



# Arbitrage, Convergence, and Long Run Equilibrium between Futures and Spot Markets: A Cointegration Approach

**KEYWORDS**

Cointegration test, Convergence of basis, Cost-of-carry model, Unit root test.

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**ABSTRACT**

This paper studies the plausibility of cost of carry model in index futures and the convergence between futures and spot markets at maturity, which negates any opportunity for index arbitrage in long term. This paper investigates the long run relationship between futures and spot market by cointegration approach. The empirical analysis has been conducted for Bank Nifty Index futures and its underlying CNX Bank Index from June'2005 to December'2013. Unit root tests have been applied to check for the stationarity and the order of integration of two price series. A bivariate cointegration test reveals that the futures and spot prices are cointegrated, and thus it confirms convergence of the two price series, very low opportunity for arbitrage, and a stable long run equilibrium relation between the futures and spot markets.

**Introduction:**

In India trading in financial derivatives was permitted from mid-2000, introducing with the index futures and gradually other segments like, index options, stock options, stock futures, etc. were kept on coming in the market. Over the time, the equity derivative market grows so overwhelmingly that in current scenario the total traded volume in equity derivatives exceeds cash market transactions. Among all the financial derivatives, futures of any asset is considered as the yard stick for expected cash market price movements in future and there lies the efficiency of the futures as a risk hedging instrument. The present paper concentrates on judging convergence of the futures and spot markets at maturity, long run equilibrium between the two, and thereby evaluating the arbitrage opportunity lies in between the two correlated markets, with respect to Indian banking index. Here we have chosen CNX-Bank Nifty Index, as the contribution of NSE in derivative segment is much higher about 98% as compared to BSE.

**Theoretical Back ground:**

Stock index futures, like most other financial futures, are also traded in a full carry market. According to Cost-of-Carry model, futures price must be equal to the spot price plus other costs of carrying charges. But in case of stock index futures valuation, two considerations are most important: carrying cost and dividend income to be received on the index underlying stocks. Hence the basic cost of carry model is modified a little bit to capture the dividend yield an investor could earn by holding the index, which is nothing but a portfolio of equity stocks, and hence fetches dividend on its underlying stocks, if and when a company declares its dividend policy from time to time. Thus the model becomes:

$$F_{t,T} = S_t(1+C) - \dots \dots \dots (I)$$

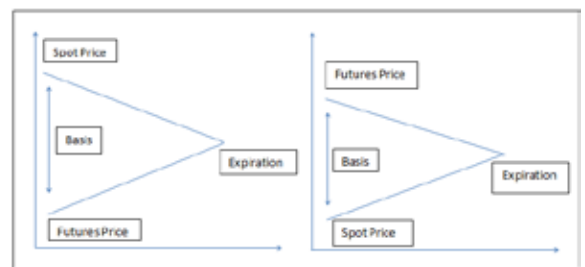
where  $F_{t,T}$  is the stock index futures price at the present time 't' for a futures contract which expires at time 'T',  $S_t$  is the spot value of the index at time 't', C is the percentage cost of carrying the asset from time 't' to the expiration at time 'T',  $D_i$  is the dividend on  $i^{th}$  stock and  $r_i$  is the interest earned on carrying the  $i^{th}$  dividend from time of receipt until the futures expiration at time 'T'. This model holds for a perfect market with unrestricted short selling. Thus the price of a stock index futures contract must be equal to the spot price of the stock index plus the cost of carrying the contract till expiration, minus the dividend yield earned from its under-

lying stocks before expiration. It should be noted that if the above relationship is not fulfilled, the traders would immediately recognize the arbitrage opportunities until prices are adjusted. But due to many imperfections in the market, during the entire contract period the futures price is either less than or greater than or equal to the spot price of the stock index plus the carrying cost, minus the dividend yield. Mathematically speaking,

$$F_{t,T} \leq S_t(1+C) - \dots \dots \dots (I.a),$$

$$\text{or, } F_{t,T} \geq S_t(1+C) - \dots \dots \dots (I.b)$$

When this (I.a) inequality holds, it gives the opportunity for 'cash and carry' trading, i.e. the arbitrageurs can book profit by taking a short position in the spot market and a long position in futures contract and when (I.b) inequality holds, the 'reverse cash and carry' trading occurs, i.e. the arbitrageurs buy the index in spot and go for short the futures contract. In both the cases, the 'basis', i.e. the difference between the current price of the futures contract and spot price of the index, is non-zero. But as the expiry approaches, this difference should also tend to zero, and finally on the day of expiration, the basis should be equal to zero, i.e. the equation (I) should hold, which leaves no scope for index arbitrage. The graphical presentation of the convergence of the 'basis' is given below in figure.1.



**Figure.1: Convergence of Futures Price to Spot Price**

Now if we consider a real life example from CNX- Bank Index spot price and Bank Nifty Futures price for December'2013 contract, we get the picture as following.



**Figure.2: Convergence of CNX-Bank Index Futures Price to Spot Price**

From the above figure.2 we can see that there are evidences of non-equality, near-equality, and equality of futures and spot prices. But nearby the expiration period there is close convergence between the two.

Now our aim is to study whether this convergence property holds in reality and whether in long run these two markets maintain their equilibrium and thus ensures no major discrepancies as scope for long term arbitrage opportunity.

**Brief Review of Cointegration based Analysis:**

Various researchers have investigated stationarity (absence of unit roots) and cointegration in financial time series over the decades. Engle and Granger (1987) reported that the short-term and the long-term rates in the US are cointegrated and such behaviour is expected in an efficient market. Campbell and Shiller (1987) established that the one month Treasury bill rate and the twenty-year yield series are cointegrated from January 1959 to October 1983. Mougoue (1992) reported that the Eurocurrency deposit rates of similar maturities are cointegrated across six currencies. He pointed out that in the absence of time varying risk premia this result is inconsistent with the weak form market efficiency. Bradley and Lumpkin (1992) investigated the US Treasury yield curve as a cointegrated system and showed how to exploit such knowledge in improving forecasting performance. Heaney and Layton (1996) tested the cost of carry relationship for the Australian 90 day bank accepted bill futures market starting with the contract maturing in June’1980 extending to the contract maturing in March’1990. There was little evidence of cointegration prior to 1985 except around 1983, when exchange rate regulations were removed. The introduction of foreign banks and the increase in competition provides one reason for the substantial reduction in arbitrage opportunities across the bank accepted bill futures and bank accepted bill spot markets evident in the period following 1985. Bhar (1996) studied the interest rate futures contracts on 90 day bank bills, ranging in delivery periods from the near quarter month up to five quarters ahead, by using unit roots and cointegration tests and developed proper error correction model. In the recent time, there are several studies which reveal price discovery efficiency of futures market and lead-lag relationship between futures and spot market in Indian context.

**Data and Methodology:**

This study considers the daily closing price data for Bank Nifty Index Futures since it started trading in NSE, i.e. from 13<sup>th</sup> June 2006 to 26<sup>th</sup> December 2013 and its corresponding Bank Nifty Index Data in spot market. For index futures only the near month contract is accounted for its comparatively high trading volume than the other two months’ contracts available in the market. There is 2124 number of observations for each price series. All the data are collected from official website of NSE ([www.nseindia.com](http://www.nseindia.com)).

**4.1 Unit Root Analysis:**

Unit root analysis is a technique to check for the stationary property of any time series data. Financial time series data generally happens to be non-stationary in nature. In that

case, by identifying the existence of the unit root, the order of integration for the series can be found out. In the present study, we have applied two types of unit root tests: ADF test and KPSS test.

**4.1.1 Augmented Dickey-Fuller Test (1979):**

ADF test is a version of the Dickey-Fuller test to account for more dynamic specification of the regression of time series models used in DF test. The model without intercept and linear trend is:  $\Delta P_t = \delta P_{t-1} + \sum \beta_j \Delta P_{t-j} + \epsilon_t \dots\dots$  (II), where  $\epsilon_t$  i.i.d (0,  $\sigma^2$ ). The null hypothesis for the model is  $H_0: P_t$  is non-stationary, i.e.  $\delta=0$ , against  $H_1: P_t$  is stationary, i.e.  $\delta \neq 0$ .

**4.1.2 Kwiatkowski, Phillips, Schmidt, and Shin Test (1992):**

The KPSS method differs from ADF test in a way that it considers the stationarity in null hypothesis against the alternative of non-stationarity. Here the model is represented as a sum of deterministic trend, random walk and stationary error term, as follows:  $P_t = \alpha + \beta t + \epsilon_t \dots\dots\dots$  (III) and KPSS statistic is computed as,

$$KPSS = \frac{T^{-2} \sum_{t=1}^T S_t^2}{S^2(L)} \dots\dots\dots (IV)$$

where

$S_t = \sum_{i=1}^t \epsilon_i, t=1,2,\dots,T$ , L is the lag parameter and  $S^2$  is the long run variance of residuals. The null hypothesis for KPSS test is  $H_0: P_t$  is stationary, against the alternative  $H_1: P_t$  is non-stationary.

**4.2 Cointegration Analysis:**

Cointegration analysis provides important information about the long term relationship among any group of time series data whose degree of integration is same. The economic interpretation of cointegration is that if two or more variables are linked to form an equilibrium relationship spanning the long-run, even though the series themselves in the short run may deviate from the equilibrium, they will move closer together in the long run equilibrium. Thus, if futures and spot price series are found to be cointegrated, it ensures the non-existence of high basis, either positive or negative, for long term. Accordingly, cointegration can establish whether there exists a stable long-run relationship between Bank Nifty futures and the underlying CNX Bank Nifty index.

In this paper we have adopted Johansen-Juselius (1990) maximum likelihood method of testing, over Engle-Granger (1987) method, as it facilitates treatment of multivariate analysis, and is considered to be more powerful and efficient. If both the futures and cash price series have same degree of integration, then only we can go for testing their cointegration property. Consider a general  $k^{th}$  order VAR model:

$$\Delta Y_t = D + \Pi Y_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta Y_{t-i} + \epsilon_t \dots\dots\dots (V)$$

where  $Y_t$  is an (n x 1) vector to be tested for cointegration, and  $\Delta Y_t = Y_t - Y_{t-1}$ ; D is the deterministic term which may take different forms such as a vector of zeros or non-zero constants depending on properties of the data to be tested;  $\Pi$  and  $\Gamma$  are matrices of coefficients; and k is chosen so that  $\epsilon_t$  is a multivariate normal white noise process with mean zero and finite covariance matrix. The cointegration relationship can be detected by examining the rank of the coefficient matrix  $\Pi$ , because the number of cointegration vectors equals the rank of  $\Pi$ . In particular, the 0 rank i.e.  $\Pi = 0$  implies no cointegration. In a bivariate case, i.e. n = 2, the two variables are cointegrated only if the rank of  $\Pi$  equals 1. Johansen (1998) suggested two test statistics to test the null hypothesis that there are at most ‘r’ cointegration vectors. The null hypothesis can be equivalently stated as the rank of  $\Pi$  is at most ‘r’, for r = 0, 1... n-1. The two test statistics are based on trace and maximum eigenvalues, respectively,

$$\lambda_{Trace} = -T \sum_{i=r+1}^{k-1} \ln(1 - \lambda_i) \dots\dots\dots (VI),$$

$$\text{and } \lambda_{Max} = -T \ln(1 - \lambda_{r+1}) \dots\dots\dots (VII),$$

where  $\lambda_1, \dots, \lambda_r$  are the 'r' largest squared canonical correlations between the residuals obtained by regressing  $\Delta Y_t$  and  $Y_{t-1}$  on  $\Delta Y_{t-1}, \Delta Y_{t-2}, \dots, \Delta Y_{t-k-1}$  and 1 respectively.

In the present study,  $Y_t = (S_t, F_t)$ ;  $n = 2$ , and the null hypothesis should be tested for  $r = 0$  and  $r = 1$ . If  $r = 0$  cannot be rejected, we will conclude that there is no cointegration vector, and therefore, no cointegration. On the other hand, if  $r = 0$  is rejected, and  $r = 1$  cannot be rejected, we will conclude that there is a cointegration relationship.

**Empirical Findings:**

For all the analyses, we have transformed the futures and spot price series to its natural logarithmic form, i.e. lnFP and lnSP.

**5.1 ADF Test Results:**

ADF test has been conducted for both the price series in their level as well as first difference form of data. Moreover, in testing we applied both the models: with intercept and with intercept and linear trend.

**Table.1: ADF Test Results for Futures and Spot Price Series**

Price Series	Futures Price Result		Spot Price Result	
Data Form	Test Model Specification			
	Intercept	Trend	Intercept	Trend
Level	-1.953	-2.823	-1.939	-2.827
First Difference	-41.382*	-41.379*	-40.418*	-40.414*

\*Significant at 1% level.

The critical values of ADF test statistic at 1% level are -3.433 in case of model with intercept, and -3.962 in model with intercept and trend. If we consider the results in above table.1, we can see that the estimated test statistic values are significant at 1% level, in first difference form of data for both the futures and spot price series. This implies that futures and spot price series are non-stationary at their level form, but after first differencing they achieve the stationarity. Hence, both the price series are having same order of integration, i.e. I(1).

**5.2 KPSS Test Results:**

Similar to ADF test, this KPSS test has also been conducted for both the price series in their level as also in first difference form, with two model specification: with intercept and another with intercept and linear trend. The critical values of KPSS test statistic at 1% level are 0.739 in case of model with intercept, and 0.216 in model with intercept and trend. If we compare these critical values with the estimated values in following table.2, we see that the level form of both the futures and spot price series are significant at 1% level. This rejects the null hypothesis of stationarity in level form of both the price series. But after first differencing both futures and spot price series become stationary. This result is similar to the above finding from ADF test results that both futures and spot price series have I(1) order of integration.

**Table.2: KPSS Test Results for Futures and Spot Price Series**

Price Series	Futures Price Result		Spot Price Result	
Data Form	Test Model Specification			
	Intercept	Trend	Intercept	Trend
Level	4.474*	0.233*	4.479*	0.236*
First Difference	0.064	0.034	0.064	0.035

\*Significant at 1% level.

Thus by deploying two different ways of unit root tests, we confirm that the futures and spot price series are both I(1) series. Now we can proceed to check their cointegration property.

**5.3 Cointegration Test Results:**

By following Johansen-Juselius method, we have conducted both the trace test and maximum eigenvalue test for a bivariate VAR model with variables lnFP and lnSP. Here we have taken the optimal lag length to be 2, following Schwartz Information Criteria (SIC).

**Table.3: Cointegration Test of Futures and Spot Price Series**

Rank Test	$\lambda_{Trace}$ Test Results		$\lambda_{Max}$ Test Results	
	Trace Statistic	P-value	Max-Eigenvalue Statistic	P-value
$r = 0$	157.026*	0.0001	152.335*	0.0001
$r = 1$	4.691	0.3190	4.691	0.3190
Cointegrating Equation: $\ln FP_t = 0.021797 - 1.002529 \ln SP_t$ (0.00624) (0.00070)				

\*Significant at 1% level. Standard errors are in parentheses ().

From the above table.3, we can see that both the  $\lambda_{Trace}$  and  $\lambda_{Max}$  estimated values are statistically significant at 1% level, for  $r = 0$ . Thus both trace and max-eigenvalue tests reject the null hypothesis that the rank order of  $\Pi$  matrix is zero. Alternatively, both the tests accept that  $r = 1$ , i.e. there exists one cointegrating equation between lnFP and lnSP, at 0.01 level of significance. The estimated cointegrating equation is given in the last row of the table.3. Here for cointegration test, we have considered the model which assumes intercept but no deterministic trend. Among all the five models available for test, this model fits the best to our data according to SIC. Thus we can conclude that futures and spot price series are cointegrated with rank 1.

**Conclusions:**

The empirical findings of the paper bring forth the fact that the Bank Nifty futures and underlying CNX Bank Index are two cointegrated series, bearing one cointegrating linear relation. This produces a statistical equilibrium which can be interpreted as a long run economic relation. Thus futures and spot price series indicate a common long run development and they do not drift away from each other except for transitory fluctuations, which ensure convergence of the non-zero basis. Hence it is proven that the Bank Nifty Futures is enough efficient to leave no long term scope for index arbitrage. There is scope for further studying the short term fluctuations and speed of adjustments between futures and spot prices and their causal relationship.

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