RESEARCH PAPER | Statistics | Volume : 3 | Issue : 9 | Sept 2013 | ISSN - 2249-555X

About One Approach to Defuzzifikation of Outputs of Fuzzy Time Series Model

KEYWORDS: semi-structured time series, fuzzy set, fuzzy relationship, membership function

Ramin Rzayev	Musa Agamaliyev	Nijat Askerov
Institute of Cybernetics of National Academy of Science, B. Vahabzade str., 9, Baku, Azerbaijan	Odlar Yurdu Univercity, K. Rahimov str., 13, Baku, Azerbaijan	Institute of Cybernetics of National Academy of Science, B. Vahabzade str., 9, Baku, Azerbaijan

ABSTRACT *By specific example of the semi-structured time series there are considered known fuzzy forecasting models which differ in rules of fuzzification and/or defuzzification. In the context of this study this paper presents a new approach to defuzzification of outputs of fuzzy time series on the base of applying the fuzzy set point-estimation method. As compared with some well-known defuzzification rules proposed method improves the statistical quality of semi-structured time series forecasting.*

Introduction
Currently, the efforts of many researchers have developed various fuzzy models of semi-structured time series forecasting which are specified by its fuzzification and/or defuzzification rules. Validity of the time series forecasting and obtained predictions depends directly on how well these rules allow to describe adequately semi-structured historical data by fuzzy sets and, respectively, adequately to interpret the obtained fuzzy outputs in a traditional numerical manner.

In [5] on the basis of a specific example the fuzzy model of the time series was offered. As showed the comparative analysis, this model profitably differs from existing correspondent models [1-4, 6, 7] by its rule of the fuzzification of historical data and rules of defazzification of fuzzy outputs. In the present article within the general concept of fuzzy time series forecasting it is offered point-estimation method based the rule of defuzzification the outputs forecasted by Poulsen's algorithm [5].

Problem definition. As a basic semi-structured time series we selected ordered in time historical data describing the dynamics of student enrollments at Alabama University (USA) from 1971 to 1992 year (Table 1). In [5], along with consideration of fuzzy time series models of Chen [1, 2] and Song-Chissom [6, 7] based on the same rule of historical data fuzzification it was offered the algorithm, which in comparison with these models can significantly improve the quality of time series forecasting, i.e. increases the validity of induced predictions. In the context of tested in [5] operations it is necessary to formulate and to test a rule of defuzzification the outputs of fuzzy time series models based on point-estimation method of fuzzy sets.

Table – 1
Historical enrollments

Year	Student enrollments	Year	Student enrollments	Year	Student enrollments
1971	13055	1979	16807	1987	16859
1972	13563	1980	16919	1988	18150
1973	13867	1981	16388	1989	18970
1974	14696	1982	15433	1990	19328
1975	15460	1983	15497	1991	19337
1976	15311	1984	15145	1992	18876
1977	15603	1985	15163		
1978	15861	1986	15984		

Source: Historical student enrollments at Alabama University (USA), 1971 – 1992

Point-estimation of fuzzy set
Let $\tilde{A}\subset U$ be a fuzzy set, where U is a certain universum. Then its α-level set (αÎ[0;1]) is presented by

$$A_\alpha=\{u|\mu_{\tilde{A}}(u)\geq\alpha,\ u\in U\}, \qquad (1)$$

which is specified by average number of own elements: cardinal number $M(\tilde{A}_\alpha)$. Specifically:

· for α-level set composed of n elements

$$M(A_{\acute{a}})=\sum_{k=1}^{n}\frac{u_k}{n}; \qquad (2)$$

· for $A_\alpha=\{a\leq u\leq b\}$

$$M(A_\alpha)=(a+b)/2; \qquad (3)$$

· for

$$M(A_{\acute{a}})=\frac{\sum_{k=1}^{n}\frac{a_k+b_k}{2}(b_k-a_k)}{\sum_{k=1}^{n}(b_k-a_k)}, \qquad (4)$$

under $0\leq a_1\leq b_1\leq a_2\leq b_2\leq\ldots\leq a_n\leq b_n\leq 1$.

As a result, the point estimate of the fuzzy set $\tilde{A}$ can be obtained from the equation:

$$F(\tilde{A})=\frac{1}{\acute{a}_{\max}}\int_0^{\acute{a}_{\max}}M(A_{\acute{a}})d\acute{a}, \qquad (5)$$

where α_{max} is the maximal value over set $\tilde{A}$.

Poulsen's approach
Poulsen's approach to the fuzzy modeling of time series (Table 1) provides the following results [5]:

1. Universe of time series data: U=[12861, 19531].

2. The number of fuzzy subsets of universe U: n=17, which of them is described by trapezoidal membership function in the form

Volume : 3 | Issue : 9 | Sept 2013 | ISSN - 2249-555X

$$\mu_{\tilde{A}_k}(x)=\begin{cases}\dfrac{x-a_{1k}}{a_{2k}-a_{1k}}, & a_{1k}\le x\le a_{2k}; \\ 1, & a_{2k}\le x\le a_{3k}; \\ \dfrac{a_{4k}-x}{a_{4k}-a_{3k}}, & a_{3k}\le x\le a_{4k}; \\ 0, & otherwise,\end{cases} \quad (k=1\div17), \ (6)$$

which produces *k*-th fuzzy analog $\tilde{A}_k$ (k=1÷17) correspondent semi-structured data of time series.

3. Showed in Table 2 parameters of trapezoidal membership functions to correspondent fuzzy subsets.

Table – 2
Generated fuzzy subsets by processing the enrollment data

Fuzzy subset	Parameters of membership function			
	a_1	a_2	a_3	a_4
$\tilde{A}_1$	12861	13055	13245	13436
$\tilde{A}_2$	13245	13436	13626	13816
$\tilde{A}_3$	13626	13816	14007	14197
$\tilde{A}_4$	14007	14197	14388	14578
$\tilde{A}_5$	14388	14578	14768	14959
$\tilde{A}_6$	14768	14959	15149	15339
$\tilde{A}_7$	15149	15339	15530	15720
$\tilde{A}_8$	15530	15720	15910	16101
$\tilde{A}_9$	15910	16101	16291	16482
$\tilde{A}_{10}$	16291	16482	16672	16862
$\tilde{A}_{11}$	16672	16862	17053	17243
$\tilde{A}_{12}$	17053	17243	17433	17624
$\tilde{A}_{13}$	17433	17624	17814	18004
$\tilde{A}_{14}$	17814	18004	18195	18385
$\tilde{A}_{15}$	18195	18385	18576	18766
$\tilde{A}_{16}$	18576	18766	18956	19147
$\tilde{A}_{17}$	18956	19147	19337	19531

4. Showed in Table 3 fuzzy analogs of enrollment data.

Table – 3
FUZZIFYING ANNUAL ENROLLMENTS

Year	Enrollment	Fuzzy analog	Year	Enrollment	Fuzzy analog
1971	13055	$\tilde{A}_1$	1982	15433	$\tilde{A}_7$
1972	13563	$\tilde{A}_2$	1983	15497	$\tilde{A}_7$
1973	13867	$\tilde{A}_3$	1984	15145	$\tilde{A}_6$
1974	14696	$\tilde{A}_5$	1985	15163	$\tilde{A}_6$
1975	15460	$\tilde{A}_7$	1986	15984	$\tilde{A}_8$
1976	15311	$\tilde{A}_7$	1987	16859	$\tilde{A}_{11}$
1977	15603	$\tilde{A}_7$	1988	18150	$\tilde{A}_{14}$
1978	15861	$\tilde{A}_8$	1989	18970	$\tilde{A}_{16}$
1979	16807	$\tilde{A}_{11}$	1990	19328	$\tilde{A}_{17}$
1980	16919	$\tilde{A}_{11}$	1991	19337	$\tilde{A}_{17}$
1981	16388	$\tilde{A}_{10}$	1992	18876	$\tilde{A}_{16}$

5. Identified and localized on groups of 1st and 2nd orders the internal relationships (see Tables 4 and 5).

Table – 4
Fuzzy relationships groups of first order

Group No	Fuzzy Relationship	Group No	Fuzzy Relationship
Group 1:	$\tilde{A}_1\to\tilde{A}_2$	Group 7:	$\tilde{A}_8\to\tilde{A}_{11}$
Group 2:	$\tilde{A}_2\to\tilde{A}_3$	Group 8:	$\tilde{A}_{10}\to\tilde{A}_7$
Group 3:	$\tilde{A}_3\to\tilde{A}_5$	Group 9:	$\tilde{A}_{11}\to\tilde{A}_{10}$, $\tilde{A}_{11}$, $\tilde{A}_{14}$
Group 4:	$\tilde{A}_5\to\tilde{A}_7$	Group 10:	$\tilde{A}_{14}\to\tilde{A}_{16}$
Group 5:	$\tilde{A}_6\to\tilde{A}_6$, $\tilde{A}_8$	Group 11:	$\tilde{A}_{16}\to\tilde{A}_{17}$
Group 6:	$\tilde{A}_7\to\tilde{A}_6$, $\tilde{A}_7$, $\tilde{A}_8$	Group 12:	$\tilde{A}_{17}\to\tilde{A}_{16}$, $\tilde{A}_{17}$

Table – 5
Fuzzy relationships groups of second order

Group No	Fuzzy Relationship
Group 1:	$\tilde{A}_1$, $\tilde{A}_2\to\tilde{A}_3$
Group 2:	$\tilde{A}_2$, $\tilde{A}_3\to\tilde{A}_5$
Group 3:	$\tilde{A}_3$, $\tilde{A}_5\to\tilde{A}_7$
Group 4:	$\tilde{A}_5$, $\tilde{A}_7\to\tilde{A}_7$
Group 5:	$\tilde{A}_7$, $\tilde{A}_7\to\tilde{A}_6$, $\tilde{A}_7$, $\tilde{A}_8$
Group 6:	$\tilde{A}_7$, $\tilde{A}_8\to\tilde{A}_{11}$
Group 7:	$\tilde{A}_8$, $\tilde{A}_{11}\to\tilde{A}_{11}$, $\tilde{A}_{14}$
Group 8:	$\tilde{A}_{11}$, $\tilde{A}_{11}\to\tilde{A}_{10}$
Group 9:	$\tilde{A}_{11}$, $\tilde{A}_{10}\to\tilde{A}_7$
Group 10:	$\tilde{A}_{10}$, $\tilde{A}_7\to\tilde{A}_7$
Group 11:	$\tilde{A}_7$, $\tilde{A}_6\to\tilde{A}_6$
Group 12:	$\tilde{A}_6$, $\tilde{A}_6\to\tilde{A}_8$
Group 13:	$\tilde{A}_6$, $\tilde{A}_8\to\tilde{A}_{11}$
Group 14:	$\tilde{A}_{11}$, $\tilde{A}_{14}\to\tilde{A}_{16}$
Group 15:	$\tilde{A}_{14}$, $\tilde{A}_{16}\to\tilde{A}_{17}$
Group 16:	$\tilde{A}_{16}$, $\tilde{A}_{17}\to\tilde{A}_{17}$
Group 17:	$\tilde{A}_{17}$, $\tilde{A}_{17}\to\tilde{A}_{16}$

Defuzzification of outputs of Poulsen's model by point-estimation method

Defuzzification of fuzzy outputs is a key step of time series forecasting prosess. It affects greatly the accuracy of the prediction in the ordinary numbers. In this section it is proposed to use point-estimation method of fuzzy predictions. The essence of this approach is as follows.

Suppose that a fuzzy subset $\tilde{A}_t$ of the universe U ($\tilde{A}_t\subset U$) is the fuzzy predictions obtained by the application, for example, Poulsen's model with fuzzy relationships of first order. As a rule, this set consolidates by combining one or more elementary fuzzy sets from the list of sets that describe the historical data of the given time series. For example, according to the Poulsen's algorithm the fuzzy output for 1978 year is a group of fuzzy relationships of the first order: $\tilde{A}_7\to\tilde{A}_6$, $\tilde{A}_7$, $\tilde{A}_8$. In the notation of fuzzy inference mechanism, this implies following implicative rule:

"if predicate is $\tilde{A}_7$,

then prediction will be $\tilde{A}_6$ or $\tilde{A}_7$ or $\tilde{A}_8$".

Taking into account existence of OR operator in the right part of this rule, common membership function is defined as:

$$\mu_{\tilde{A}_{1978}}(u)=\mu_{\tilde{A}_6\cup\tilde{A}_7\cup\tilde{A}_8}(u)=$$
$$\max\{\mu_{\tilde{A}_6}(u)\ \mu_{\tilde{A}_7}(u)\ \mu_{\tilde{A}_8}(u)\}.$$

Volume : 3 | Issue : 9 | Sept 2013 | ISSN - 2249-555X

Here, as membership function one can use trapezoidal function in the form (6).

For point-estimation of fuzzy predictions are obtained by Poulsen's model initially determine α-level sets ($\alpha \in [0; 1]$) in the form (1): $A_\alpha = \{i | \mu_{\tilde{A}}(i) \geq \alpha,\ i \in I\}$, where I is a finite aggregate of numbers from U_{min} to U_{max}, which form the arithmetical progression. Further, for each level set it is determined correspondent cardinal number $M(A_\alpha)$ by Eq. (2). Finnaly, point-estimation of fuzzy set $\tilde{A}_t$ is calculated by Eq. (5).

Apply formulated point-estimation method to fuzzy outputs of Poulsen's model, which in most cases are offered the union of several elementary fuzzy sets from the list $\{\tilde{A}_k\}$ ($k=1\div17$). For the construction of these sets as the support vector one can choose a suitable set of numbers from the universe U. Let this be a set of 51-th numbers varying from 12861 to 19531 by step of 133.4:

C={12861, 12994.4, 13127.8, 13261.2, 13394.6, 13528, 13661.4, 13794.8, 13928.2, 14061.6, 14195, 14328.4, 14461.8, 14595.2, 14728.6, 14862, 14995.4, 15128.8, 15262.2, 15395.6, 15529, 15662.4, 15795.8, 15929.2, 16062.6, 16196, 16329.4, 16462.8, 16596.2, 16729.6, 16863, 16996.4, 17129.8, 17263.2, 17396.6, 17530, 17663.4, 17796.8, 17930.2, 18063.6, 18197, 18330.4, 18463.8, 18597.2, 18730.6, 18864, 18997.4, 19130.8, 19264.2, 19397.6, 19531}.

As an example, we choose a fuzzy output of Poulsen's model for 1978 year, which is the union of fuzzy sets $\tilde{A}_6$, $\tilde{A}_7$ and $\tilde{A}_8$ (see Tables 3 and 4). Restoring these sets with the appropriate trapezoidal membership functions of the form (6), on the basis of the support vector C we obtain the following interpretation of a fuzzy set $\tilde{A}_{1978}$:

$$\widetilde{A}_{1978} = \frac{0}{12861} + \frac{0}{12994.4} + \frac{0}{13127.8} + \frac{0}{13261.2} + \frac{0}{13394.6} + \frac{0}{13528} + \frac{0}{13661.4} + \frac{0}{13794.8} + \frac{0}{13928.2} + \frac{0}{14061.6} + \frac{0}{14195} + \frac{0}{14328.4} + \frac{0}{14461.8} + \frac{0}{14595.2} + \frac{0}{14728.6} + \frac{0.4921}{14862} + \frac{1}{14995.4} + \frac{1}{15128.8} + \frac{0.5958}{15262.2} + \frac{1}{15395.6} + \frac{1}{15529} + \frac{0.6968}{15662.4} + \frac{1}{15795.8} + \frac{0.8995}{15929.2} + \frac{0.2010}{16062.6} + \frac{0}{16196} + \frac{0}{16329.4} + \frac{0}{16462.8} + \frac{0}{16596.2} + \frac{0}{16729.6} + \frac{0}{16863} + \frac{0}{16996.4} + \frac{0}{17129.8} + \frac{0}{17263.2} + \frac{0}{17396.6} + \frac{0}{17530} + \frac{0}{17663.4} + \frac{0}{17796.8} + \frac{0}{17930.2} + \frac{0}{18063.6} + \frac{0}{18197} + \frac{0}{18330.4} + \frac{0}{18463.8} + \frac{0}{18597.2} + \frac{0}{18730.6} + \frac{0}{18864} + \frac{0}{18997.4} + \frac{0}{19130.8} + \frac{0}{19264.2} + \frac{0}{19397.6} + \frac{0}{19531}.$$

Level set A_α and correspondent cardinal number $M(A_\alpha)$ are determined as following:

- for $0<\alpha<0.201$, $d\alpha=0.201$, A_α={14862, 14995.4, 15128.8, 15262.2, 15395.6, 15529, 15662.4, 15795.8, 15929.2, 16062.6}; $M(A_\alpha)$=15462.3;
- for $0.201<\alpha<0.4921$, $d\alpha=0.2911$, A_α={14862, 14995.4, 15128.8, 15262.2, 15395.6, 15529, 15662.4, 15795.8, 15929.2}; $M(A_\alpha)$=15395.6;
- for $0.4921<\alpha<0.5958$, $d\alpha=0.1036$, A_α={14995.4, 15128.8, 15262.2, 15395.6, 15529, 15662.4, 15795.8, 15929.2}; $M(A_\alpha)$=15462.3;
- for $0.5958<\alpha<0.6968$, $d\alpha=0.1011$, A_α={14995.4, 15128.8, 15395.6, 15529, 15662.4, 15795.8, 15929.2}; $M(A_\alpha)$=15490.9;
- for $0.6968<\alpha<0.8995$, $d\alpha=0.2026$, A_α={14995.4, 15128.8, 15395.6, 15529, 15795.8, 15929.2}; $M(A_\alpha)$=15462.3;
- for $0.8995<\alpha<1$, $d\alpha=0.1005$, A_α={14995.4, 15128.8, 15395.6, 15529, 15795.8}; $M(A_\alpha)$=15368.9.

Then in accordance with (5) point-estimation of fuzzy prediction $\tilde{A}_{1978}$ will be:

$$F(\widetilde{A}_{1978}) = \frac{1}{1}\int_0^1 M(A_{\acute{a}})d\acute{a} = \frac{1}{1}(0.201\cdot 15462.3 + 0.2911\cdot 15395.6 + 0.1036\cdot 15462.3 + 0.1011\cdot 15490.9 + 0.2026\cdot 15462.3 + 0.1005\cdot 15368) = 15436.$$

Thus, using the procedure of the point-estimation of fuzzy sets to the output of Poulsen's model induced by relationships of first and second orders we get the correspondent target predictions (see Tables 6 and 7).

Table – 6
Point-estimation of outputs of Poulsen's model induced by relationships of 1st order

Year	Actual data	Fuzzy relationships group of first order	Point-estimation of Poulsen's model outputs
1971	13055	$\tilde{A}_1$□$\tilde{A}_2$	
1972	13563	$\tilde{A}_2$□$\tilde{A}_3$	13532
1973	13867	$\tilde{A}_3$□$\tilde{A}_5$	13905
1974	14696	$\tilde{A}_5$□$\tilde{A}_7$	14670
1975	15460	$\tilde{A}_7$□$\tilde{A}_6$, $\tilde{A}_7$, $\tilde{A}_8$	15443
1976	15311	$\tilde{A}_7$□$\tilde{A}_6$, $\tilde{A}_7$, $\tilde{A}_8$	15436
1977	15603	$\tilde{A}_7$□$\tilde{A}_6$, $\tilde{A}_7$, $\tilde{A}_8$	15436
1978	15861	$\tilde{A}_8$□$\tilde{A}_{11}$	15436
1979	16807	$\tilde{A}_{11}$□$\tilde{A}_{10}$, $\tilde{A}_{11}$, $\tilde{A}_{14}$	16949
1980	16919	$\tilde{A}_{11}$□$\tilde{A}_{10}$, $\tilde{A}_{11}$, $\tilde{A}_{14}$	17231
1981	16388	$\tilde{A}_{10}$□$\tilde{A}_7$	17231
1982	15433	$\tilde{A}_7$□$\tilde{A}_6$, $\tilde{A}_7$, $\tilde{A}_8$	15443
1983	15497	$\tilde{A}_7$□$\tilde{A}_6$, $\tilde{A}_7$, $\tilde{A}_8$	15436
1984	15145	$\tilde{A}_6$□$\tilde{A}_6$, $\tilde{A}_8$	15436
1985	15163	$\tilde{A}_6$□$\tilde{A}_6$, $\tilde{A}_8$	15432
1986	15984	$\tilde{A}_8$□$\tilde{A}_{11}$	15432
1987	16859	$\tilde{A}_{11}$□$\tilde{A}_{10}$, $\tilde{A}_{11}$, $\tilde{A}_{14}$	16949
1988	18150	$\tilde{A}_{14}$□$\tilde{A}_{16}$	17231
1989	18970	$\tilde{A}_{16}$□$\tilde{A}_{17}$	18860
1990	19328	$\tilde{A}_{17}$□$\tilde{A}_{16}$, $\tilde{A}_{17}$	19235
1991	19337	$\tilde{A}_{17}$□$\tilde{A}_{16}$, $\tilde{A}_{17}$	19051
1992	18876	$\tilde{A}_{16}$□$\tilde{A}_{17}$	19051

Volume : 3 | Issue : 9 | Sept 2013 | ISSN - 2249-555X

Table – 7
Point-estimation of outputs of Poulsen's model induced by relationships of 2[d] order

Year	Actual data	Fuzzy relationships group of first order	Point-estimation of Poulsen's model outputs
1971	13055		
1972	13563	$\tilde{A}_{1}, \tilde{A}_{2}$□$\tilde{A}_{3}$	
1973	13867	$\tilde{A}_{2}, \tilde{A}_{3}$□$\tilde{A}_{5}$	13905

TABLE – 7 (continued)

Year	Actual data	Fuzzy relationships group of first order	Point-estimation of Poulsen's model outputs
1974	14696	$\tilde{A}_{3}, \tilde{A}_{5}$□$\tilde{A}_{7}$	14670
1975	15460	$\tilde{A}_{5}, \tilde{A}_{7}$□$\tilde{A}_{7}$	15443
1976	15311	$\tilde{A}_{7}, \tilde{A}_{7}$□$\tilde{A}_{6}, \tilde{A}_{7}, \tilde{A}_{8}$	15443
1977	15603	$\tilde{A}_{7}, \tilde{A}_{7}$□$\tilde{A}_{6}, \tilde{A}_{7}, \tilde{A}_{8}$	15436
1978	15861	$\tilde{A}_{7}, \tilde{A}_{8}$□$\tilde{A}_{11}$	15436
1979	16807	$\tilde{A}_{8}, \tilde{A}_{11}$□$\tilde{A}_{11}, \tilde{A}_{14}$	16949
1980	16919	$\tilde{A}_{11}, \tilde{A}_{11}$□$\tilde{A}_{10}$	17527
1981	16388	$\tilde{A}_{11}, \tilde{A}_{10}$□$\tilde{A}_{7}$	16569
1982	15433	$\tilde{A}_{10}, \tilde{A}_{7}$□$\tilde{A}_{7}$	15443
1983	15497	$\tilde{A}_{7}, \tilde{A}_{7}$□$\tilde{A}_{6}, \tilde{A}_{7}, \tilde{A}_{8}$	15443
1984	15145	$\tilde{A}_{7}, \tilde{A}_{6}$□$\tilde{A}_{6}$	15436
1985	15163	$\tilde{A}_{6}, \tilde{A}_{6}$□$\tilde{A}_{8}$	15056
1986	15984	$\tilde{A}_{6}, \tilde{A}_{8}$□$\tilde{A}_{11}$	15823
1987	16859	$\tilde{A}_{8}, \tilde{A}_{11}$□$\tilde{A}_{11}, \tilde{A}_{14}$	16949
1988	18150	$\tilde{A}_{11}, \tilde{A}_{14}$□$\tilde{A}_{16}$	17527
1989	18970	$\tilde{A}_{14}, \tilde{A}_{16}$□$\tilde{A}_{17}$	18860
1990	19328	$\tilde{A}_{16}, \tilde{A}_{17}$□$\tilde{A}_{17}$	19235
1991	19337	$\tilde{A}_{17}, \tilde{A}_{17}$□$\tilde{A}_{16}$	19235
1992	18876		18860

Comparison of forecasting results
To compare the considered approaches to semi-structured time series forecasting we use the following statistical evaluation criteria (Table 8): Mean Absolute Percentage Error (*MAPE*) and Mean Squared Error (*MSE*), which calculated as:

$$MAPE = \frac{1}{n}\sum_{j=1}^{n}\frac{\left|forecast_j - actual_j\right|}{actual_j}\times 100 \quad (7)$$

$$MSE = \frac{1}{n}\sum_{j=1}^{n}\left(forecast_j - actual_j\right)^2 . \quad (8)$$

Table – 8
Comparing forecast results

Year	Actual data (1st order)	Chen [1,2] (1st order)	Cheng et al. [3] (1st order)	Poulsen [5]		Point-estimation method	
				1st order	2nd order	1st order	2nd order
1971	13055						
1972	13563	14000	13531	14230		13532	
1973	13867	14000	13912	14230	13912	13905	13905
1974	14696	14000	14673	14230	14673	14670	14670
1975	15460	15500	15435	15541	15435	15443	15443
1976	15311	16000	15435	15541	15435	15436	15443
1977	15603	16000	15435	15541	15435	15436	15436
1978	15861	16000	15435	16196	15435	15436	15436
1979	16807	16000	16958	16196	16958	16949	16949
1980	16919	16833	17211	16196	17529	17231	17527
1981	16388	16833	17211	17507	16577	17231	16569
1982	15433	16833	15435	16196	15435	15443	15443
1983	15497	16000	15435	15541	15435	15436	15443
1984	15145	16000	15435	15541	15435	15436	15436
1985	15163	16000	15435	15541	15054	15432	15056
1986	15984	16000	15435	15541	15815	15432	15823
1987	16859	16000	16958	16196	16958	16949	16949
1988	18150	16833	17211	17507	17529	17231	17527
1989	18970	19000	18861	18872	18861	18860	18860
1990	19328	19000	19242	18872	19242	19235	19235
1991	19337	19000	19052	18872	19242	19051	19235
1992	18876	19000	19052	18872	18861	19051	18860
MSE		407507	119096	261162	60562	119384	60182
MAPE (%)		3.11	1.42	2.66	1.03	1.42	1.02

Conclusion
Comparison of forecasting results obtained by point-estimation method with the results obtained by known forecasting methods showed that defuzzification method of outputs of fuzzy TSM have a right to exist. In the illustrated variant of the application of point-estimation method the outputs of fuzzy TSM described by the fuzzy set on support vector, which includes 50 components of the specified universe. Further experiments showed that an increase of number of the support vector components (for example, up to 100 units and more) significantly improves the prediction quality.

Volume : 3 | Issue : 9 | Sept 2013 | ISSN - 2249-555X

REFERENCE [1]Chen, S.M. (1996), "Forecasting enrollments based on fuzzy time series." Fuzzy Sets and Systems, ELSEVIAR, 81, 311-319. | [2]Chen, S.M. (2002), "Forecasting enrollments based on high-order fuzzy time series." Cybernetics and Systems: An int. Journal, 33, 1-16. | [3]Cheng, C.H., Chang, J.R., and Yen, C.A. (2006), "Entropy-based and trapezoid fuzzification fuzzy time series approaches for forecasting IT project cost." Technological Forecasting & Social Change, 73, 524-542. | [4]Kumar, N., Ahuja, S., Kumar, V., and Kumar, A. (2010), "Fuzzy time series forecasting of wheat production." International Journal on Computer Science and Engineering 2(3), 635-640. | [5]Poulsen, J.R. (2009), "Fuzzy Time Series Forecasting – Developing a new forecasting model based on high order fuzzy time series." Aalborg University Esbjerg, Semester: CIS 4. | [6]Song, Q., and Chissom, B.S. (1993), "Forecasting enrollments with fuzzy time series." Part I, Fuzzy Sets and Systems, ELSEVIAR, 54, 1-9. | [7]Song Q., and Chissom B.S. (1994), "Forecasting enrollments with fuzzy time series." Part II, Fuzzy Sets and Systems, ELSEVIAR, 62, 1-8.