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Calculation of the Wall Shear Stress in the Case of a Stenosed Internal Carotid Artery

KEYWORDS: blood flow; non-Newtonian model; stenosed artery; wall shear stress; rupture of vascular vessels

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ABSTRACT *In this paper we use a non-Newtonian mathematical model for the blood flow in large vessels – elaborated and presented in a previous paper (Albert, Vacaras, Trif & Petrila, 2013), for making some numerical simulations in the case of a human internal carotid artery with a stenosis. We concentrate on the calculation of the wall shear stress, which is believed to have a special importance in the possible ruptures of vascular vessels. The numerical simulations are made using COMSOL Multiphysics 3.3, and the results are compared to some already existing in the literature.*

1. Introduction

In this research for blood we accept a non-Newtonian rheological behavior with a variable coefficient of viscosity under the conditions of an unsteady (pulsatile) flow regime, connected with the rhythmic pumping of the blood by the heart. At the same time we admit the incompressibility and homogeneity of the blood while the exterior body forces are neglected. We also take into consideration the viscoelastic behavior of the limiting walls of the vessels, the whole configuration accepting an axial symmetric geometry versus the vertical axis Oz.

We use a non-Newtonian mathematical model, elaborated in a previous research and presented by Albert et al. (2013). According to this model the motion equations result from the general Cauchy equations

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=div\mathbf{T}$$

,where the stress tensor $\mathbf{T}=-p\mathbf{I}+2(\mu_s+\mu_{RBC})\mathbf{D}$, where I being the unit tensor, the scalar p is the physical pressure, D is the rate of strain tensor, μ_s representing the (constant) plasma viscosity while μ_{RBC} is given by the Cross model, i.e.

$$\mu_{RBC}=\frac{\mu_0^*}{1+(k\dot{\gamma})^{1-n}}$$

with $\dot{\gamma}$ being the shear rate, μ_0^* the viscosity coefficient of the blood, k is a time constant and n is the index for a shear thinning behavior.

The evolution equations are joined to some boundary conditions which express the existence of a pressure gradient along Oz axis according to the heart beats and implicitly to the rhythmic blood pushing into the vessel (feature which is important in large vessel). Precisely we have

$\frac{\partial v}{\partial r}=0$ and $u=0$ for $r=0$.

At r = R, due to the viscoelastic behavior of the vessel's wall, the velocity of the blood must be equal to the displacement velocity of the wall. The boundary conditions at "edges" z = 0 and z = L of the vessel agree with a physiological pulse velocity given by a periodic time-varying function.

To describe the viscoelastic behavior of the vessel's wall we have used the generalized Maxwell model, which is the most general form of the linear model for viscoelasticity (Götz, 2012).

2. Numerical experiments and results

We are now analyzing a real case of a human internal carotid artery (ICA), which has a stenosis of 69%. In figure 1 it can be seen the velocity field of the stenosed ICA.

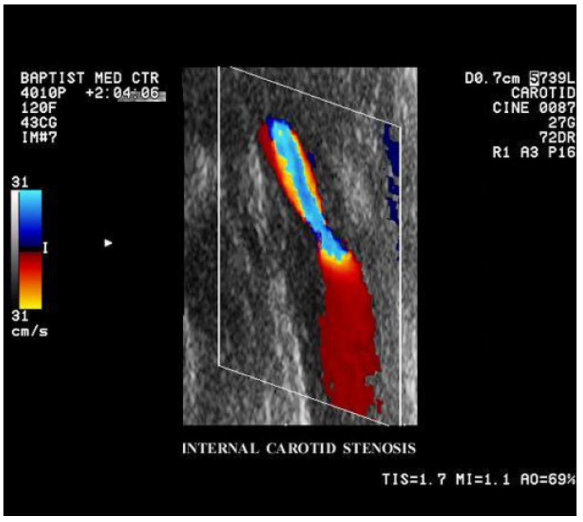

Figure 1. Velocity field of the blood in a stenosed ICA. (Medical measurement (Albert et al., 2013))

The length of the stenosed arterial segment is 3cm, the internal diameter of the blood vessel is 7mm and the thickness of the vessel wall is 0.8mm. The mass density of the blood has been fixed at ρ =1060kg /m^3.

For the numerical calculations we are using the Navier-Stokes equations (Petrila & Trif, 2005, Albert et al., 2013),

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=\nabla\ p+\mu\nabla^2\mathbf{u}$$

where the dynamic viscosity, according to the Cross model, is

$$\mu(\dot{\gamma})=\mu_s+\frac{\mu_0^*}{1+(k\dot{\gamma})^{1-n}}$$

The boundary conditions are those mentioned in the previous section. To avoid the transient effect of the initial conditions the results are presented only for the last 5 periods, although the time integration interval is t∈[0,10s]. The numerical simulations are made using the programming package COMSOL 3.3. For the constants k, n, and the vis-

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cosities μ_s, μ_0^* we have taken respectively the values k = 1.036, n = 0.2, μ_s = 0.00345 $Pa \cdot s$, μ_0^* = 0.0465 $Pa \cdot s$ In the case of this real ICA we present some results concerning the wall shear stress ($WSS = \mu\left(\frac{\partial u}{\partial z}+\frac{\partial v}{\partial r}\right)$), which is believed to have a special importance in the possible ruptures of vascular vessels. We have chosen 4 particular points on the vessel wall of the stenosed ICA (see figure 2) in which the values of the WSS are evaluated.

On figure 3 the variation of the wall shear stress (through 5 seconds) – evaluated in those four particular points mentioned in figure 2, is presented. It can be clearly seen, that the WSS reaches very high values in the middle of the stenosis (the red point on figure 2).

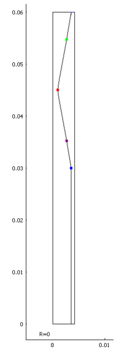

Figure 2. The 4 particular points on wall of the stenosed ICA (coordinates on axes are expressed in m)

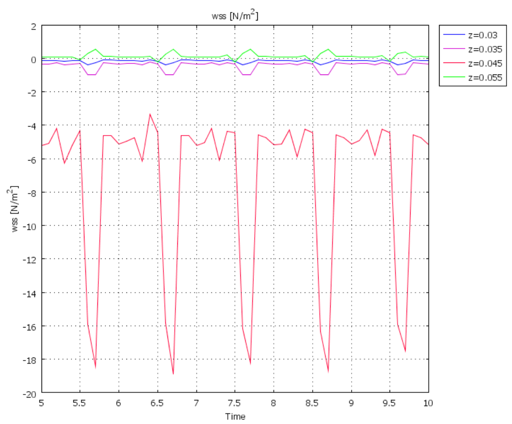

Figure 3. Value of the WSS (through 5 seconds) in the 4 particular points

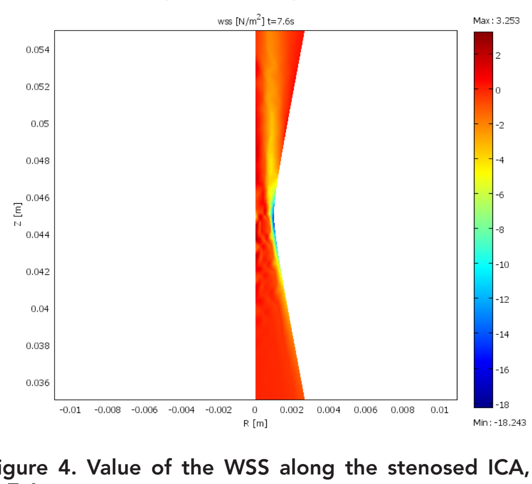

Figure 4. Value of the WSS along the stenosed ICA, at t=7.6s

In figures 3 and 4 one can see that the absolute maximum value of the WSS is around 18 N/m^2. The "-" sign shows that the wall shear stress acts in opposite direction to that of the blood flow.

Figure 5 presents the normal values of the WSS in some different vascular vessels (according to Papaioannou & Stefanos (2005)). In this figure it can be clearly seen, that the normal value of the WSS in the case of the arteries is around 1 N/m^2. The comparison of these values with those obtained numerically shows the fact that the value of the WSS (obtained numerically) has a significant increasing indeed in the stenosed section, against the normal value.

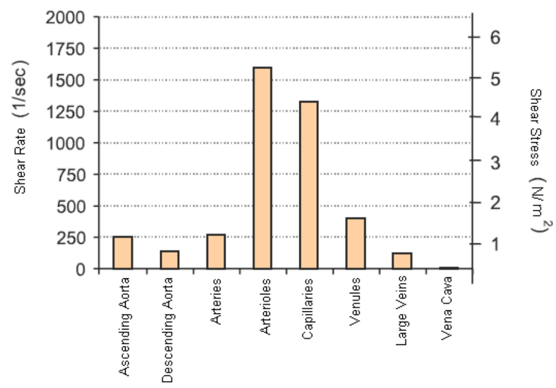

Figure 5. Shear stress values in different blood vessels Papaioannou & Stefanos (2005)

Similarly, in Tu, Yeuh & Liu (2013), page 12, one appears, that in the case of arteries with stenosis the values of the wall shear stress may increase from 10 N/m^2 to 50 N/m^2. A similar result can be found also in Minea (2003), page 169, where – in the case of an ICA with a stenosis of 30 % – the value of the WSS is about 20 N/m^2.

3. Conclusions

Using a non-Newtonian mathematical model, developed by us in a previous paper, we may calculate the values of the wall shear stress in some particular points of a human internal carotid artery with stenosis. These values obtained numerically are in total accord with those already presented in the international literature. This fact validates the accuracy of the used Cross type non-Newtonian mathematical and numerical model for the blood flow together with the viscoelastic behavior of the vessel walls. At the same time our algorithm of calculation of WSS provides a useful tool for assessment of

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