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The Non-Simplicity of Simple Pendulum

KEYWORDS: Nonlinear, Elliptic function, feasible, large amplitudes, sine of an angle, linearization.

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ABSTRACT *Perchance one of the nonlinear systems the majority premeditated and investigated is the simple pendulum. The periodic steps forward revealed by a simple pendulum is harmonic only for tiny angle oscillations. Further than this limit, the equation of motion is nonlinear. The simple harmonic motion is inadequate to sculpt the oscillation motion for hefty amplitudes and in such cases the period depends on amplitude. Appliance of Newton's second law to this physical system furnishes a differential equation with a nonlinear term (the sine of an angle). It is feasible to discover the integral articulation for the period of the pendulum and to articulate it in terms of elliptic functions. Even though it is potential in numerous cases to restore the nonlinear differential equation by an analogous linear differential equation that approximates the source equation, such linearization is not forever reasonable. In such cases, the genuine nonlinear differential equation ought to be honestly treated with.*

1. Introduction:

There have been two chief fashions in the chronological maturity of differential equations [1]. The first trend is illustrated by challenges to discover explicit elucidations, either in congested form which is infrequently possible or in requisites of power series. In the second, one abandons all hope of solving equations in any traditional sense, and instead concentrates on a search for qualitative information about the general behavior of solutions [2-3]. We imposed this tip of scrutiny to linear equations. But the qualitative hypothesis of nonlinear differential equations is utterly special [4]. The hypothesis of linear differential equations has been premeditated genuinely and expansively for the past 200 years, and is a comparatively complete and pleasing body of awareness [5]. However, exceptionally miniature of general scenery is branded about nonlinear equations.

2. The purpose of study:

Our purpose here is to survey some of the innermost dreams and processes of this focus, and furthermore to make obvious that it presents a ample variety of fascinating and idiosyncratic novel phenomena that do not materialize in the linear hypothesis. Most of these occurrences can be handled somewhat straightforwardly with the exclusive of the aids of chic mathematical utensils and in reality necessitate little more than straightforward differential equations and two dimensional vector algebra [6].

3. Why should one be fascinated in nonlinear differential equations?

The indispensable basis is that numerous physical systems and the equations that portray them are minimally nonlinear from the outset [6]. The accustomed linearization is approximating contrivance that is somewhat affirmation of trounce in the ambush of the original nonlinear problems and partly expressions of the sensible view that half a loaf is better than none [6]. It should be added at once that there are numerous physical circumstances in which a linear approximation is precious and passable for a good number of principles [6]. This does not amend the reality that in various other circumstances linearization is unpardonable [6]. It has been still recommended by Einstein that since the essential equations of physics are nonlinear, all of mathematical physics will have to be prepared for a second time [6].

4. Simple pendulum without the damping force:

It is somewhat uncomplicated to bestow straightforward paradigms of problems that are fundamentally nonlinear [6]. For instance, if '$\theta(t)$' is the angle of deviation of an un-damped pendulum of length 'l_0' whose bob has mass 'm_0', then its equation of motion has been solved and studied earlier using elliptic functions [7]. Therefore we will furnish the exact solutions for the damped pendulum.

5. Simple pendulum with the damping force:

When viscous damping proportional to velocity of the bob is taken into description, the equation of motion has not been deciphered exactly hitherto and hence transform procedures are espoused (Laplace, Fourier...etc.,) to turn up an exact solution using favorable boundary values or initial conditions. If the sine function is not approximated, meticulousness can be brought about, knowing the mass 'm_0' and the damping co-efficient 'C' [6-10].

If there is present a damping force proportional to the velocity of the bob, the equation of motion becomes [6]

$$\frac{d^2[\theta(t)]}{dt^2}+\frac{C}{m_0}\frac{d[\theta(t)]}{dt}+\frac{g}{l_0}\sin[\theta(t)]=0$$

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$$[\ddot{\theta}(t)] + \frac{C}{m_0}[\dot{\theta}(t)] + \frac{g}{l_0}\sin[\theta(t)] = 0$$

- (5.1)

Since Fourier transform is inapplicable in this context, the appropriate transform shall be Laplace transform only. ∴ Applying the Laplace transform on both sides,

$$L[\ddot{\theta}(t)] + \frac{C}{m_0}L[\dot{\theta}(t)] + \frac{g}{l_0}\mathrm{L}[sin[\theta(t)]] = L[0]$$

$$\{s^2L[\theta(t)] - s\,[\theta(0)] - [\dot{\theta}(0)]\} + \frac{C}{m_0}\{sL[\theta(t)] - [\theta(0)]\} + \frac{g}{l_0}\left\{\frac{1}{s^2+1}\right\} = 0$$

- (5.2)

To solve the above equation we will use the following conditions, initial angular displacement $\theta(0) = 0$, and initial angular velocity $\dot{\theta}(0) = 0$.

$$\left[s^2 + \frac{C}{m_0}s\right]L[\theta(t)] = -\frac{g}{l_0}\left[\frac{1}{s^2+1}\right]$$

$$L[\theta(t)] = -\frac{g}{l_0}\left[\frac{1}{\left[s^2 + \frac{C}{m_0}s\right][s^2+1]}\right]$$

$$\theta(t) = -\frac{g}{l_0}L^{-1}\left(\frac{1}{[s]\left[s + \frac{C}{m_0}\right][s^2+1]}\right)$$

- (5.3)

Let $\left(\frac{1}{[s]\left[s + \frac{C}{m_0}\right][s^2+1]}\right) = \left(\frac{A_1}{s} + \frac{A_2}{s + \frac{C}{m_0}} + \frac{A_3 s + A_4}{s^2+1}\right)$

After solving the above, we get

$$A_1 = \frac{m_0}{C},$$

$$A_2 = -\frac{{m_0}^3}{C(C^2 + {m_0}^2)},$$

$$A_3 = \frac{{m_0}^3}{C(C^2 + {m_0}^2)} - \frac{m_0}{C} = -\frac{m_0 C}{(C^2 + {m_0}^2)},$$

$$A_4 = -\frac{{m_0}^2}{(C^2 + {m_0}^2)}$$

Using these values in equation (5.3),

$$\theta(t) = -\frac{g}{l_0}L^{-1}\left(\frac{A_1}{s} + \frac{A_2}{s^2 + \frac{C}{m_0}} + \frac{A_3 s}{s^2+1} + \frac{A_4}{s^2+1}\right)$$

$$\theta(t) = -\frac{g}{l_0}\left[A_1(1) + A_2\exp\left(-\frac{Ct}{m_0}\right) + A_3\cos t + A_4\sin t\right]$$

$$\boldsymbol{\theta(t) = -\frac{g}{l_0}\left[A_1 + A_2\exp\left(-\frac{Ct}{m_0}\right) + A_3\cos t + A_4\sin t\right]}$$

- (5.4)

Using the values of $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$ **and** $\mathbf{A_4}$ in equation (5.4) and simplifying, we get

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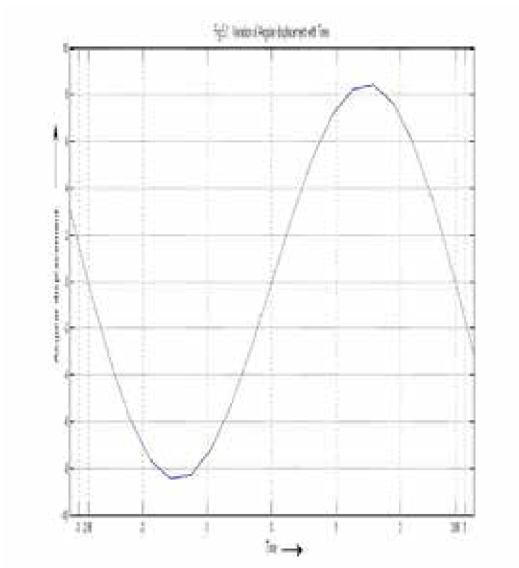
$$\theta(t) = \frac{m_0 g}{l_0}\left[\frac{m_0^{2}\left(\exp\left(-\frac{Ct}{m_0}\right)-1\right)+C^2(\cos t-1)+m_0 C \sin t}{C\,(C^2+m_0^{2})}\right]$$

- (5.5)

Selecting $m_0 = 0.1\,Kg, l_0 = 1m$ in equation (5.5) and also we know the values $g = 9.8\,ms^{-2}$, C = 1.983 x 10^{-5} Nsm^{-2}. Hence we are plotting the graph between '$\theta(t)$'and't' (Fig 5.1).



6. Conclusion:

We can conclude from the equation (5.5) that $\theta(t)$ is **explicitly expressed**, so it is unique. And also we can verify the initial condition by putting t=0 in equation (5.5), then we get $\theta(0) = 0$. Yet another value of 't' is also calculable by graphing the explicit function thus obtained in one cycle ($-\pi$ to π) itself. The interesting problem of how to have a simple pendulum whose maximum swing remains constant for all time, in spite of the presence of damping, does not seem to have been treated before in the literature. By varying the length of the string, the mass of the bob or both the maximum swing of the bob can be held constant.

A. Appendix:

We have used the following formulae for solving the simple pendulum equation with the damping force, F is proportional to $\dot{\theta}(t)$

F = $C\dot{\theta}(t)$

C = Proportionality constant = Viscosity of Air = 1.983 x 10^{-5} Nsm^{-2}

g = $9.8ms^{-2}$

$L[\ddot{\theta}(t)] = s^2 L[\theta(t)] - s[\theta(0)] - [\dot{\theta}(0)]$

$L[\dot{\theta}(t)] = sL[\theta(t)] - [\theta(0)]$

$L[sin\theta(t)] = \frac{1}{s^2+1}$

$L[cos\theta(t)] = \frac{s}{s^2+1}$

$L[\exp(-at)] = \frac{1}{s+a}$

$L[1] = \frac{1}{s}$

The various constants used in equation (5.4) are

$A_1 = \frac{m_0}{C}$,

$$A_2 = -\frac{m_0^{\,3}}{C(C^2+m_0^{\,2})},$$

$$A_3 = \frac{m_0^{\,3}}{C(C^2+m_0^{\,2})} - \frac{m_0}{C},$$

$$A_4 = -\frac{m_0^{\,2}}{(C^2+m_0^{\,2})}$$

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