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Domination of Complement of A Splitted Graphs

KEYWORDS: Domination, Splitted graph, complement of splitted graphs

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ABSTRACT *Let G be (p,q) graph. A set D of vertices in a complement of any splitted graph $[S(G)]^C=(V,E)$ is called a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number $\gamma[[S(G)]^C]$ of $[S(G)]^C$ is minimum cardinality of a domination set of $[S(G)]^C$*

1. INTRODUCTION

Let G be a (p,q) graph, By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G denoted are by V (G) and E(G) respectively. Let G be a graph. For each vertex v of a graph G , take a new vertex u . Join u to those vertices of G adjacent to v . The graph thus obtained is called the splitting graph of G . It is denoted by S(G) . For a graph G , the splitting graph S(G) is obtained by adding a new vertex v corresponding to each vertex u of G such that N(u) = N(v) and it is denoted by S(G) .

2. PRELIMINARIES

Let G be (p,q) graph.A set D of vertices in a complement of any splitted graph $[S(G)]^C$ = (V,E) is called a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number γ $[[S(G)]^C$] of $[S(G)]^C$ is minimum cardinality of a domination set of $[S(G)]^C$

3.RESULTS

Theorem 3.1. Let $[S(G)]^C$ be a Complement of splitted graph. If D is a minimal domination set, then V-D is also a domnating set.

Proof. Let S(G) be a splitted graph. Let $[S(G)]^C$ be a Complement of S(G). Let D be a minimal dominating set of $[S(G)]^C$. Suppose V-D is not a dominating set. Then there exists a vertex u such that u is not dominated by any vertex in V-D. u is dominated by atleast one vertex in D-{u}. which is contradicts the minimality of D.Thus every vertex in D is adjacent with atleast one vertex in V-D. Hence V-D is a dominating set.

For example complement of $S(P_2)$ is shown in the following figure 3.2 and figure 3.3 respectively

Result 3.4 Complement of all complete graph is disconnected, It does not have a dominating set.

Result 3.5. Complement of all complete bipartite graph is disconnected, It does not have a dominating set.

Result 3.6. Complement of a cycle c_n (n < 4)is disconnected, It does not have a dominating set.

Result 3.7. Complement of a cycle c_n (n ≥ 5) is connected graph, It has a dominating set.

Result 3.8. Complement of path is disconnected, It does not have a dominating set.

Theroem 3.9. For any splitted graph G, γ [$[S(G)]^C$] = 2.

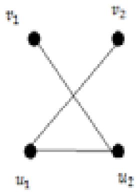

Figure 3.2 $S(P_2)$

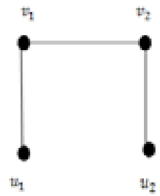

Figure 3.3$[S(P_2)]^c$

Proof. Let S(G) besplitted graph of a graph G. Let $[S(G)]^C$ be complement of S(G). Let D be a minimal dominating set of $[S(G)]^C$. V-Dis a domination set of $[S(G)]^C$.

Hence ,γ $[[S(G)]^C]$ = min{ |D|, |V-D|}= min { 2,|V-D|} =2.

Theorem 3.10.γ $[[S(G)]^C]$ = P − Δ $[[S(G)]^C]$

Proof. Let S(G) be splitted graph of a graph G. Let $[S(G)]^C$ be complement of S(G). Let v be a vertex of maximum degree Δ$[[S(G)]^C]$.Then v is adjacent to N(v) vertices such thatΔ$[[S(G)]^C$]= N(v). Hence V-N(v) is a dominating set.

Thus γ $[[S(G)]^C]$ =| V −N(v)|

= P − Δ $[[S(G)]^C]$

Theorem 3.11.δ $[[S(G)]^C]$ = P − (Δ [S(G)] +1)

Proof. Let S(G) be splitted graph of a graph G. Let $[S(G)]^C$ be complement of S(G). Let v be a vertex of maximum degree inS(G).Then v is adjacent to N(v) vertices ,such that Δ [S(G)] = deg(v). It implies v is a vertex of minimum degree in $[S(G)]^C$.

Then δ $[[S(G)]^C]$ = P − (Δ [S(G)] +1)

Theorem 3.12. Δ $[[S(G)]^C]$ = P − (δ[S(G)] +1)

Proof. Let S(G) be splitted graph of a graph G. Let $[S(G)]^C$ be complement of S(G). Let v be a vertex of minimum degree in S(G).Then v is adjacent to N(v) vertices such thatδ [S(G)] = deg(v). It implies v is a vertex of maximum degree$in[S(G)]^C$ of complement of splitted graph,

Then Δ $[[S(G)]^C]$ = P − (δ [S(G)] +1)

Observation 3.13. $\gamma\,[[S(G)]^C] \leq \left\{P + 1 - (\delta\,[[S(G)]^C] - 1)\,\frac{\Delta\,[[S(G)]^C]}{\delta\,[[S(G)]^C]}\right\}/2$

Observation 3.14. $\gamma\,[[S(G)]^C] < \{P + 2 - (\delta\,[[S(G)]^C]\}/2$

Observation 3.15. $\gamma\,[[S(G)]^C] \leq \{P + 1 - \delta[[S(G)]^C]\}/2$

Observation 3.16. $q < \left\lfloor \frac{1}{2}\ (P - \gamma\,[[S(G)]^C)(P - \gamma\,[[S(G)]^C + 2) \right\rfloor$

Theorem 3.17. $\gamma\,[[S(G)]^C] < P + 1 - \sqrt{1 + 2q}$

Proof. By observation 3.16,$2q < (P - \gamma[[S(G)]^C])^2 + 2(P - \gamma[[S(G)]^C])$

Adding 1 to both sides $2q + 1 < (P - \gamma[[S(G)]^C])^2 + 2(P - \gamma[[S(G)]^C]) + 1$

$= \left((P - \gamma[[S(G)]^C]) + 1\right)^2$

Since$(P - \gamma[[S(G)]^C]) \geq 0$it follows that

$$\sqrt{1 + 2q} < P - \gamma[[S(G)]^C] + 1$$

$$\gamma[[S(G)]^C] < P + 1 - \sqrt{1 + 2q}$$

Observation 3.18. $S(P_2) \cong S(P_2)^c$

For examplecomplement of$S(P_2)$ is shown in the following figure 3.19 and igure3.20

Define 3.21.isomorphism

we define θ : $V(S(P_2)) \rightarrow V(S(P_2)^c)$

by$\begin{pmatrix} v_1 & u_2 & u_1 v_2 \\ u_1^1 & v_1^1 & v_2^1 u_2^1 \end{pmatrix}$and

we define $\emptyset : \; E\,(S(P_2) \rightarrow E\,(\,S(P_2)^c\,)$

by$\begin{pmatrix} e_1 & e_2 e_3 \\ \varepsilon_1 & \varepsilon_2\ \varepsilon_3 \end{pmatrix}$

$\varphi_{S(G)}(e_1) = v_1 u_2$, $\varphi_{S(G)^c}(\emptyset\ (e_1)\ ) =$
$\varphi_{S(G)^c}(\varepsilon_1) = u_1^1 v_1^1 = \theta(v_1)\,\theta(u_2)$

$\varphi_{S(G)}(e_2) = u_2 u_1$, $\varphi_{S(G)^c}(\emptyset\ (e_2)\ ) =$

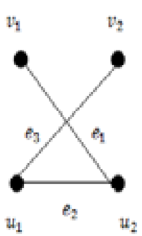

Figure 3.19$S(P_2)$

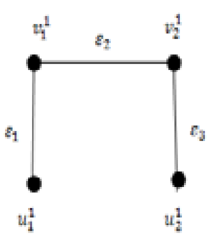

Figure 3.20$[S(P_2)]^c$

$\varphi_{S(G)^c}(\varepsilon_2) = v_1^1 v_2^1 = \theta(u_2)\,\theta(u_1)$

$\varphi_{S(G)}(e_3) = u_1 v_2$, $\varphi_{S(G)^c}(\emptyset\ (e_3)\ ) =$
$\varphi_{S(G)^c}(\varepsilon_3) = v_2^1 u_2^1 = \theta(u_1)\,\theta(v_2)$

$(\,\theta,\ \emptyset\,)$ is an isomorphism

$S(P_2) \;\cong\; S(P_2)^c$

Note 3.22. No other graph example, wil satisfy the above isomorphism observation 3.18.

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