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Experimental and Numerical Comparison Between the Natural Frequency of Cracked Simply Supported Hollow and Solid Beams

KEYWORDS: Natural Frequency, Solid Beam, Hollow Beam, Cracked Beam, Experimental Beam Vibration, Finite Element Method, ANSYS, Rayleigh Method.

Luay S. Al-Ansari

University of Kufa / Faculty of Mechanical Engineering

ABSTRACT *This research presents a comparison of the natural frequency between solid and hollow simple supported cracked beam for different crack depths and positions. Three methods utilized in this research experimental and two numerical method (Rayleigh Method and Finite Element Method (using ANSYS)). The beam is made of iron with dimensions of L*W*H=(0.84*0.02* 0.02m), and density = 7680kg/m3, E=200Gpa. A comparison made between ANSYS results with experimental results, the biggest error percentage is about (17%) in crack position (42 cm) and depth (10 mm) for the hollow beam. While the biggest error percentage between Rayleigh method with experimental results is about (29 %) for the same crack position and depth and for the hollow beam. The effect of the depth of the crack on the natural frequency of hollow beam is nearly the same effect as in the solid beam as a ratio.*

INTRODUCTION

Cracks present a serious threat to the performance of structures since most of the structural failures are due to material fatigue. For this reason, many methods allowing early detection and localization of cracks have been the subject of intensive investigation the last two decades. As a result, a variety of analytical, numerical and experimental investigations now exist. Cracks found in structural elements have various causes. They may be fatigue cracks that take place under service conditions as a result of the limited fatigue strength. They may be also due to mechanical defects, as in the turbine blades of jet engines. In these engines the cracks are caused by sand and small stones sucked from the surface of runway. Another group involves cracks which are inside the material. They are created as a result of manufacturing processes. The presence of vibrations on structures and machine components leads to cyclic stresses resulting in material fatigue and failure. Most of the failures of the equipment are due to material fatigue. It is very essential to detect the cracks in structures and machine members from very early stage.

For crack identification change in natural frequency and modal value has been studied. In some cases crack properties are used to obtain the dynamic behavior as in [1-4] and sometimes inverse methods are used [5-8].

The first two natural frequencies were used by Narkis [9] to identify the crack and later Morassi [10] used it on simply supported beam and rods. Although it can be solved by using 2D or 3D finite element method (FEM), analysis of this approximate model results in algebraic equations which relate the natural frequencies of beam and crack characteristics. These expressions are then applied to study of the inverse problem identification of crack location from frequency measurements. It is found that the only information required for accurate crack identification is the variation of the first two natural frequencies due to the crack, with no other information needed concerning the beam geometry or material and the crack depth or shape. The proposed method is confirmed by comparing it with results of numerical finite element calculations the researchers still try to detect it with the help of physical parameters of the crack i.e. crack depth, position and support condition to the beam.

Freud and Herrmann [17] modeled the problem using a torsional spring in the place of crack whose stiffness is related. The first model is used to Euler-Bernoulli cracked beam with different end conditions [4, 11, 19-23, 18] and recently on Timoshenko beams [24, 25].

Chondros et. al [26] used a continuous cracked beam vibration theory for predication of changes in transverse vibration of simply supported beam with a breathing crack. They found that the changes in vibration frequencies for fatigue breathing crack are smaller than the ones caused by open cracks. Utilizing aluminum beams with fatigue cracks for experimental setup they compared the results with the analytical.

Chondros [27] used a continuous cracked beam vibration theory for predication of changes in dynamic characteristic due to loading conditions and vibration amplitude. He used the numerical results to correlate the analytical results for lumped crack beam vibration analysis for aluminum and steel beams with open cracks. He supported the theoretical result by experimental results for the same cases.

Cam et. al. [28] studied ,experimentally and theoretically, the effect of the crack on vibration of cracked beam. They used echo method for predication the size and location of the crack in cracked beam. They found that the theoretical results (ANSYS) were agreed with experimental results.

In this paper three approaches are employed, an analytical approach compared with experimental result and with that gained numerically by ANSYS program to verify the results. The objective of this paper is the study of the effect crack depth and position on the natural frequency of the simple supported hollow and solid beam and comparing between them and finding the best method that give good results comparing with experimental results.

EXPERIMENTAL WORK

The (TM16 universal vibration apparatus) from TQ company is employed at this study shown in Fig.(1). The dimensions of the solid beam specimen used are (L*W*H=0.84*0.02 *0.02 m) shown in Fig.(2). While the dimensions of the cross section area of hollow beam specimen used are shown in Fig.(3). The material of the specimen was stainless steel (Code No.: 314, Robert L. Norton [16]) with density of (7680 kg/m3), Young modulus (200 GPa) and Poisson's Ratio (0.3). The crack was created in the specimen with the certain dimensions of crack (see Table (1)).

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Table (1): Dimensions of The Cracks that Used Experimentally.

Specimen No.	Crack Location (m)	Crack Length (m)	Crack Width (m)	Crack Depth (mm)		
1	0.12	0.02	0.002	0	5	10
2	0.22	0.02	0.002	0	5	10
3	0.32	0.02	0.002	0	5	10
4	0.42	0.02	0.002	0	5	10
5	0.54	0.02	0.002	0	5	10
6	0.64	0.02	0.002	0	5	10
7	0.74	0.02	0.002	0	5	10

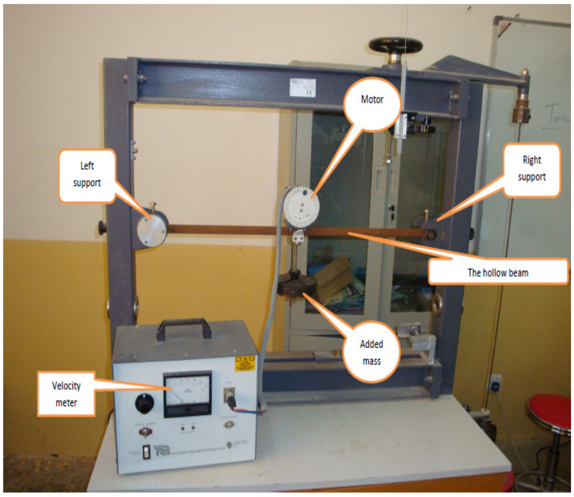


Fig.(1): The Universal Vibration Apparatus.

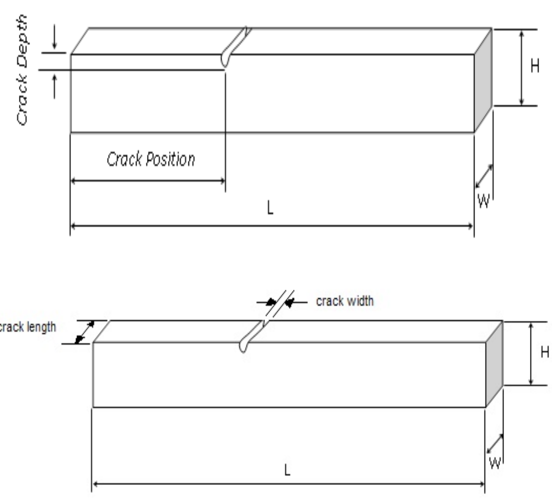
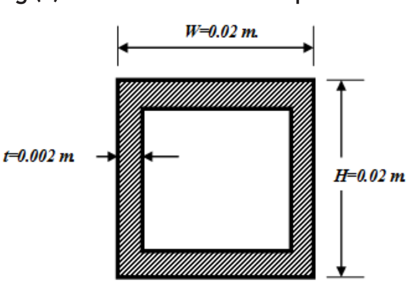
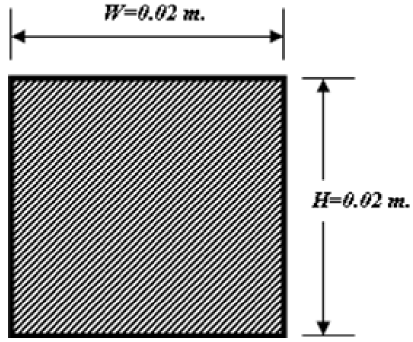


Fig.(2):Dimensions of the Sample.



(a) Cross Section Area of Hollow Beam.



(b) Cross Section Area of Solid Beam

Fig.(3):Dimensions of the Cross Section Area of the Samples.

THEORETICAL APPROACH

1- Rayleigh Method

Rayleigh method is a good method and simpler than the other numerical methods for finding the natural frequencies of uniform beam. It includes calculating the kinetic energy and potential energy of the system. The kinetic energy can be calculated by integrating the mass through length of the beam and the potential energy by integrating the stiffness through the length of the beam. So one can get [29, 30]:

$$\omega^2 = \frac{\int_0^l E \left(\frac{d^2y(x)}{dx^2}\right)^2 dx}{\int_0^l \rho A (y(x))^2 dx} = \frac{g\sum_{i=1}^{n+1} m_i y_i}{\sum_{i=1}^{n+1} m_i y_i^2} \quad (1)$$

Where: (ω) is frequency, (E) is Modulus of Elasticity,(I) is Moment of Inertia, (ρ)is Density, (A) is Cross Section Area, (m) mass, and (y) is Deflection.

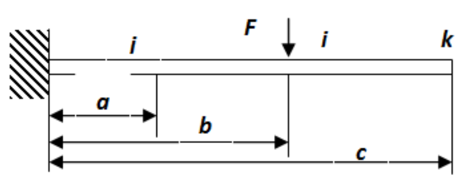
By calculating the deflection of the beam(y(x)) using the following steps [29, 30]:

(1) Dividing the beam into (n) parts (i.e. (n+1) nodes).
(2) Calculate the delta matrix [δ]((n+1)* (n+1)) using Table (2).
(3) Calculate the mass matrix [m]((n+1)).
(4) Calculate the deflection at each node by multiplying delta matrix and mass matrix ([y] (n+1)= [δ]((n+1)* (n+1)) [m] ((n+1)) after applying the boundary conditions.

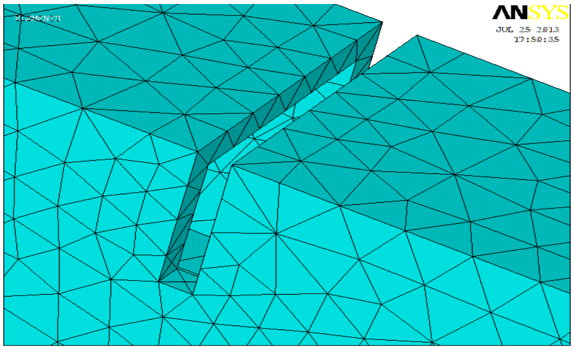
The analytical results are solved using MATLAB. Where a MATLAB program simulated the Rayleigh method were written in order to calculate the first natural frequency of any beam (Different materials, different dimensions and different shape).

Table (2): Formulae of the Deflections of the Cantilever Beams[29 and30].

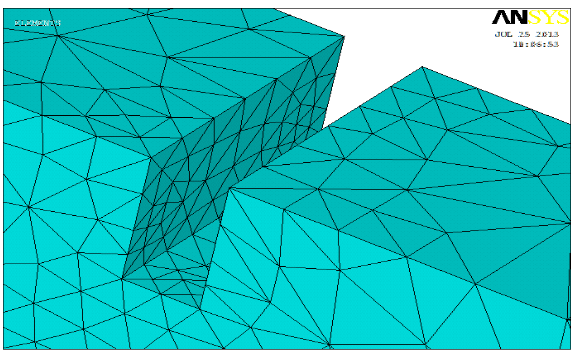
$$\delta_{ji} = \frac{Fa^2(3b-a)}{6EI}, \quad \delta_{ii} = \frac{Fb^3}{3EI}, \quad \delta_{ki} = \frac{Fb^2(3c-b)}{6EI}$$



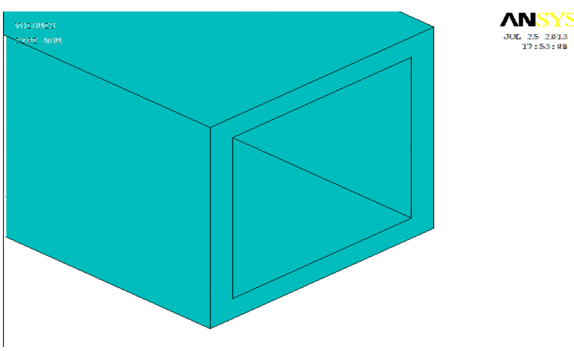
2-Numerical Approach (Finite Elements Method)
In this method, the finite elements method was applied by using the ANSYS program(ver.14). The three dimensional model were built and the element (Solid Tet 10 node 187) were used. Generally the number of nodes was approximately (51000-54000) and the number of elements was (25000-27000). A sample of meshed beam is shown in Fig. (4).



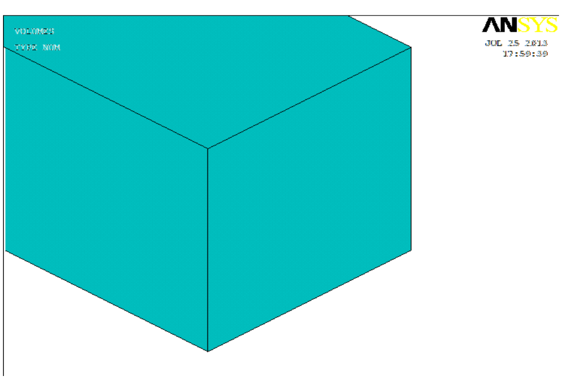
(a) Mesh of the Hollow Beam.



(b) Mesh of the Solid Beam.



(c) Cross Section Area of the Hollow Beam.



(d) Cross Section Area of the Solid Beam.
Fig.(4):The Cross Section Area and Mesh of the Samples.

RESULTS AND DISCUSSION
Figure (5) and (6) show the comparison of the experimental results of the natural frequency for solid and hollow beam when the crack position change and the crack depth is (5 mm) and (10 mm) respectively. In both figures, the natural frequency of the hollow beam is larger than that of the solid beam. In Fig. (6) , the value of natural frequency, when the crack position is (42 cm), is smaller than the other values. This happened because the motor of the device technically must be put in the midpoint of the beam and this make the experimental reading is not correct for this point. This appears sharply when the crack depth is (10 mm) for hollow beam more than that of the other types of beams and crack depths.

Figures (7-10) show the comparison between the ANSYS and Rayleigh results of the natural frequency of the solid and hollow beam when the crack position change and the crack depth is (5 mm) and (10 mm). The same ratio between the natural frequency values of solid and hollow beam can be found.

Figures (11-13) show the comparison between the experimental, ANSYS and Rayleigh results of natural frequency for solid beam (when the crack depth are (5 mm) and (10 mm)) and hollow beam (when the crack depth are (5 mm) and (10 mm)). Figures (14-17) show the comparison between the natural frequency of the three methods (experimental, ANSYS and Rayleigh) for solid beam with crack depth (5 mm), solid beam with crack depth (10 mm), hollow beam with crack depth (5 mm) and hollow beam with crack depth (10 mm) respectively.

The following points can be seen:
- The values of natural frequency of the solid and hollow beam, when the crack depth are (5 mm) and (10 mm), are close to each other.
- For the three methods, the same ratio can be shown, approximately, between the value of natural frequency of solid and hollow beam for the same crack depth.
- There is a good agreement between ANSYS results and experimental results for each types of beam (see Table (3)) where the biggest error percentage is about (17 %) for hollow beam and about (10 %).
- The Rayleigh method is not sensitive to the crack depth effect for the hollow beam where the biggest error percentage is about (29 %) for hollow beam and about (9 %).
- The natural frequency decreases with increasing crack depth for different crack position, this is because the changing in stiffness beam and for the same reason the natural frequency decreases when the crack go away from the support.
- The natural frequency of the hollow beam is greater than that of solid beam, this is because of the difference in stiffness and mass of beam.

CONCLUSIONS
From the results, the following concluding marks have been observed:
1. A comparison made between ANSYS results with experimental results shows a good approximation where the biggest error percentage is about (17 %) in crack position (42 cm) and depth (10 mm) for hollow beam.
2. The comparison between Rayleigh method with experimental results shows a good approximation where the biggest error percentage is about (29 %) in crack position (42 cm) and depth (10 mm) for hollow beam.
3. From the error percentages in Table 3. the ANSYS method give close results to experimental than Rayleigh method.
4. The crack in the beam has an effect on the stiffness of the beam, this will affect the frequency of the beam. So, with increasing the crack depth the stiffness of beam will

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decreases, this causes a decreasing the natural frequency of the beam.

5. The position of crack in the beam near the midpoint of the beam has more effect on the stiffness and natural frequency of beam in comparison with the other positions (near the ends of the beam), i.e. frequency of beam when the crack in the middle position it has a lower frequency with respect to the cracks near to the end position.

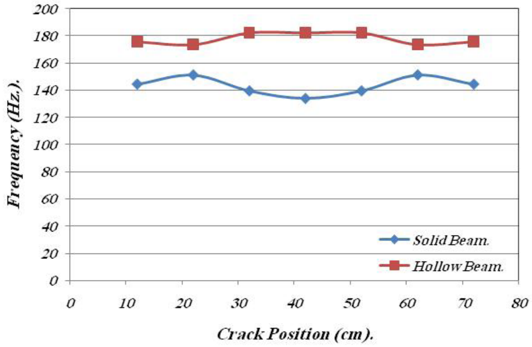

Fig.(5): The Comparison Between the Experimental Results of Natural Frequency of Solid and Hollow Beam Varying with Crack Position When the Crack Depth is (5 mm).

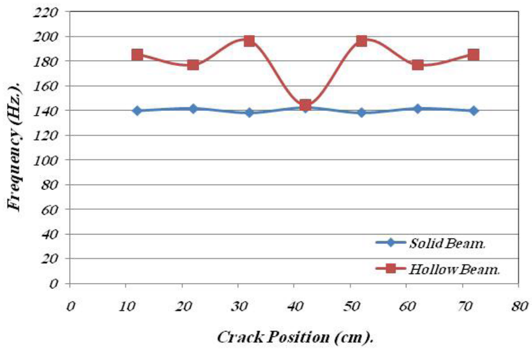

Fig.(6): The Comparison Between the Experimental Results of Natural Frequency of Solid and Hollow Beam Varying with Crack Position When the Crack Depth is (10 mm)

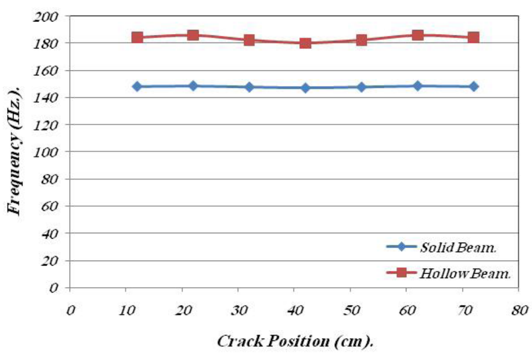

Fig.(7): The Comparison Between the ANSYS Results of Natural Frequency of Solid and Hollow Beam Varying with Crack Position When the Crack Depth is (5 mm).

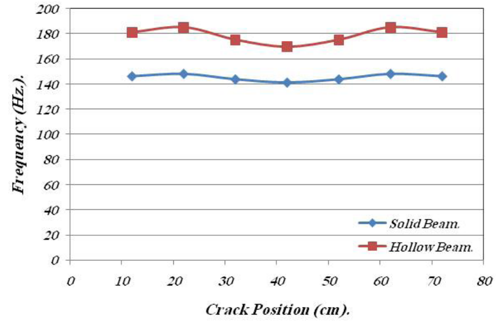

Fig.(8): The Comparison Between the ANSYS Results of Natural Frequency of Solid and Hollow Beam Varying with Crack Position When the Crack Depth is (10 mm).

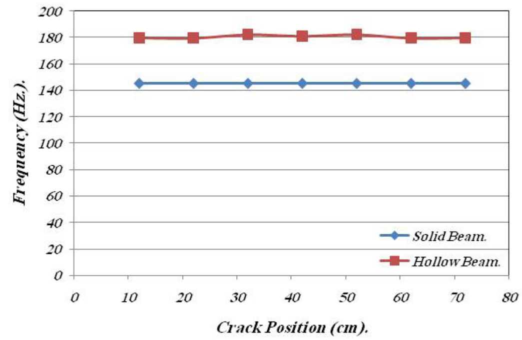

Fig.(9): The Comparison Between the Rayleigh Results of Natural Frequency of Solid and Hollow Beam Varying with Crack Position When the Crack Depth is (5 mm).

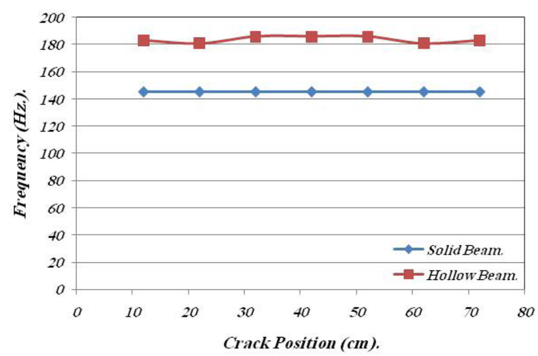

Fig.(10): The Comparison Between the Rayleigh Results of Natural Frequency of Solid and Hollow Beam Varying with Crack Position When the Crack Depth is (10 mm).

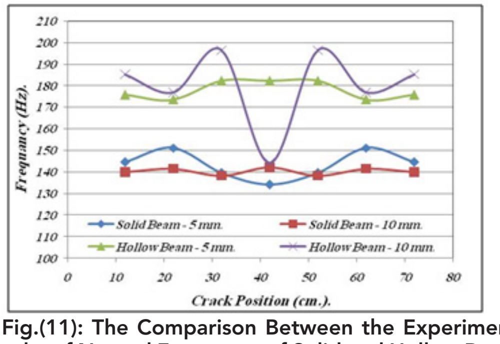

Fig.(11): The Comparison Between the Experimental Results of Natural Frequency of Solid and Hollow Beam Varying with Crack Position for Different Crack Depths

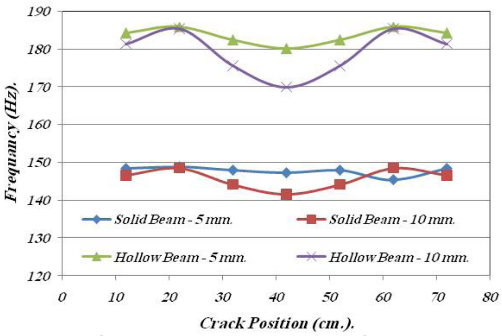

Fig.(12): The Comparison Between the ANSYS Results of Natural Frequency of Solid and Hollow Beam Varying with Crack Position for Different Crack Depths .

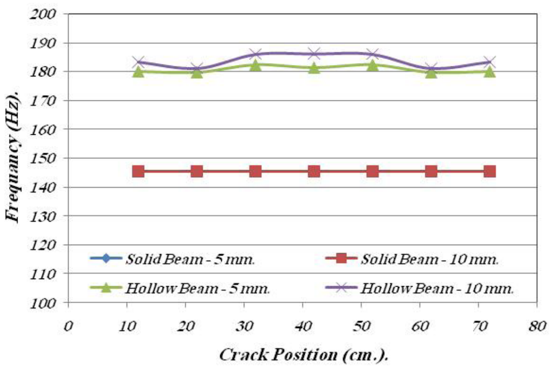

Fig.(13): The Comparison Between the Rayleigh Results of Natural Frequency of Solid and Hollow Beam Varying with Crack Position for Different Crack Depths .

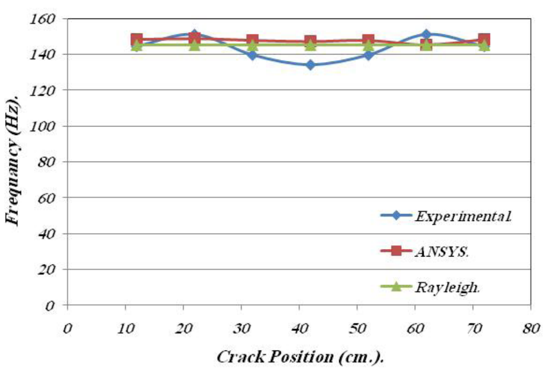

Fig.(14): The Comparison Between the Experimental, ANSYS and Rayleigh Results of Natural Frequency of Solid Beam Varying with Crack Position When the Crack Depth is (5 mm).

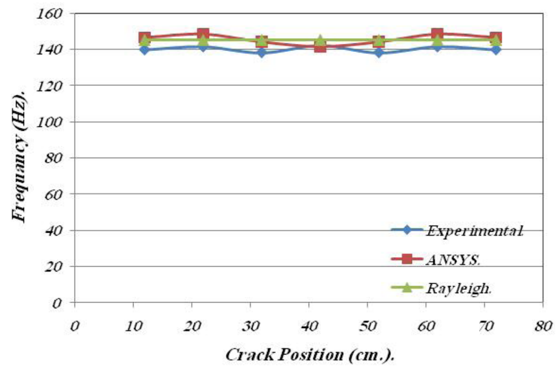

Fig.(15): The Comparison Between the Experimental, ANSYS and Rayleigh Results of Natural Frequency of Solid Beam Varying with Crack Position When the Crack Depth is (10 mm).

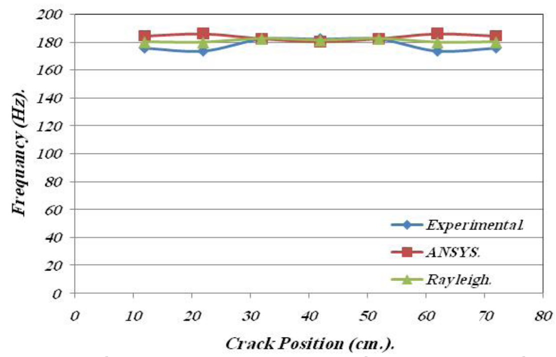

Fig.(16): The Comparison Between the Experimental, ANSYS and Rayleigh Results of Natural Frequency of Hollow Beam Varying with Crack Position When the Crack Depth is (5 mm).

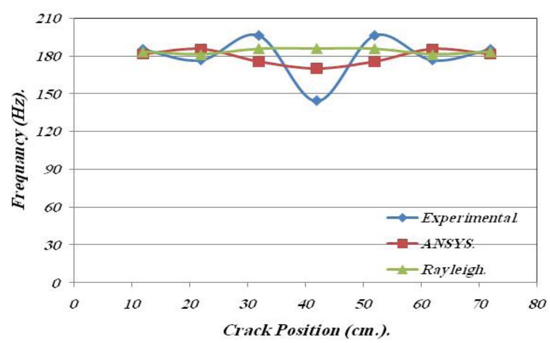

Fig.(17): The Comparison Between the Experimental, ANSYS and Rayleigh Results of Natural Frequency of Hollow Beam Varying with Crack Position When the Crack Depth is (10 mm).

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Table (3): Natural Frequency and Error Percentage for Different Crack Position and Crack Depth.

Crack Position (cm)	Beam Type.	Crack Depth (mm)	Frequency (Hz)			Error Exp. And ANSYS (%)	Error Exp. And Rayleigh (%)
			Experimental	ANSYS	Rayleigh method		
12	Solid (5 mm).	5	144.4	148.235	145.2824	2.658	0.613
	Solid (10 mm)	10	139.75	146.5	145.2888	4.8	3.96
	Hollow (5 mm)	5	175.62	184.2	179.9112	4.88	2.442
	Hollow (10 mm)	10	185.19	181.28	183.2373	-2.114	-1.05
22	Solid (5 mm)	5	151.125	148.73	145.2903	-1.585	-3.861
	Solid (10 mm)	10	141.42	148.43	145.3046725	4.95	2.7
	Hollow (5 mm)	5	173.40	185.88	179.6416	7.195	3.598
	Hollow (10 mm)	10	176.73	185.27	180.9998	4.832	2.416
32	Solid (5 mm)	5	139.475	147.76	145.2903	5.94	4.169
	Solid (10 mm)	10	138.144	144.03	145.3046725	4.26	5.18
	Hollow (5 mm)	5	182.21	182.24	182.2269	0.014	0.0072
	Hollow (10 mm)	10	196.332	175.41	185.8714	-10.66	-5.33
42	Solid (5 mm)	5	133.95	147.105	145.3078	9.825	8.483
	Solid (10 mm)	10	141.99	141.44	145.3397282	0.39	2.4
	Hollow (5 mm)	5	182.14	180.03	181.0836	-1.157	-0.5785
	Hollow (10 mm)	10	144.17	169.89	186	17.84	29.02

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