RESEARCH PAPER | Physics | Volume : 4 | Issue : 12 | Dec 2014 | ISSN - 2249-555X

General Solution of Static Sphere of Perfect Fluid And Dust of Uniform Density Using Isotropic Line Element

KEYWORDS: (4, 4) equation - is an equation which has - 8πT44 term on one side Dust - The distribution of matter in which the only interaction is that of gravity

Dr. MOHAN AMRUTRAO GAIKWAD

ASSST.PROFESSOR OF PHYSICS, DEPART MENT OF PHYSICS, N. B. MEHTA (VALWADA) SCIENCE COLLEGE, BORDI, TAL.DAHANU, , DIST.PALGHAR-401701., MAHARASHTRA STATE

ABSTRACT *The general solution of a static sphere of perfect fluid of uniform density using isotropic line element has been obtained using the additional condition of continuity of $\frac{\partial}{\partial r}\left(e^{\mu}\right)$ at the boundary $r = a$. It is shown that $k^2 f(r)$ is a solution of the (4, 4) equation of Wyman[1], where $f(r)$ is the solution obtained by Wyman and k^2 is an additional integrating constant. If we do not put the condition of continuity of $\frac{\partial}{\partial r}\left(e^{\mu}\right)$ at the boundary $r = a$, then it can be shown that $p \to 0$ as $k^2 \to \infty$ using equation 1.1 of Wyman so the solution of static sphere of dust can be obtained using Wyman's solution (Equation 1.8) by putting $p = 0$ in the solution so that R^2 becomes $R^2 = \frac{3}{8\pi\rho_0}$ for dust instead of $R^2 = \frac{3}{8\pi\rho}$ of fluid obtained by Wyman. Some anomalies discussed in earlier papers of Kelkar and Shrivastav[2] and Kelkar.et .al[3] in 2000 and 2001 and also anomalies discussed in the present paper can be removed by new field equation. The new proposed field equation is given and it is shown that the new proposed equation can bring Newtonian approximation.*

INTRODUCTION: Wyman (1946) has obtained an exact solution in the case of static sphere of perfect fluid of uniform density using isotropic line element;

$$ds^2 = -e^{\mu}\left(dr^2 + r^2 \cdot d\theta^2 + r^2 \mathrm{Sin}^2\theta \cdot d\phi^2\right) + e^{\nu} dt^2 \quad \text{------------- (1)}$$

He has obtained the solution of (4, 4) equation as

$$e^{\mu} = \frac{4R^2}{\left(e^{c} r^2 + e^{-c}\right)^2} \quad \text{------ (2)}$$

with $R^2 = \frac{3}{8\pi\rho}$ and $\rho = \rho_0 + 3p$, where ρ_0 is the proper density[5], e^c is an arbitrary constant, while $\rho = \rho_0 + 3p$ is regarded as a constant in his analysis.

Volume : 4 | Issue : 12 | Dec 2014 | ISSN - 2249-555X

His equation (4, 4) is a second order equation. It will have two constants of integration in the general solution and hence the solution obtained by Wyman[6], though exact, is only a particular solution of (4, 4) equation, since it contains only one constant of integration, namely e^c.

GENERAL SOLUTION OF (4, 4) EQUATION

It is possible to get the second constant of integration by using Lemma given below.

Lemma: If e^{μ} is a solution of the (4, 4) equation of Wyman then any constant multiple of e^{μ} is also a solution of the same equation.

Proof: Suppose $e^{\mu} = f(r)$

then $e^{\mu} \cdot \mu' = f'(r)$

$$\therefore \mu' = \frac{f'(r)}{f(r)}$$

Now put $e^{\mu} = k \cdot f(r)$

$$e^{\mu} \cdot \mu' = k \cdot f'(r) \quad \left[\text{Hint: } e^{\mu}(8\pi\rho) = \mu' + \frac{\mu'^2}{4} + \frac{2\mu'}{r}\right]$$

$$\therefore \mu' = \frac{f'(r)}{f(r)}$$

Hence μ', μ'^2 and μ'' which occur in RHS of (4, 4) equation of Wyman are not altered by putting $e^{\mu} = k \cdot f(r)$

Therefore using this lemma we put

$$e^{\mu} = \frac{4R^2 \cdot k^2}{\left(e^c r^2 + e^{-c}\right)^2} \text{ -------------- (3)}$$

where k^2 is a numerical constant so that we now have two constants of integration

namely k^2 and e^c. These two constants can be determined using continuity of

e^{μ} and $\frac{\partial\left(e^{\mu}\right)}{\partial r}$ at the boundary $r = a$. Latter condition may not be .

Volume : 4 | Issue : 12 | Dec 2014 | ISSN - 2249-555X

necessary This condition was suggested to us by Newtonian approximation given by Eddington[7].

Eddington has shown[8] that the metric $ds^2 = -(1+2\Omega)(dx^2 + dy^2 + dz^2) + (1-2\Omega)dt^2$

gives Newtonian approximation and Ω is approximately Newtonian potential.

Continuity of e^{μ} and its derivative gives

$$e^{c} = \left(\frac{m}{2 \cdot a^3}\right)^{\frac{1}{2}} \text{ and}$$

$$k = \frac{1}{R}\left[\left(\frac{a^3}{2 \cdot m}\right)^{\frac{1}{2}}\left(1 + \frac{m}{2a}\right)^3\right] \qquad \text{------ (4)}$$

Since $R^2 = \dfrac{3}{8\pi\rho}$

therefore $$k = \left(\frac{4\pi\rho a^3}{3m}\right)^{\frac{1}{2}}\left(1 + \frac{m}{2a}\right)^3$$

so that $k \cong 1$

Also it can be seen that e^{ν} given by Wyman, namely

$$e^{\nu} = \left[\frac{Ar^2 + B}{e^{c} r^2 + e^{-c}}\right]^2$$ has already two constants of integration and hence is a general solution.

CONCLUSION

Although we have found the value of the second constant of integration k^2 by matching $\dfrac{\partial\left(e^{\mu}\right)}{\partial r}$ at r = a, there is nothing in Einstein's equation which makes it necessary to match the derivative of e^{μ} at r = a. Hence k^2 can have any numerical value, excluding negative value because e^{μ} is a potential and hence e^{μ} has to be real. In particular we can have $k^2 \to \infty$ and we can show that the pressure tends to zero using the (1.1) equation of Wyman[9]. Hence there can be a static sphere of dust according to Einstein. This is also corroborated by the solution e^{μ} (equation 1.8) of

Volume : 4 | Issue : 12 | Dec 2014 | ISSN - 2249-555X

Wyman[10] which holds well for P = 0 because in that case

$R^2 = \frac{3}{8\pi\rho_0}$ instead of $R^2 = \frac{3}{8\pi\rho}$.

Further k^2 can have any positive value starting from 0 onwards, so that the pressure is

left undecided. It is interesting to note that pressure is given by Eddington[11]

in the case of anisotropic co-ordinate system and is given by the equation.

$$P = \frac{\alpha}{8\pi} \frac{\left[\frac{3}{2}\left(1-\alpha r^2\right)^{\frac{1}{2}} - \frac{3}{2}\left(1-\alpha a^2\right)^{\frac{1}{2}} \right]}{\left[\frac{3}{2}\left(1-\alpha a^2\right)^{-\frac{1}{2}} - \frac{1}{2}\left(1-\alpha r^2\right)^{-\frac{1}{2}} \right]} \quad \text{------------} \quad (5)$$

Also Kelkar and Shrivastav (1999) [12] have found the pressure according to Newton's

theory. They have used the relation $V = \frac{2\pi\rho\left(3a^2 - r^2\right)}{3}$ -------- (6)

$$\frac{\partial V}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}$$

giving $P = \frac{2\pi\rho^2\left(a^2 - r^2\right)}{3}$ ---------- (7)

so that the pressure is given uniquely. Another interesting point in this regard is the

fact that Einstein has converted the hydrodynamic equation $T_{i\ ,k}^{\ k} = 0$ of the G R into

an identity $T_{i\ ,k}^{\ k} \equiv 0$ .But Einstein's field equation for anisotropic line element can be

solved (as will be shown in the papers to follow) without using the equation $T_{i\ ,k}^{\ k} = 0$

. If we put pressure P=0 in the solution of the anisotropic case we can still get the

solution of field equation. Hence for anisotropic line eleme nt also there can be static

sphere of dust according to Einstein. These anomalies obviously can be avoided by taking $T_{i,k}^{k} = 0$ as an independent equation of G.R and making the field equation

different from Einstein's equation. This has been done by Kelkar and Shrivastav[13,14]

by proposing two new field equations as (i) $T^{k}_{i,k} = 0$

$$\text{(ii)}\quad R^{k}_{i} - \frac{1}{2} g^{k}_{i} R = 4\pi\rho_0 g^{k}_{i} + \eta^{k}_{i}$$

where $\eta^{ik} = 4\pi P\left(4\frac{dx^i}{ds}\frac{dx^k}{ds} - g^{ik}\right)$ so that $g^{ik}\eta_{ik} = 0$ and pressure is negligible as

compared with proper density ρ_0

Hence $R^{ik} \cong -4\pi\rho_0 g^{ik}$ and

$R_{44} \cong -4\pi\rho_0 g_{44} \cong -4\pi\rho_0$

This brings about Newtonian approximation.

It is interesting to note here that Einstein's theory is not capable of Newtonian approximation in the case of rotation and radial motion of a star discussed already by Kelkar et al[15]

Volume : 4 | Issue : 12 | Dec 2014 | ISSN - 2249-555X

REFERENCE

[1] Wyman : Phy. Review. Volume 70. 1946 page no. 74. Equation 1.3
[1] Wyman : Phy. Review. Volume 70. 1946 page no. 74. Equation 1.3
$$8\pi p = e^{-\mu}\left(\mu'' + \frac{\mu'^2}{4} + \frac{2\mu'}{r}\right)$$

[2] V. B. Kelkar and M. K. Shrivastav, Bull. Cal. Math Soc., 92, (5), pp 385-388(2000) [3] V. B. Kelkar, V. D. Deshpande. J. J. Rawal and M. K. Shrivastav. Bull. Cal. Math. Soc., 93 (3) pp 197-204(2001).

[4] V. B. Kelkar and M. K. Shrivastav, Bull. Cal. Math Soc., 92, (5), 385-388(2000) V. B. Kelkar, V. D. Deshpande. J. J. Rawal and M. K. Shrivastav. Bull. Cal. Math. Soc., 93 (3) pp 197-204 (2001).

[5] Eddington A.S :The Mathematical Theory of Relativity ,Cambridge, At The University Press 1963, Article 53 pp 116, 117.

[6] Wyman (1946) Ref.1 pp75 solution is $e^{\mu} = \frac{4R^2}{\left(e^{c} r^2 + e^{-c}\right)^2}$

[7] Eddington A.S Ref.6 Equation 46.2 pp 101
[8] and equation 46.5pp102 , also equation 55.4 pp123

[9] Wyman Ref.3 pp74. Equation 1.1 $8\pi p = e^{-\mu}\left(\frac{\mu'^2}{4} + \frac{\mu'\nu'}{2} + \frac{\mu' + \nu'}{r}\right)$

[10] Wyman Ref.3 pp. 75 Equation 1.8 $e^{\mu} = \frac{4R^2}{\left(e^{c} r^2 + e^{-c}\right)^2}$

[11] Eddington A.S –Ref. 6 Equation 72.4 pp169 [12]
[13] Ph.D Thesis of Manoj Shrivastav,
[14] University of Mumbai 1999

[15] Ref 4&5