plastic Deformation of Solids and Structural Materials, Third Edition' E. J. Hearn, 1997. |