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Popular Satisfactory Matching Problem

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ABSTRACT *An instance of the satisfactory marriage problem with strict preferences is considered for computing a maximum cardinality popular satisfactory matching in a bipartite graph G = (AUB, E) where each vertex u ϵ A U B ranks its neighbors in a strict order of preference. A matching M* is satisfactory popular if for every matching M in G, the satisfactory level of vertices that prefer M to M* is at most the satisfactory level of vertices that prefer M* to M. In this paper, an algorithm is described to find a maximum cardinality popular satisfactory matching in G= G= (A U B, E)*

INTRODUCTION

Let A be a set of applicants and B be a set of jobs. Every applicant a ϵ A has a strict ranking over the jobs that a is interested in and similarly, every job b ϵ B has a strict ranking over the applicants that b is interested in. The problem is to find an assignment of applicants to jobs that reflects the preferences of applicants and jobs in an optimal way. This problem can be easily modeled as a bipartite graph G = (A U B, E) where every vertex u ϵ A U B seeks to be assigned to one of its neighbors and ranks its neighbors in a strict order of preference. Preference lists can be incomplete, which means that a vertex may be adjacent to only some of the vertices on the other side. We assume without loss of generality that a belongs to b's list if and only if b belongs to a's list, for any a and b. Such a graph is an instance of the stable marriage problem [4] with strict preferences and incomplete lists and it is customary to call the two sides of the graph men and women, respectively. Let V denote the entire vertex set A U B and let |V| =n and |E| =m. We assume that no vertex is isolated, so m ≥n/2.

A matching M is a set of edges no two of which share an endpoint. For any vertex u that is matched in M, let M(u) denote u's partner in M. An edge (u, v) is said to be a blocking edge for a matching M if by being matched to each other, both u and v are better-off than their respective assignments in M: that is, u is either unmatched in M or prefers v to M(u)and similarly, v is either unmatched in M or prefers u to M(v). A matching that admits no blocking edge is called a stable matching. It is known that every instance G admits a stable matching and such a matching can be computed in linear time by a straightforward generalization[7] of the well-known Gale–Shapley algorithm[4] for complete lists.

POPULAR MATCHINGS

For any two matchings M and M*we say that vertex u prefers M to M* if u is better-off in M than M* (i.e., u is either matched in M and unmatched M* or matched in both and prefers M(u) to M*(u). We say that M is more popular M* if the number of vertices that prefer M to M* is more than the number of vertices that prefer M* to M.

PREVIOUS WORKS

Popularity is an attractive notion of optimality since it captures global stability as there is no matching where more vertices are better-off than in a popular matching. Gardenfors [6] introduced the notion of popularity in the context of stable matchings. Abraham et al. [1] considered the popular matching problem in the domain of one-sided preference lists; they described efficient algorithms to determine if a given instance admits a popular matching or not and if so, to compute one with maximum cardinality. For one-sided preference lists (both for strict lists and for lists with ties), they gave a structural characterization of instances that admit popular matching. The work in [1] on one-sided popular matching was generalized to the capacitated version by Manlove and Sng [13], the weighted version by Mestre [16], and Mahdian studied random popular matchings [12]. Kavitha and Nasre [11] as well as McDermid and Irving [15] independently studied the problem of computing an optimal popular matching for several notions of optimality in strict instances. For instances that do not admit popular matchings, McCutchen [14] considered the problem of computing a least unpopular matching and showed this problem to be NP-hard, while Kavitha, Mestre, and Nasre [10] showed the existence of popular mixed matchings and efficient algorithms for computing them.Gardenfors [6], who originated the notion of popular matchings, considered this problem in the domain of two-sided preference lists.

When ties are allowed in preference lists, it has recently been shown by Biró, Irving, and Manlove [2] that the problem of computing an arbitrary popular matching in the stable marriage problem is NP-hard. It has very recently been shown in [9] that a maximum cardinality popular matching in G = (A U B, E) can be computed in linear time. There are simple examples in the one-sided preference lists domain that admit no popular matching. In the world of two-sided strict preference lists, i.e. in G = (A U B, E) where every u ϵ A U B ranks its neighbors in a strict order of preference, popular matchings always exist since stable matching always exist and every stable matching is popular. But not all popular matchings are stable.

PROBLEM FORMULATION

Given G = (A U B, E) with two-sided preference lists, a stable matching has usually been considered the optimal way of matching the vertices. The fact that stable matchings always exist and can be computed efficiently using the Gale–Shapley algorithm makes them natural candidates for optimality. In fact, there may be many stable matchings in a given instance G = (A U B, E). However the fact that a stable matching admits no blocking edge is a very strong condition and in order to ensure that there is no blocking edge, the size of the matching may suffer. It is known [5] that all stable matchings in G = (A U B, E) have the same size and match exactly the same vertices in V.A stable matching is a minimum cardinality popular matching.

It is easy to show that the size of any stable matching is at least 1/2(size of a maximum cardinality matching). Biró, Manlove, and Mittal [3] considered the problem of computing a maximum cardinality matching that minimized the number of blocking edges; they showed that this problem is NP-hard. So on one hand we have stable matchings where no blocking edge is permitted and whose size could be just half the size of a maximum cardinality matching and on the other hand, we have maximum cardinality matchings whose size is the best possible but the preferences of the vertices play no role here. What we seek here is a matching that is somewhere in between these two extremes – we are willing to weaken to some extent the notion of stability for the sake of obtaining a larger matching.

The notion of popularity captures a natural relaxation of stability, where pairwise stability is weakened to global satisfactory. Hence in problems where, for the sake of increasing the size of the resulting matching, we are willing to weaken stability to popularity, what we seek is a maximum cardinality popular satisfactory matching. In other words, we want a largest matching M in G such that there is no matching where more vertices are better-off than in M. There are instances where a maximum cardinality popular satisfactory matching can be twice as large as a stable matching. The size of a maximum cardinality popular matching is at least 2/3 (size of a maximum cardinality matching) and this bound is tight.

POPULAR SATISFACTORY MATCHING

For any two matchings M and M* we say that vertex u prefers M and M* if u is better-off in M than in M* (i.e., u is either matched in M and unmatched in M* or matched in both and prefers M(u) to M* (u)). We say that M is more satisfactory popular than M* if the satisfactory level of vertices that prefer M to is more than the satisfactory level of vertices that prefer M* to M.

We now give an overview of how we obtain these results here. The following definition will be useful to us:

Definition 3. For any u A B & neighbors x,y,z of u, u's preference value between x,y & z as:

Preference value $u(_{x,y,z})$={1 if u prefers x to y,z: 2/3 if u prefers y to z :1/3 if u prefers z}.

Similarly,

Preference value $v(_{x,y,z})$={1 if v prefers x to y,z: 2/3 if v prefers y to z :1/3 if v prefers z}.

Label every edge e=(u,v) in E by the sum of preference value of u on v and v on u.

The Related terminologies like Preference value, satisfactory level and Hungarian method of Assignment model are discussed in [8].

In the next section we show the following results

• We give an algorithm to find popular satisfactory matchings in G = (A U B, E).

• We then give a sufficient condition for a satisfactory popular matching to be a maximum cardinality popular satisfactory matching.

ALGORITHM

In this section we present algorithm for computing a maximum cardinality popular satisfactory matching in G = (A U B, E).

A bipartite weighted graph of n^2 vertices are described by n by n matrix P=[pij],where p_{ij}= weight of edge between a_i and b_j.

- Input the weighted bipartite graph
- Construct a matrix P
- Apply Hungarian method on the matrix P

A sufficient condition for a satisfactory popular matching to be a maximum cardinality popular satisfactory matching is given by Hungarian algorithm, which is an algorithm for one-one matching with maximum cardinality. So it results a maximum cardinality popular satisfactory matching.

EXAMPLE 1:

Consider an instance of the satisfactory marriage problem with strict preferences and incomplete list. Let M be set of men and W be set of women. Every man m϶M has a strict ranking over the women, that m is interested in and similarly, every women w϶W has a strict ranking over the men that women is interested in. This problem can be easily modeled as a bipartite graph G = (M U W, E), where every vertex u ϵ M U W seeks to be assigned to one of its neighbors. The problem is to find Maximum cardinality popular satisfactory matching. The preference lists of the vertices are given below:

m1 :w1	w1 :m2 m1
m2 :w1 w5 w2	w2 :m2 m3
m3 : w4 w2 w3	w3 : m3
m4 : w4	w4 : m5 m3 m4
m5 : w5 w4	w5 : m2 m6 m5
m6 : w5 w6	w6 : m6

The satisfactory matching for the above problem is (a_1,b_1) (a_2,b_2) (a_3,b_3) (a_4,b_4) (a_5,b_5) (a_6,b_6) and satisfactory level of set M and set W is 50% & 50% respectively. The size of the satisfactory matching is 6 but size of Stable Matching is 4.So the satisfactory matching is the maximum cardinality popular satisfactory matching with respect to the satisfactory level.

CONCLUSION

To conclude, in this study, a simple characterization of popular satisfactory matching in a satisfactory marriage instance in a bipartite graph G = (A∪B,E) with strict preferences and incomplete lists is discussed. Now that a linear time algorithm is known for computing a maximum cardinality popular matching in G = (A∪B, E).A linear time algorithm was studied with a real life example. It was found that both men and women gains high satisfactory level

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and gets maximum cardinality popular satisfactory matching. This algorithm helps to find popular matching in matching problems and to take absolute decisions in a best possible manner.

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