



Fuzzy Time Series Model and ARIMA Model - A Comparative Study

KEYWORDS

ARIMA model, Fuzzy time series model, Export data.

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ABSTRACT Forecasting or predicting is an essential tool in any decision making processes. The quality of forecasts management can make is strongly related to the information that be extracted and used from the past data. A comparative study has been carried out for Indian export data with two vast applications of forecasting namely, ARIMA time series Model and Fuzzy time series Model. Models are discussed for the fuzzy time series method includes the Heuristic Model, First Order Time – Invariant Fuzzy Time Series and Fuzzy Markov Chain. In This paper concluded that the fuzzy time series models is predict the values more accurately when the small set of data (when the sample period is shorter) than ARIMA models.

INTRODUCTION

India is the country which is relative to the small domestic market and nature resources. The economic development of India relies heavily on international trades. Foreign trade has been the engine of India's rapid growth during past forty years. India's economy relies related to export, thus it depends on an open world trade system and relies susceptible to down turns in the world economy. Indian countenances many of the economic concerns as other developed countries. With the panorama of continued repositioning of labor-intensive industries to economies with low-priced work forces, such as in China and Southeast Asia. Taiwan has recovered quickly from the global financial crisis of 2007-10 and its economy has been growing steadily and the unemployment rate had fallen to 4.73% in the year of 2011. Industrial output reached to 19.37% in 2010, it indicated that the strong experts and a growing local economy. Export composition changed from

predominantly agricultural commodities to industrial goods (presently 98%). Indian's most important industrial export sector and is the largest recipient of U.S. investment. Given the highly dependent nature on the export business of Indian, be aware of towards the future trends in the international trade to boost up the market competition is an essential task. Hence, the most accurate forecast has a major role and predicting a suitable model to support and endorse the economic policies and business strategies particularly in the international export trade.

Forecasting plays a vital role in any decision making processes in many of the fields like industries, government policies and so on. Time series analysis is one of the quantitative methods used to determine patterns in data collected over time period. It is used to detect patterns of change in statistical information over regular intervals of time. In recent years, due to

increasing number of innovations in forecasting techniques and improvement of forecasting accuracy, the forecasting methodology becomes an important one to many of the fields. Traditionally, most of the forecasting methods usually construct a statistical model as a tool for forecasting future value and data analysis. In forecasting, the main objective is to recognize a best model through the historical data. However, the high level of uncertainty occurs due to error caused in data collection, time lag and reciprocal effect between variables. These time series data does not appear with the accurate number, however they represent credible values contained by their intervals. In this situation the traditional forecasting methods may possibly results the over-fit of the model and over-explained about the data. New methods have to be developed to evaluate a model for the data.

Zadeh (1965) initially describes the Fuzzy Logic system to solve the problem of linguistic data (like medium, high or low). He defines "A fuzzy algorithm is an ordered set of fuzzy instructions which upon execution yield an approximate solution to a specified problem". The notion of fuzzy set is broadly applied in many of the fields like education, medicine, economics and so on to obtain a elucidation for the linguistic data. In recent times, the initiative of fuzzy logic has been fruitfully well-designed with the forecasting methods. In This Paper, the export data for India is considered. The data is obtained from Handbook of reserve bank of India, during June 1, 2002 to March 3, 2014.

RELATED LITERATURES

Numerous literatures are available based on fuzzy theory, Song and Chissom (1993a, 1993b, 1994) evaluate the model with the one-factor fuzzy time series, Sullivan and Woodall (1994) reviewed two related models, the first-order-time-invariant fuzzy time series model and the first-invariant model proposed by Sing and Chissom, where the two models are with each other and with a time-invariant Markov model

using linguistic tag with probability distributions. Chen (1996) projected a new notion of time series method more efficient model than the Sing and Chissom (1993a). The projected method of Chen (1996) used to simplify the arithmetic operations relatively than the thorny of the maximum composition operation which is defined by Sing and Chissom (1993a). Using Chen's method one can define the variations of enrolment of the current year with the past year data and also he defined the systematic calculations for evaluate the degree of variations in the enrolment data. He can forecast the enrolments with the derived relation. Lee, Wang and Chen (2006), has proposed the high-order fuzzy time series and genetic simulated annealing techniques. Huarng (2001a 2002b) proposed heuristic models by chen's model is easy to calculate, straightforward to integrate heuristic knowledge, Box Jenkins (2006) Explain stochastic models, forecasting and intervention analysis models and outlier detection, since integrating problem-specific heuristic knowledge with Chen's model to improve forecasting, and could forecast better than others. These proposed techniques are used to adjust the length of each internal in the universe of discourse to increase the forecasting accuracy rate. Though, some of the studies concentrated to consider the aptitude of traditional time series and fuzzy time series.

In this paper, an endeavor has been made with the use of export values in India. With the use of the India export data shown that the fuzzy time series analysis is certainly forecast the macro economic variables. Fuzzy time series has been compared with the traditional forecasting method ARIMA time series method, it results a clear way of understanding the suitability of forecasting models. In addition, this article intentions to recognize which model gives more information for decision making, predominantly the data is inadequate. The consequences of the data may perhaps to improve the quality of perceptive of how the time periods affects the

forecasting performance, through analyzing the ARIMA time series model and Fuzzy time series model.

METHODOLOGY

3.1. ARIMA model

The ARIMA time series model was initiated by Box-Jenkins in 1970, which scrutinize each variable by using auto regression, AR (P) and Moving average, MA (Q) to explore the chronological data and economic vacillations.

Definition 3.1.1

A time series model for the observed data $\{X_t\}$ is a specification of the joint distributions (or possibly only the means and covariance) of a sequence of random variables $\{X_t\}$ of which is postulated to be a realization.

The algorithm of the ARIMA time series model is as follows:

1. Data preparation.
2. Developing a Box-Jenkins model to resolve if the series is stationary and seasonality to be modelled if there is any significant. The autocorrelation functions (ACF) is used to characterize the distribution of the given sample data.
3. Stationary attains via differencing.
4. Identification: Identifies the period of the series by the use of autocorrelation functions (ACF) and partial autocorrelation functions (PACF).
5. Estimation: Parameters are estimated using the condition likelihood and exact likelihood methods.
6. Model diagnosing: The method of model diagnosing is to test the assumptions of the given model and also to recognize the areas where the model is insufficient.
7. Statistical credentials: This process includes two steps is define as follows,
 - i. Test whether the parameters attain the statistical significance or multicollinearity.

- ii. Test whether the residual term is white noise or not.

3.2. Fuzzy Time Series model

Definition 3.2.1

If X is a collection of objects denoted generically by x , then a fuzzy set \bar{A} in X is a set of ordered pairs:

$$\bar{A} = \left\{ \left(x, \mu_{\bar{A}}(x) \right) / x \in X \right\}$$

$\mu_{\bar{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in \bar{A} that maps X to the membership space M (When M contains only the two points 0 and 1, \bar{A} is nonfuzzy and $\mu_{\bar{A}}(x)$ is identical to the characteristic function of a nonfuzzy set). The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

Definition 3.2.2

A fuzzy set is a class of objects with a continuum of grade of membership. Let U be the universe of discourse with $U = \{u_1, u_2, \dots, u_n\}$ where u_i are possible linguistic values of U , then a fuzzy set of linguistic variables A_i of U is defined by

$$A_i = f_{A_i} \left(\frac{u_1}{u_1} \right) + f_{A_i} \left(\frac{u_2}{u_2} \right) + \dots + f_{A_i} \left(\frac{u_n}{u_n} \right),$$

where, f_{A_i} is membership function of fuzzy set, $A_i, f_{A_i} : U \rightarrow [0,1]$. u_k is an element of fuzzy set A_i , and $f_{A_i}(u_k)$ is the degree of belongingness of u_k to A_i . $f_{A_i}(u_k) \in [0,1]$ and $1 \leq k \leq n$.

Definition 3.2.3

Let $Y(t)$, ($t = 0, 1, 2, \dots$), is a subset of real number R be the universe of discourse on which fuzzy sets $f_i(t)$, $i = 1, 2, \dots$ are defined and $F(t)$ is the group of $f_i(t)$, $i = 1, 2, \dots$ then $F(t)$ is called a fuzzy time series of $Y(t)$, $t = 0, 1, 2, \dots$,

Definition 3.2.4

Suppose $F(t)$ is caused only by $F(t-1)$ and is denoted by $F(t-1) \rightarrow F(t)$, then there is a fuzzy relationship between $F(t)$ and $F(t-1)$ which can be expressed as the fuzzy relational equation, $F(t) = F(t-1) \circ R(t, t-1)$ where "0" is Max-Min composition operation. The relation R is called the first-order of $F(t)$. Further if fuzzy relation $R(t, t-1)$ of $F(t)$ is independent of time t , meaning for different times t_1 and t_2 , $R(t_1, t-1)$, then $F(t)$ is called a time invariant fuzzy time series.

3.3 Markov Chain Model

Definition 3.3.1

Let U be the universe of discourse, with $G = (l_1, \dots, l_r)$, $H = (v_1, \dots, v_i)$, and $\{P_i, i = 1, 2, \dots, r\}$ defined as an ordered partition of U . μ_i and ν_i are the membership functions on the universe of the fuzzy set U . The relation between fuzzy groups G and H is defined as

$$R = G^T \circ H = [R_{ij}]_{r \times r}$$

where '0' is the max - min operator, T the transpose, R_{ij} the membership function between G and H .

Definition 3.3.2

(First-order multi-variant fuzzy auto-regressive series). The series $(FX_{1,t}, FX_{2,t}, \dots, FX_{k,t})$ is defined as first-order multi-variant fuzzy auto-regressive if the equation below is satisfied:

$$(FX_{1,t}, FX_{2,t}, \dots, FX_{k,t}) = (FX_{1,t-1}, FX_{2,t-1}, \dots, FX_{k,t-1})$$

$$\begin{bmatrix} \mathcal{F}_{11} & \dots & \mathcal{F}_{1k} \\ \vdots & \ddots & \vdots \\ \mathcal{F}_{k1} & \dots & \mathcal{F}_{kk} \end{bmatrix}, i = 1, 2, \dots, k, j = 1, 2, \dots, k$$

where R_{ij} is the Markov fuzzy relation matrix. Since the series $(FX_{1,t}, FX_{2,t}, \dots, FX_{k,t})$ completely depends on the status of $(FX_{1,t-1}, FX_{2,t-1}, \dots, FX_{k,t-1})$, Eq. (11) is defined as a Markov multivariate fuzzy process.

The algorithm for the Markov fuzzy time series model

- Step 1: Define the universe of discourse.
- Step 2: Maximize fuzzy membership function.
- Step 3: Decide the order of the fuzzy auto-regressive model.
- Step 4: Compute Markov fuzzy relation matrix.
- Step 5: Forecast.

3.4 First-Order Time-Invariant Fuzzy Time Series Model

Definition 3.4.1 Time - Invariant Fuzzy Time Series

Suppose $F(t)$ is caused only by $F(t-1)$ and is denoted by $F(t-1) \rightarrow F(t)$; if there exists a fuzzy relationship between $F(t)$ and $F(t-1)$ can be expressed as the fuzzy relational equation

$$F(t) = F(t-1) \circ R(t, t-1)$$

Here "o" is max-min composition operator. The relation R is called first-order model of $F(t)$. Further, if fuzzy relation $R(t, t-1)$ of $F(t)$ is independent of time t , that is to say for different times t_1 and t_2 , $R(t_1, t-1) = R(t_2, t-1)$, then $F(t)$ is called a time - invariant fuzzy time series. Otherwise is called a time - variant fuzzy time series.

The algorithm of the First order time invariant fuzzy time series model is as follows

- Step 1: Define the universe of discourse U .
- Step 2: Divide U into several equal-length intervals.
- Step 3: Define the fuzzy sets on U and fuzzify the historical data.
- Step 4: Derive the fuzzy logical relationships based on the historical data.
- Step 5: Classify the derived fuzzy logical relationships into groups.
- Step 6: Utilize three defuzzification rules to calculate the forecasted values.

3.5 Heuristic Fuzzy Time Series Model

Definition 3.5.1 (Heuristic fuzzy logical relationship).

Suppose that there is a relationship between series $F(t-1)$ and $F(t)$, it is defined as $F(t) = F(t-1) \times R(t-1, t)$, where \times is an operator. Further, the fuzzy logical relationship between series $F(t)$ and $F(t-1)$ is expressed by

$$F(t-1) \rightarrow F(t),$$

Let $F(t) = A_j$ and $F(t-1) = A_i$, a fuzzy logical relationship is defined as $A_i \rightarrow A_j$, where A_j is named as left-hand side, A_i the right-hand side.

Definition 3.5.2 (Heuristic fuzzy logical relationships groups).

The fuzzy logical relationship can be further integrated into fuzzy groups because on the same left-hand sides. For example, there are fuzzy logical relationships with the same left-hand sides (A_i):

$$A_i \rightarrow A_{j1}, A_i \rightarrow A_{j2}, A_i \rightarrow A_{j3},$$

Therefore, these fuzzy logical relationships can be expressed as follows:

$$A_i \rightarrow A_{j1}, A_{j2}, A_{j3}.$$

Definition 3.5.3

In the heuristic models, heuristic function takes fuzzy logical relationship groups and relevant variables as parameters. From these fuzzy logical relationship groups, the heuristic functions use the variables to select proper fuzzy sets to establish heuristic fuzzy logical relationship groups.

All the fuzzy sets $A_1, A_2, A_3, \dots, A_k$ are well ordered. This condition greatly facilitates the selection of proper fuzzy sets by the heuristic function.

Suppose $F(t-1) = A_i$ and the fuzzy logical relationship group for A_i is $A_i \rightarrow A_{j1}, A_{j2}, A_{j3}, \dots$

Proper fuzzy sets, $A_{p1}, A_{p2}, A_{ps}, \dots, A_{pk}$, can be selected by heuristic function $h(x)$, $h(x; A_{j1}, A_{j2}, A_{j3}, \dots) = A_{p1}, A_{p2}, A_{ps}, \dots, A_{pk}$, where x are heuristic variable; $A_{p1}, A_{p2}, A_{ps}, \dots, A_{pk}$, are selected from $A_{j1}, A_{j2}, A_{j3}, \dots$, by the heuristic function. A heuristic fuzzy logical relationship group is obtained below:

The model algorithm of heuristic fuzzy time series can be presented as follows:

Step 1: Decide the universe of discourse and the length of interval.

Step 2: Define the fuzzy sets and establish the fuzzy logical relationships.

Step 3: Define the second and third variable.

Step 4: Redefine the fuzzy logical groups and the threshold auto-regression model.

Step 5: Forecast and defuzzify.

4 EMPIRICAL RESULTS:

All the data which are considered for the study was congregated from the AREMOS economic database. The given data were the amount of Indian export during the period between January 2002 and 30 March 2014 and 147 data were collected and shown in the table 1. The fuzzy time series model considered comprises three models, namely, Heuristic model, Fuzzy Markov Chain and First-Order Time-Invariant. For the purpose of similarity, the ARIMA time series model was used to compare the forecast prophecies, the mean square error (MSE) was calculated to measure the performance capability of the models.

The ARIMA model

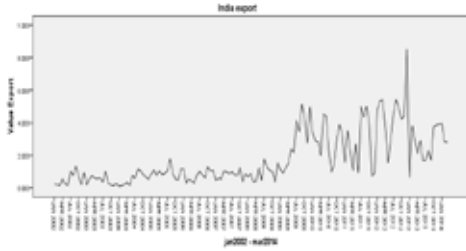


Figure 1: Indian export January 2002 to March 2014

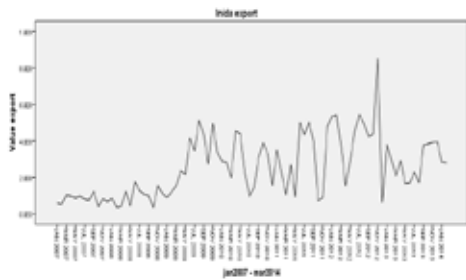


Figure 2: Indian export May 2007 to March 2014

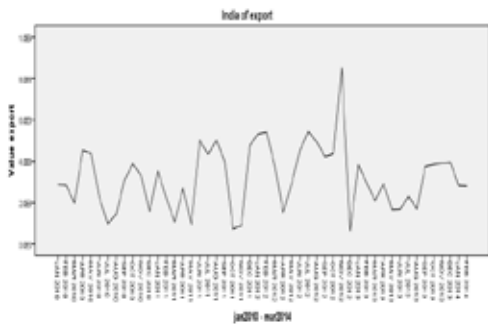


Figure 3: Indian export January 2010 to March 2014



Figure 4: Indian export January 2012 to March 2014

TABLE -1
FOUR MODELS COMPARISON RESULT OF
MSE

ARIMA	First order time invariant model	Fuzzy Markov chain	Heuristic Model
4452894.6849	6136930.7329	5805277.5	7452554.651

TABLE 2: THIS THREE SAMPLE TEST PERIODS
MEASURED FROM FOUR DIFFERENT MODELS
OF THE MSE

Period	ARIMA	First order time invariant model	Fuzzy Markov Chain	Heuristic Model
200701-201403	5757373.08	1805277.5	744280.570	576661.605
201001-201403	350082565.2	1905277.5	7375715.78	308068.360
201201-201403	3529698.02	2805277.5	728546.827	656686.925

TABLE D1: Prediction of export data using ARIMA, First-order time-invariant model, Fuzzy Markov chain and Heuristic model the period of 200201 to 201403.

Date	Export	ARIMA	First-order time-invariant model	Fuzzy Markov chain	Heuristic Model
200201	24086626				
200202	19989695	31897214.39	18822381	15635000	19993720
200203	13776585	32038158.31	11333344	13301142	13783439
200204	56101709	30936706.7	54934395	33091307	56109555
200205	23330592	53936363.19	20887351	22119240	23337581
200206	15496356	44652884.28	14329042	11254231	15504342

200207	102328528	43189511.74	99885287	93001756	102338286
200208	78521430	82005275.03	77354116	70400199	78531306
200209	132798173	80559025.91	130354932	122009514	132805067
200210	73208159	108283821.8	72040845	68853464	73216055
200211	19249084	92912435.93	16805843	18773641	19255563
200212	94850229	74486831.51	93682915	71839827	94857216
200301	19048456	34266193.12	16605215	17837104	19058331
200302	56173952	36314056.58	55006638	51931827	56183819
200303	73542539	54532754.08	71099298	64215767	73552211
200304	59471526	67147612.03	58304212	51350295	59541399
200305	56016486	68036377.12	53573245	45227827	56024338
200306	62991640	71392021.41	61824326	58636945	62999605
200307	36607617	78045903.12	34164376	36132174	36614509
200308	102050557	71847829.54	100883243	79040155	102058453
200309	29025724	98913468.68	26582483	27814372	29035492
200310	17419033	76748158.76	16251719	13176908	17429030
200311	13444094	72762292.14	11000853	4117322	13451883
200312	27904523	70907696.1	26737209	23549828	27911282
200401	9415603	6671209.021	6972362	8940160	9425492
200402	13524116	26677028.62	12356802	-9486286	13533113
200403	21152783	32049754.59	18709542	19941431	21162648
200404	34710359	38641031.46	33543045	30468234	34717347
200405	15784132	47622308.66	13340891	6457360	15794109
200406	74911146	44616115.77	73743832	66789915	74920833
200407	58311262	69885404.52	55868021	47522603	58320141
200408	115802459	69695354.11	114635145	111447764	115812334
200409	105503278	96200700.48	103060037	105027835	105512157
200410	83224854	99989580.1	82057540	60214452	83233411
200411	66373366	97672451.03	63930125	65162014	66381225
200412	53839773	95305042.04	52672459	49597648	53846730
200501	84714769	23789084.22	82271528	75387997	84721663
200502	109700983	63686492.09	108533669	101579752	109708847
200503	77317000	81172914.53	74873759	66528341	77326647
200504	103520351	77436987.78	102353037	99165656	103530035
200505	83131468	93130028.34	80688227	82656025	83141152
200506	88333976	92060533.57	87166662	65323574	88340951
200507	105275107	98750439.06	102831866	104063755	105282038
200508	178509540	109787498.5	177342226	174267415	178517431
200509	88948973	143064454.8	86505732	79622201	88955945
200510	53499986	118119689.2	52332672	49145291	53506928
200511	49293547	107042264.2	46850306	48818104	49300342
200512	115684499	105353663.3	114517185	92674097	115691474
200601	116159444	61237250.57	113716203	114948092	116166375
200602	31366680	92069463.12	30199366	27124555	31373577
200603	53225152	67741886.78	50781911	43898380	53232046
200604	42067798	77892886.37	40900484	33946567	42073674
200605	30314173	76621390.32	27870932	19525514	30317870
200606	80665260	74064775.67	79497946	76310565	80667755
200607	101213556	94364867.31	98770315	96858861	101217245
200608	80263620	106435709	79096306	79788177	80265206
200609	63191381	103482931.2	60748140	40180979	63193866
200610	129178948	100095757.7	128011634	127967596	129182069
200611	104254069	127129485.8	101810828	100011944	104255270
200612	106989229	123427831.3	105821915	97662457	106990350

200701	47257682	58817752.1	44814441	39136451	47264573
200702	59560807	65868640.88	58393493	48772148	59562055
200703	54148486	74249772.6	51705245	49793791	54150597
200704	103759200	76249755.94	102591886	103283757	103760210
200705	98584222	98578726.64	96140981	75573820	98585452
200706	88108645	102792728.3	86941331	86897293	88109895
200707	98813690	103968914.4	96370449	94571565	98815821
200708	81387591	112030505.8	80220277	72060819	81388711
200709	76546065	109576215.8	74102824	68424834	76547167
200710	123604669	110352959.9	122437355	112816010	123605770
200711	40793160	130451620.9	38349919	36438465	40794261
200712	85290232	103579308.5	84122918	84814789	85291353
200801	67559125	50340125.55	65115884	44548723	67560137
200802	87512042	72100536.7	86344728	86300690	87513256
200803	36146593	84921322.35	33703352	31904468	36147714
200804	41350123	71128126.48	40182809	32023351	41351244
200805	123109607	75034954.11	120666366	118754912	123110728
200806	45429138	108466805.7	44261824	44953695	45430239
200807	175790470	85860105.99	173347229	152780068	175791595
200808	127639402	137227302.4	126472088	126428050	127641612
200809	107365300	128511370.9	104922059	103123175	107365513
200810	100477317	126282818.7	99310003	91150545	100478440
200811	35752304	127264571.2	33309063	27631073	35753406
200812	154210699	105204393.2	153043385	143422040	154211900
200901	111724295	79643516.32	109281054	107369600	111724307
200902	91664629	96134259.06	90497315	91189186	91666641
200903	121021394	95746869.25	118578153	98010992	121025419
200904	153814588	112226439.7	152647274	152603236	153821442
200905	237366492	131400942.7	234923251	233124367	237374338
200906	218074145	171514554.5	216906831	208747373	218081134
200907	415717435	176791836.6	413274194	407596204	415725421
200908	345841762	262716122.2	344674448	335053103	345851520
200909	514135536	257887979.1	511692295	509780841	514145412
200910	435924361	337532789.9	434757047	435448918	435931255
200911	274486140	332026660.1	272042899	251475738	274494036
200912	495269484	286805285.7	494102170	494058132	495275963
201001	336815638	306626732.5	334372397	332573513	336822625
201002	287223344	290790824.6	27870932	277896572	287233219
201003	284136865	282318894	79497946	279782170	284146732
201004	199641619	287305087.9	98770315	199166176	199651291
201005	454096531	260179510.8	79096306	431086129	454166404
201006	438515074	357032517	60748140	437303722	438522926
201007	155913154	265592540.1	286056030	155437711	155919913
201008	351880573	230783852.5	281693624	328870171	351890462
201009	222363702	303654710.3	198474305	221152350	222372699
201010	105813239	264738399.5	451653290	101571114	105823104
201011	269574017	221119290.7	437347760	260247245	269581005
201012	93951224	277307194.7	220313211	85829993	93961201
201101	501535271	214714007.4	96526262	490746612	501544958
201102	435064887	363911978.9	142657485	430710192	435073766
201103	501378285	358747943.5	307893345	500902842	501388160

201104	399403539	398003636.8	385956049	376393137	399412418
201105	72593907	375551803.4	327743516	71382555	72602464
201106	91223498	258299232.4	153469913	86981373	91231357
201107	478209654	181997603.9	350713259	468882882	478216611
201108	532489834	345678507.6	219920461	524368603	532496728
201109	540132221	386321268.5	104645925	529343562	540140085
201110	375496808	410518884.6	267130776	371142113	375506455
201111	154100637	366817783.6	92783910	153625194	154110321
201112	292955029	288407728.6	499092030	269944627	292964713
201201	453803363	333005033.2	433897573	452592011	453810338
201202	543093613	396115558.4	498935044	538851488	543100544
201203	494532748	442200666.6	398236225	485205976	494540639
201204	422906846	439986931.8	70150666	418552151	422913818
201205	437252680	423967672.9	90056184	436777237	437259622
201206	851956964	435008020.8	475766413	828946562	851963759
201207	64484792	530823269.1	531322520	63273440	64491767
201208	381623687	287690485.5	537688980	377381562	381630618
201209	293765465	391225215.2	374329494	284438693	293772362
201210	209168229	362121385.3	151657396	201046998	209175123
201211	291568311	327991810.3	291787715	280779652	291574187
201212	167339754	352573080.7	451360122	162985059	167343451
201301	169560799	304016195.1	541926299	169085356	169563294
201302	229788376	295912789.7	492089507	206777974	229792065
201303	170350916	311178648.2	421739532	169139564	170352502
201304	375669158	285314180.9	434809439	371427033	375671643
201305	384798879	357490800.8	850789650	375472107	384802000
201306	391055968	368584483	62041551	382934737	391057169
201307	395516643	309107298.8	380456373	384727984	395517764
201308	282841269	344407630.4	291322224	278486574	282848160
201309	280313940	312264077.6	208000915	279838497	280315188
201310	155913154	265592540.1	289125070	155437711	155919913
201311	351880573	230783852.5	166172440	328870171	351890462
201312	222363702	303654710.3	167117558	221152350	222372699
201401	105813239	264738399.5	228621062	101571114	105823104
201402	269574017	221119290.7	167907675	260247245	269581005
201403	93951224	277307194.7	374501844	85829993	93961201
MSE		4452894.6849	6136930.7329	5805277.5	7452554.651

Table D2: Prediction of export data using ARIMA, First-order time-invariant model, Fuzzy Markov chain and Heuristic model the period of 200701 to 201403.

Date	Export	ARIMA	First-order time-invariant model	Fuzzy Markov chain	Heuristic Model
200701	47257682				
200702	59560807	65868640.88	58393493	48772148	59562055
200703	54148486	74249772.6	51705245	49793791	54150597
200704	103759200	76249755.94	102591886	103283757	103760210
200705	98584222	98578726.64	96140981	75573820	98585452
200706	88108645	102792728.3	86941331	86897293	88109895
200707	98813690	103968914.4	96370449	94571565	98815821

200708	81387591	112030505.8	80220277	72060819	81388711
200709	76546065	109576215.8	74102824	68424834	76547167
200710	123604669	110352959.9	122437355	112816010	123605770
200711	40793160	130451620.9	38349919	36438465	40794261
200712	85290232	103579308.5	84122918	84814789	85291353
200801	67559125	50340125.55	65115884	44548723	67560137
200802	87512042	72100536.7	86344728	86300690	87513256
200803	36146593	84921322.35	33703352	31904468	36147714
200804	41350123	71128126.48	40182809	32023351	41351244
200805	123109607	75034954.11	120666366	118754912	123110728
200806	45429138	108466805.7	44261824	44953695	45430239
200807	175790470	85860105.99	173347229	152780068	175791595
200808	127639402	137227302.4	126472088	126428050	127641612
200809	107365300	128511370.9	104922059	103123175	107365513
200810	100477317	126282818.7	99310003	91150545	100478440
200811	35752304	127264571.2	33309063	27631073	35753406
200812	154210699	105204393.2	153043385	143422040	154211900
200901	111724295	79643516.32	109281054	107369600	111724307
200902	91664629	96134259.06	90497315	91189186	91666641
200903	121021394	95746869.25	118578153	98010992	121025419
200904	153814588	112226439.7	152647274	152603236	153821442
200905	237366492	131400942.7	234923251	233124367	237374338
200906	218074145	171514554.5	216906831	208747373	218081134
200907	415717435	176791836.6	413274194	407596204	415725421
200908	345841762	262716122.2	344674448	335053103	345851520
200909	514135536	257887979.1	511692295	509780841	514145412
200910	435924361	337532789.9	434757047	435448918	435931255
200911	274486140	332026660.1	272042899	251475738	274494036
200912	495269484	286805285.7	494102170	494058132	495275963
201001	336815638	306626732.5	334372397	332573513	336822625
201002	287223344	290790824.6	286056030	277896572	287233219
201003	284136865	282318894	281693624	279782170	284146732
201004	199641619	287305087.9	198474305	199166176	199651291
201005	454096531	260179510.8	451653290	431086129	454166404
201006	438515074	357032517	437347760	437303722	438522926
201007	222756452	367069038.9	220313211	218514327	222764417
201008	97693576	297529207.3	96526262	88366804	97700468
201009	145100726	247181050.4	142657485	136979495	145108622
201010	309060659	255053843.9	307893345	298272000	309070427
201011	388399290	311890118.2	385956049	384044595	388409287
201012	328910830	347689390.1	327743516	324556135	328918619
201101	155913154	265592540.1	153469913	155437711	155919913
201102	351880573	230783852.5	350713259	328870171	351890462
201103	222363702	303654710.3	219920461	221152350	222372699
201104	105813239	221119290.7	104645925	101571114	105823104
201105	269574017	277307194.7	267130776	260247245	269581005
201106	93951224	214714007.4	92783910	85829993	93961201
201107	501535271	363911978.9	499092030	490746612	501544958
201108	435064887	358747943.5	433897573	430710192	435073766
201109	501378285	398003636.8	498935044	500902842	501388160
201110	399403539	375551803.4	398236225	376393137	399412418
201111	72593907	258299232.4	70150666	71382555	72602464
201112	91223498	181997603.9	90056184	86981373	91231357
201201	478209654	345678507.6	475766413	468882882	478216611
201202	532489834	386321268.5	531322520	524368603	532496728

201203	540132221	410518884.6	537688980	529343562	540140085
201204	375496808	366817783.6	374329494	371142113	375506455
201205	154100637	288407728.6	151657396	153625194	154110321
201206	292955029	333005033.2	291787715	269944627	292964713
201207	453803363	396115558.4	451360122	452592011	453810338
201208	543093613	442200666.6	541926299	538851488	543100544
201209	494532748	439986931.8	492089507	485205976	494540639
201210	422906846	423967672.9	421739532	418552151	422913818
201211	437252680	435008020.8	434809439	436777237	437259622
201212	851956964	530823269.1	850789650	828946562	851963759
201301	64484792	287690485.5	62041551	63273440	64491767
201302	381623687	391225215.2	380456373	377381562	381630618
201303	293765465	362121385.3	291322224	284438693	293772362
201304	209168229	327991810.3	208000915	201046998	209175123
201305	291568311	352573080.7	289125070	280779652	291574187
201306	167339754	304016195.1	166172440	162985059	167343451
201307	169560799	295912789.7	167117558	169085356	169563294
201308	229788376	311178648.2	228621062	206777974	229792065
201309	170350916	285314180.9	167907675	169139564	170352502
201310	375669158	357490800.8	374501844	371427033	375671643
201311	384798879	368584483	382355638	375472107	384802000
201312	391055968	309107298.8	389888654	382934737	391057169
201401	395516643	344407630.4	393073402	384727984	395517764
201402	282841269	312264077.6	281673955	278486574	282848160
201403	280313940	221119290.7	277870699	279838497	280315188
MSE		5757373.081	1805277.5	744280.570	576661.605

Table D3: Prediction of export data using ARIMA, First-order time-invariant model, Fuzzy Markov chain and Heuristic model the period of 201001 to 201403.

Date	Export	ARIMA	First-order time-invariant model	Fuzzy Markov chain	Heuristic Model
201001	336815638				
201002	287223344	281146.0012	286056030	279782170	287233219
201003	284136865	288240.3798	281693624	199166176	284146732
201004	199641619	305727.7536	198474305	431086129	199651291
201005	454096531	309744.2873	451653290	437303722	454166404
201006	438515074	315216.9991	437347760	218514327	438522926
201007	222756452	319918.7283	220313211	88366804	222764417
201008	97693576	322581.1622	96526262	136979495	97700468
201009	145100726	330185.3437	142657485	298272000	145108622
201010	309060659	335194.0219	307893345	384044595	309070427
201011	388399290	330899.4452	385956049	324556135	388409287
201012	328910830	338424.0553	327743516	155437711	328918619
201101	155913154	284193.7166	153469913	328870171	155919913
201102	351880573	296641.4691	350713259	221152350	351890462
201103	222363702	290345.8422	219920461	101571114	222372699
201104	105813239	240435.5351	104645925	260247245	105823104
201105	269574017	259659.9203	267130776	85829993	269581005
201106	93951224	284867.9774	92783910	490746612	93961201
201107	501535271	298388.8039	499092030	430710192	501544958
201108	435064887	309037.0521	433897573	500902842	435073766
201109	501378285	322305.6375	498935044	376393137	501388160
201110	399403539	323517.8579	398236225	71382555	399412418
201111	72593907	337879.1596	70150666	86981373	72602464

201112	91223498	349578.4498	90056184	468882882	91231357
201201	478209654	305550.6429	475766413	524368603	478216611
201202	532489834	324953.7844	531322520	529343562	532496728
201203	540132221	319845.0908	537688980	371142113	540140085
201204	375496808	335878.7021	374329494	153625194	375506455
201205	154100637	349830.3707	151657396	269944627	154110321
201206	292955029	367112.4633	291787715	452592011	292964713
201207	453803363	375799.5921	451360122	538851488	453810338
201208	543093613	390695.2738	541926299	485205976	543100544
201209	494532748	397270.7682	492089507	418552151	494540639
201210	422906846	403780.1196	421739532	436777237	422913818
201211	437252680	414392.5856	434809439	828946562	437259622
201212	851956964	418297.9807	850789650	63273440	851963759
201301	64484792	370097.9093	62041551	377381562	64491767
201302	381623687	365088.2356	380456373	284438693	381630618
201303	293765465	362172.6469	291322224	201046998	293772362
201304	209168229	371608.7711	208000915	280779652	209175123
201305	291568311	373634.4581	289125070	162985059	291574187
201306	167339754	370103.6436	166172440	169085356	167343451
201307	169560799	371201.5419	167117558	206777974	169563294
201308	229788376	368788.0871	228621062	169139564	229792065
201309	170350916	364866.5542	167907675	371427033	170352502
201310	375669158	357095.7439	374501844	375472107	375671643
201311	384798879	371028.9861	382355638	382934737	384802000
201312	391055968	371932.305	389888654	384727984	391057169
201401	395516643	323970.0107	393073402	278486574	395517764
201402	282841269	332698.3428	281673955	279838497	282848160
201403	280313940	326758.5085	277870699	277896572	280315188
MSE		320082565.2	1905277.5	7375715.78	358068360

Table D4: Prediction of export data using ARIMA, First-order time-invariant model, Fuzzy Markov chain and Heuristic model the period of 201201 to 201403.

Date	Export	ARIMA	First-order time-invariant model	Fuzzy Markov chain	Heuristic Model
201201	478209654				
201202	532489834	370838.4224	531322520	524368603	532496728
201203	540132221	327673.6333	537688980	529343562	540140085
201204	375496808	344094.8448	374329494	371142113	375506455
201205	154100637	354847.3062	151657396	153625194	154110321
201206	292955029	367735.4679	291787715	269944627	292964713
201207	453803363	373397.1459	451360122	452592011	453810338
201208	543093613	383117.4054	541926299	538851488	543100544
201209	494532748	388064.4024	492089507	485205976	494540639
201210	422906846	392988.1137	421739532	418552151	422913818
201211	437252680	399745.2257	434809439	436777237	437259622
201212	851956964	403833.1733	850789650	828946562	851963759
201301	64484792	342978.7874	62041551	63273440	64491767
201302	381623687	344867.4561	380456373	377381562	381630618
201303	293765465	347037.5593	291322224	284438693	293772362
201304	209168229	352617.146	208000915	201046998	209175123
201305	291568311	356239.6219	289125070	280779652	291574187
201306	167339754	358300.0064	166172440	162985059	167343451
201307	169560799	361269.3168	167117558	169085356	169563294
201308	229788376	363340.142	228621062	206777974	229792065

201309	170350916	364917.6548	167907675	169139564	170352502
201310	375669158	365504.1371	374501844	371427033	375671643
201311	384798879	370231.2826	382355638	375472107	384802000
201312	391055968	372778.7781	389888654	382934737	391057169
201401	395516643	311541.8312	393073402	384727984	395517764
201402	282841269	316016.6226	281673955	278486574	282848160
201403	280313940	318101.9411	277870699	279838497	280315188
MSE		3529698.02	2805277.5	728546.827	656686.923

CONCLUSION

The past literature discloses that India export values is preponderate influenced by the exterior factors but not by the past experience. The export values confirm an increasing trend. This present article suggested that the time periods are influenced by the lag periods lag1, lag 12 and lag 13 which are the prior period to test. The first lag period found that it is having the greatest power. The results shown that, the testing time periods are large that provides smaller MSE while using the ARIMA time series model. In this situation, it is recommended that ARIMA time series model be able to reimburse and equilibrium vacillations of export values over the period of time. Testing time periods occurs, the fuzzy time series model is more appropriate than the ARIMA time series model.

It gives the most accurate result when the testing period is small. In shorter testing period, the fuzzy series model represents a dwindling trend of the MSE. As a result, when dealing with the lack of data observations or when necessitates more timely forecasts, the fuzzy time series models will be a very constructive model and more appropriate tool to apply for the prediction.

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