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Influence of Newtonian Heating and Radiation on Chemically Reactive Fluid Flow Past an Accelerated Surface with Variable Mass Diffusion

KEYWORDS: Chemical reaction, Newtonian heating, radiation, accelerated plate

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ABSTRACT *An exact analysis of heat and mass transfer flow past an accelerated vertical plate with Newtonian heating and chemical reaction in the presence of radiation is presented. Equations are solved for velocity, temperature, and concentration using Laplace transforms. Graph results are presented for temperature, velocity and skin friction. The effects of various parameters on flow variables are illustrated graphically and the physical aspects of the problem are discussed.*

1. INTRODUCTION:

The study of heat generation in many fluids due to exothermic and endothermic chemical reactions and natural convection with heat generenation can be added to combustion modeling. It is found that in many chemical engineering processes, chemical reaction takes place between foreign masses (present in the form of ingredients) and the fluid. This type of chemical reaction may change the temperature and the heat content of the fluid and may affect the free convection process. However, if the presence of such foreign mass is very low then we can assume the first order chemical reaction so that heat generation due to chemical reaction can be considered to be very negligible. Here only first order chemical reaction is considered. A reaction is said to be of the first order if the rate of reaction is directly proportional to the concentration. Flow past a vertical plate with chemical reaction is analyzed by Fayez[1] and Sarada et. al. [2] under different physical situations. Bhaben et. al. [3] analyzed chemical reaction effects on flow past a vertical plate with variable temperature.

In many practical situations where the heat transfer from the surface is taken to be proportional to the local surface temperature .The proportionally condition of the heat transfer to the local surface temperature is termed as Newtonian heating .Chemical reaction and mass transfer effects on flow past a surface with Newtonian heating is analyzed by Rajesh [4].

Actually, many processes in new engineering areas occur at high temperature and knowledge of radiation heat transfer becomes imperative for the design of the pertinent equipment. Natural convective flow past a plate in the presence of radiation is studied by Chaudhary et. al. [5]. Radiation and Newtonian heating effect on flow past a vertical plate is analyzed by and Das et.al. [6]. Recently, Jain [7] pioneered effects of radiation and chemical reaction on flow past a vertical surface using Laplace Transform technique.

The aim of the present work is to provide an exact solution for the problem of chemically reactive fluid flow over a moving vertical plate in presence of radiation with Newtonian heating. The solutions are obtained numerically for various parameters entering into the problem and discussed them from the physical point of view.

2. MATHEMATICAL ANALYSIS: Consider unsteady two-dimensional flow of an Incompressible and electrically conducting viscous fluid along an infinite vertical plate. The x'-axis is taken on the infinite plate and parallel to the free stream velocity and y'-axis normal to it. Initially, the plate and the fluid are at same temperature T'_∞ with concentration level C'_∞ at all points. At time t' > 0, It accelerates with a velocity $\frac{U_R{}^3 t'}{\nu}$ in its own plane. At the same time, the heat transfer from plate to the fluid is directly proportional to the local surface temperature T' and the plate concentration is raised linearly with respect to time. It is assumed that there exist a homogeneous chemical reaction of first order with constant rate K_l between the diffusing species and the fluid. The fluid is considered to be gray absorbing-emitting radiation but non scattering medium Under these assumptions the problem can be governed by the following set of equations:

$$\frac{\partial T'}{\partial t'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} \quad \ldots(1)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - k_1 C' \quad \ldots(2)$$

$$\frac{\partial u'}{\partial t'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta\,(T' - T'_\infty) + g\beta_c\,(C' - C'_\infty{}') \quad \ldots(3)$$

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with following initial and boundary conditions

$$u' = 0\,,\ T' = T'_{\infty}\,,\ C' = C'_{\infty} \text{ for all } y', t' \le 0 \qquad \ldots(4)$$

$$\left.\begin{array}{ll} u' = \dfrac{U_R{}^3 t'}{\nu},\ \dfrac{\partial T'}{\partial y'} = -h_s T',\ C' = C'_{\infty} + (C'_w - C'_{\infty})\dfrac{u_R^2 t'}{\nu} & \text{at } y' = 0,\ t' > 0 \\ u' \to 0,\ T' \to T'_{\infty},\ C' \to C'_{\infty} & \text{as } y' \to \infty,\ t' > 0 \end{array}\right\}$$

The radiative heat flux term, by using the Rosseland's approximation is given by

$$q_r = -\frac{4\sigma'}{3\kappa^*}\frac{\partial T'^4}{\partial y'} \qquad ..(5)$$

We assume that the temperature differences within the flow are such that T'^4 may be expressed as a linear function of the temperature T'. This is accomplished by expanding T'^4 in a Taylor series about T'_{∞} and neglecting higher-order terms

$$T'^4 \simeq 4\,T'^3_{\infty}\,T' - 3T'^4_{\infty} \qquad \ldots(6)$$

By using equations (5) and (6), equation (1) gives

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma' T'^3_{\infty}}{3\kappa^*}\frac{\partial^2 T'}{\partial y'^2} \qquad \ldots(7)$$

Introducing the following dimensionless quantities

$$t = \frac{t'}{t_R},\ y = \frac{y'}{L_R},\ u = \frac{u'}{U_R},\ k = \frac{U_R^2 k_1}{\nu^2},$$

$$Pr = \frac{\mu C_p}{\kappa},\ Sc = \frac{\nu}{D},\ \theta = \frac{T' - T'_{\infty}}{T'_{\infty}},\ G = \frac{g\beta_T T'_{\infty}\nu}{U_R{}^3}$$

$$C = \frac{C' - C'_{\infty}}{C'_w - C'_{\infty}},\ Gm = \frac{\nu g\,\beta_C (C'_w - C'_{\infty})}{U_R^3},\ k = \frac{\nu k_1}{U_R^2}$$

$$R = \frac{\kappa^*\kappa}{4\sigma' T'^3_{\infty}}$$

$$\Delta T = T'_w - T'_{\infty},\ U_R = (\nu g\beta\Delta t)^{1/3},$$

$$L_R = \left(\frac{g\beta\Delta T}{\nu^2}\right)^{-1/3},\ t_R = (g\beta\Delta T)^{-2/3}\nu^{1/3} \qquad \ldots(8)$$

Where all the symbols have their usual meanings.

The governing equations (1) to (3) reduce to the following non-dimensional form

$$Pr\ \frac{\partial\theta}{\partial t} = (\mathbf{1} + \frac{\mathbf{4}}{\mathbf{3}R})\frac{\partial^2\theta}{\partial y^2} \qquad ...(9)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - kC \qquad \ldots(10)$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G\theta + Gm\ C \qquad \ldots(11)$$

with the following initial and boundary conditions

$$u = 0\,,\ \theta = 0,\ C = 0 \qquad \text{for all } y,\ t \le 0 \qquad \ldots(12)$$

$$\left.\begin{array}{ll} u = t,\ \dfrac{\partial\theta}{\partial y} = -\gamma(1+\theta),\ C = t & \text{at } y = 0,\ t > 0 \\ u \to 0,\ \theta \to 0,\ C \to 0 & \text{as } y \to \infty,\ t > 0 \end{array}\right\} \qquad \ldots(13)$$

$\gamma = \dfrac{h_s\nu}{U_R}$ is Newtonian heating Parameter.

Equation (13) gives $\theta=0$ when $\gamma=0$ which physically means that no heating from the plate exists.

On solving equations (9) to (11) by Laplace-transform technique, we get

$$\theta = \exp(-\mathbf{2}\gamma\eta\sqrt{t} + b^2 t)\,\text{erfc}\,(\eta\sqrt{a} - b\sqrt{t}) - \text{erfc}\left(\eta\sqrt{a}\right) \qquad \ldots(14)$$

$$C = \frac{t}{2}\{\exp(2\eta\sqrt{kSc\,t})\text{erfc}\,(\eta\sqrt{Sc} + \sqrt{kt}) + \exp(-2\eta\sqrt{kSc\,t})\text{erfc}\,(\eta\sqrt{Sc} - \sqrt{kt})\}$$

$$+\frac{\eta\sqrt{Sct}}{2\sqrt{k}}\{\exp(2\eta\sqrt{kSc\,t})\text{erfc}\,(\eta\sqrt{Sc} + \sqrt{kt}) - \exp(-2\eta\sqrt{kSc\,t})\text{erfc}\,(\eta\sqrt{Sc} - \sqrt{kt})\} \qquad \ldots(15)$$

For $Sc \neq 1$

$$u = t\left\{(1+2\eta^2)\text{erfc}(\eta) - \frac{\mathbf{2}\eta\exp(-\eta^2)}{\sqrt{\pi}}\right\}$$

$$+\frac{Gb}{a-1}\left\{\frac{1}{4\sqrt{\pi t}}\exp(-\eta^2) + b\exp(b^2 t - 2b\eta\sqrt{t})\text{erfc}(\eta - b\sqrt{t})\right\}$$

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$$-\frac{Gb}{a-1}\left\{\frac{1}{4\sqrt{\pi t}}\exp(-\eta^2 a)+b\exp(b^2t-2\gamma\eta\sqrt{t})\mathrm{erfc}(\eta\sqrt{a}-b\sqrt{t})\right\}$$

$$+\frac{Gm}{k\,Sc}\left\{t\left(\mathbf{1}+\mathbf{2}\eta^2\right)\mathrm{erfc}(\eta)-\frac{\mathbf{2}\eta e^{-\eta^2}}{\sqrt{\pi}}\right\}+\frac{Gm(\mathbf{1}-Sc)}{k^2Sc^2}\mathrm{erfc}(\eta)$$

$$-\frac{Gm(1-Sc)}{2k^2Sc^2}\exp\left(\frac{kSct}{1-Sc}\right)\{\exp(2\eta\sqrt{\frac{kSct}{1-Sc}})\,\mathrm{erfc}(\eta+\sqrt{\frac{kSct}{1-Sc}})$$

$$+\exp(-2\eta\sqrt{\frac{kSct}{1-Sc}})\mathrm{erfc}(\eta-\sqrt{\frac{kSct}{1-Sc}})\}$$

$$-\frac{Gm(1-Sc)}{2k^2Sc^2}\exp\left(\frac{kSct}{1-Sc}\right)\{\exp(2\eta\sqrt{\frac{kSct}{1-Sc}})\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{\frac{kt}{1-Sc}})$$

$$+\exp(-2\eta\sqrt{\frac{kSct}{1-Sc}})\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{\frac{kt}{1-Sc}})\}$$

$$-\frac{Gmt}{2kSc}(\exp(2\eta\sqrt{kSct})\ \mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{kt})$$

$$+\exp(-2\eta\sqrt{kSct})\ \mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{kt}))$$

$$-\frac{Gm\eta}{2kSc}\sqrt{\frac{tSc}{k}}(\exp(2\eta\sqrt{kSct})\ \mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{kt})$$

$$+\exp(-2\eta\sqrt{kSct})\ \mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{kt}))$$

$$-\frac{Gm(1-Sc)}{2k^2Sc^2}(\exp(2\eta\sqrt{kSct})\ \mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{kt})$$

$$+\exp(-2\eta\sqrt{kSct})\ \mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{kt}))$$

…(16)

Where

$$b=\frac{\gamma}{\sqrt{a}},\quad a=\frac{Pr}{\mathbf{1}+R},\quad \eta=\frac{y}{\mathbf{2}\sqrt{t}}\quad d=\frac{kSc}{\mathbf{1}-Sc}$$

2.1 SKIN-FRICTION:

From velocity field , skin-friction at the plate in non dimensional form is expressed as:

$$\tau=-\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

For $Sc\neq 1$

$$\frac{2t}{t\sqrt{\pi}}\left(1+\frac{Gm}{k\,Sc}\right)+\frac{Gb}{a-1}\left\{\frac{b}{\sqrt{\pi t}}+b^2\exp\left(b^2t\right)\mathrm{erfc}\left(-b\sqrt{t}\right)\right\}$$

$$+\frac{Gm(1-Sc)}{k^2Sc^2}\left(\exp\left(\frac{kSc\,t}{1-Sc}\right)\right)\sqrt{\frac{kSc}{1-Sc}}\left\{\mathrm{erf}\left(\sqrt{\frac{kSc\,t}{1-Sc}}\right)+\mathrm{erf}\left(\sqrt{\frac{kt}{1-Sc}}\right)\right\}$$

$$+\frac{2Gm(1-Sc)}{k^2Sc^2\sqrt{\pi t}}-\frac{Gb^3\sqrt{a}}{(a-1)}e^{b^2t}\mathrm{erfc}(-b\sqrt{t})-\frac{Gb^2}{(a-1)}e^{b^2t}\sqrt{\frac{a}{\pi t}}$$

$$-\frac{Gm\,t}{k\,Sc}\left\{\sqrt{kSc}\,\mathrm{erf}\left(\sqrt{kt}\right)+\sqrt{\frac{Sc}{\pi t}}\ e^{-kt}\right\}$$

$$-\frac{Gm(1-Sc)}{k^2Sc^2}\left\{\sqrt{kSc}\,\mathrm{erf}\left(\sqrt{kt}\right)\right\}+\frac{Gm}{2k^{3/2}\sqrt{Sc}}$$

...(17)

2.2 NUSSELT NUMBER

From temperature field, the rate of heat transfer in non-dimensional form is expressed as

$$Nu\ =\ -\frac{\nu}{U_R(T'-T'_\infty)}\frac{\partial T'}{\partial y'}\bigg|_{y'=0}$$

$$=\frac{\mathbf{1}}{\theta(\mathbf{0},t)}+\mathbf{1}$$

$$b\sqrt{a}\left\{\mathbf{1}+\frac{\mathbf{1}}{e^{b^2t}(\mathbf{1}+\mathrm{erf}(b\sqrt{t})-\mathbf{1}}\right\}$$

...(18)

3. DISCUSSION: Figure 1 elucidates the effect of Pr on temperature profile . It is observed that temperature is maximum at the plate then tends to zero far away from the plate. Further, thickness of thermal boundary layer decreases as Pr increases. This is due to the fact that thermal conductivity of fluid decreases with increasing Pr, resulting a decrease in thermal boundary layer thickness. It is also seen that it decreases steeply for Pr = 7 than that of Pr = 0.71.

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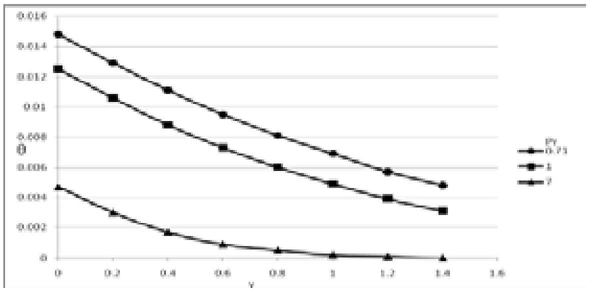

Figure 1: Temperature profile ϒ=0.01, R=5, t=0.2

Figure 2 illustrates the influences of Sc, t, Gm and Pr on the velocity against η for Pr=0.71 and 7 respectively. It is noticed that at the plate, fluid velocity is equal to time then it increases and attains maximum velocity in the vicinity of the plate(η<1) after that it decreases and vanish far away from the plate for Pr=0.71 whereas for Pr=7 same phenomenon is observed in opposite direction. Further, magnitude of velocity also increases with an increase in time at each point in the flow field for both Pr=7 and Pr=0.71 when Hydrogen gas is present in the flow. Moreover, for Pr=7 with an increase in Gm the magnitude of velocity increases. The reason is that the values of modified Grashof number has the tendency to increase the mass buoyancy effect. The change in velocity is more near the plate than away from the plate due to increase in parameters Sc, t and Gm. On the other hand the magnitude of velocity decreases with an increase in Sc for water. It is justified since increase in the value of Sc increases the viscosity of fluid which reduces the velocity of fluid.

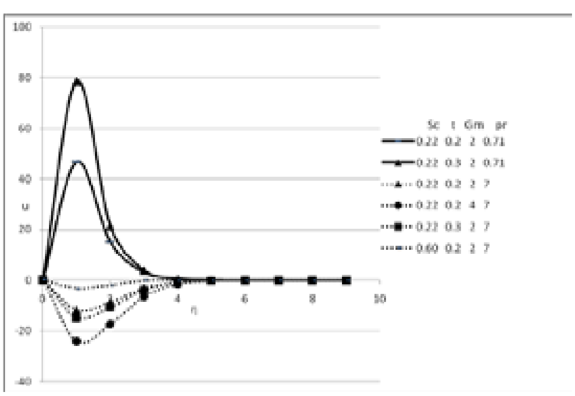

Figure 2 : Velocity profile for R=3, k=0.2, G=5,ϒ=1

The effects of G and ϒ on velocity profile for Pr=7 and Pr=0.71 is indicated in figure 3. It is clear from figure that velocity is equal to time t at the plate then increases to maximum value after that it decreases to zero value for Pr=0.71 but for Pr=7 the same shape of profile is found in reverse direction. Further, magnitude of velocity for Pr=0.71 is higher than that of Pr=7. Physically, it is possible because fluids with high Prandtl number have high viscosity and hence move slowly. Moreover, with an increase in value of G the magnitude of velocity increases for both air and water due to mass buoyant effect of G. Further, the magnitude of velocity increases with an increase in the value of ϒ for both Pr=7 and Pr=0.71.

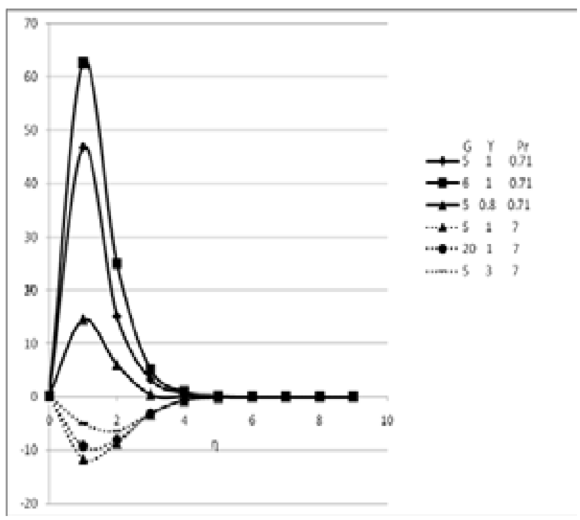

Figure 3: Velocity profile R=1, k=0.2, t=0.2, Sc=0.22, Gm=2

Skin- friction for different parameters against time t is presented in Figure 4. It is clear from figure that for smaller values of t the maximum value of skin friction occurs and then it decreases rapidly with an increase in t(≤0.5) and after this value of t the value of skin friction falls slowly. Moreover, the Figure reveals that the value of skin-friction increases with an increase of Gm. It is observed from figure that magnitude of skin friction is lower for water vapor (Sc=0.60) in comparison to for Hydrogen gas (Sc=0.22). Physically, it is correct since an increase in Sc

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serves to increase momentum boundary layer thickness. Moreover, magnitude of skin friction decreases with an increase in the value of chemical reaction parameter k, the reason is that increasing value of chemical reaction parameter reflects decrease in kinematic viscosity or viscosity of fluid which results decrease in the value of skin friction.

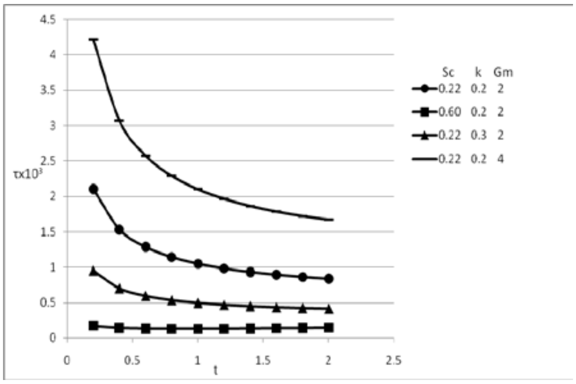

Figure 4: Skin-friction for R=3, G=5, Pr=7, ϒ=0.2

4. CONCLUSION: The results of the flow problem indicates:

1.Increasing Prandtl Number the temperature decreases .

2. Fluid velocity decreases with an increase in Schmidt number, Prandtl number but increases with an increase in Grashof number, modified Grashof number, time, and Newtonian heating Parameter.

4. There is a fall in the value of skin friction with an increase in Schmidt number, chemical reaction parameter and rise with an increase in value of modified Grashof number.

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