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Single Product - Multiple Manufactures Inventory Model For Fixed Lifetime Product with and Without Coordination

KEYWORDS: Inventory, Quantity discount, Coordination, Fixed life time products, Production

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ABSTRACT *The cost of business organization is affected by the production and inventory decision. This paper deals with the single product - multiple manufactures inventory model with fixed lifetime product with and without coordination. Each manufacturer uses an optimal inventory policy and incurs standard inventory holding cost, production cost and setup cost. The objective is to find optimal multiples of order that minimize the manufacturer's total inventory cost. The savings percentage of Manufacturer1 and Manufacturer2 is also determined. It proves that the quantity discount mechanism attains system optimization. The results are discussed with numerical examples.*

1. INTRODUCTION

The fixed life time products or products with limited shelf period like medicines are manufactured and supplied by small, medium and large scale manufacturers. Sometimes the same product may be manufactured by a small scale and large scale manufacturer at the same period. In this situation, the small manufacturer (Manufacturer1) may decide to procure the same product from the large scale manufacturer (Manufacturer2) instead of manufacturing by himself. At that time, Manufacturer2 offer discount to Manufacturer1 to increase his profit and sales. The model proposed in this paper deals with this case.

A large number of process manufacturers, mainly food and beverage companies are faced with limited shelf life of their products. These same companies are challenged with seasonal demand or supply and limited production capacities. A common way to deal with uncertain demand when capacity is limited is to stock up. Making the right decisions on when to stock up and when not to, how much, in which way at which location and at which of age of production have an impact on cost. Hence a model to be developed related to such a situation is the need of the hour.

Past researchers Biswajit Sarkar, Ilkyenog Moon [1] developed inventory model with inflation in an imperfect production system, Kit Nam Francis Leung [8] had analyzed production system with multistage. Yong He, Shou - Yang Wang, K.K Lai [13] proposed their model for production for deteriorating items with multiple market demand. Gede Agus Widyadana and Hui Ming Wee [3] developed production inventory models for deteriorating items with random machine break down and stochastic repair time. Fujiwara, Soewandi and Sedarage [2], Kanchana and Anulark [7] analyzed a two stage inventory systems for fixed life time perishable product. Goyal and Gupta [4], Kaj-Mikael Bjork [6], Mahdi Tajbakhsh, Chi – Guhn Lee and Saeed Zolfaghari [9], Saoussen Krichen, Awatef Laabidi and Fouad Ben Abdelaziz [12] and Hung -Chi Chang [5] did consider the quantity discount inventory models. Multi-Item multi-period optimal production problem was analyzed by S. Mandal et.al [10] and an economic production and remanufacturing model was analyzed by Mohamad Y. Jaber and Ahmed M.A. El Saadany [11].

The first step in the proposed model deals with the total cost incurred by the Manufacturer1 and Manufacturer2 without coordination from which optimum cost for Manufacturer1 and Manufacturer2 is determined. In the next step, the optimum cost in case of coordination between Manufacturer1 and Manufacturer2 is determined. The coordination strategy includes a quantity discount offer from Manufacturer2 to Manufacturer1. This model also helps to arrive at the multiples of order placed without coordination (m) and with coordination (n). Finally the savings percentage of Manufacturer1 and Manfacturer2 is determined. Similar inventory situation is analyzed by various authors (Yongrui Duan, Jianwen Luo and Jiazhen Huo [14]) pertaining to normal buyer and vendor. In this model the situation is entirely varied and is applicable in case of inventory decision by two manufacturers.

The detailed description of the paper is as follows. In section 2, assumptions, notations, model development of with and without coordination are formulated. Analytically easily understandable solutions are obtained in these models. It is proved that the quantity discount is the best strategy to achieve system optimization and win – win outcome. In section 3 numerical examples are given in detail to illustrate the models. Finally conclusions and summary are presented.

2. MODEL DEVELOPEMENT

The developed model deals, with and without coordination strategy for Manufacturer1 and Manufacturer2. Quantity discount is offered by the Manufacturer2 in the model with coordination.

2.1 Assumptions and Notations

Assumptions

(1) Demand is constant

(2) Production rate is greater than the demand of an item.

(3) Shortages are not allowed.

(4) Lead time is zero.

(5) During the production run the production of the item is continuous and at a constant rate until production of quantity Q is complete.

(6) Manufacturer1 and Manufacturer2 produce a same product.

(7) Under coordination strategy Manufacturer1 stop their produce and purchase the product with Manufacturer2

Notations

D - Annual demand of the Manufacturer1

L - Life time of product

P - Replenishment rate per year (P > D)

k_1, k_2- Manufacturer2 and Manufacturer1 setup costs per order, respectively

h_1, h_2- Manufacturer2 and Manufacturer1 holding costs, respectively

p_1, p_2- Delivered unit price paid by the Manufacturer2 and the Manufacturer1 respectively

Q_0 - Buyer's EOQ

m - Manufacturer2 order multiple in the absence of any coordination

n - Manufacturer2 order multiple under coordination

K - Manufacturer1 order multiple under coordination. KQ_0 buyer's new order quantity

$d(K)$- Denotes the per unit dollar discount to the buyer if he orders $K(Q_0)$ every time

TC_{M1}- Total cost of the Manufacturer1 without coordination

$TC_{M2}(m)$ - Total cost of Manufacturer2 without coordination

$TC_{M2}(n)$ - Total cost of Manufacturer2 with coordination

2.2 Model development for the system without coordination

Manufacturer1

Without coordination strategy, the Manufacturer1 order quantity is $Q_0 = \sqrt{\frac{2Dk_2}{h_2}\left(\frac{P}{P-D}\right)}$, with optimum total cost $TC_{M1} = \sqrt{2Dk_2h_2\left(1-\frac{D}{P}\right)}$.

Manufacturer2

The Manufacturer2 order is equal to some integer multiple of Q_0.

i.e., order size = mQ_0, with the fixed intervals $t_0 = \sqrt{\frac{2k_2}{Dh_2}\left(\frac{P}{P-D}\right)}$.

Here, average inventory held up per year of Manufacturer2 is given by

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$$=\frac{(m-1)Q_0+(m-2)Q_0+\ldots+Q_0+0Q_0}{m}=\frac{(m-1)Q_0}{2}$$

Now the total annual cost for the Manufacturer2 is given by

$$TC_{M2}(m)=\frac{Dk_1}{mQ_0}+\frac{(m-1)h_1Q_0}{2}\left(1-\frac{D}{P}\right)$$

$$=\frac{Dk_1}{m\sqrt{\frac{2Dk_2}{h_2}\left(\frac{P}{P-D}\right)}}+\frac{\left(1-\frac{D}{P}\right)(m-1)\sqrt{\frac{2Dk_2}{h_2}\left(\frac{P}{P-D}\right)}}{2}$$

$$=\frac{k_1}{m}\sqrt{\frac{Dh_2}{2k_2}\left(\frac{P-D}{P}\right)}+(m-1)h_1\sqrt{\frac{Dk_2}{2h_2}\left(\frac{P-D}{P}\right)}$$

So without coordination , the Manufacturer2 model can be developed as follows

$$\min TC_{M2}(m)$$

$$\text{s.t}\begin{cases}mt_0\le L,\\ m\ge 1,\end{cases}\qquad(1)$$

here $mt_0 \le L$ shows that the product is not overdue before they are sold up by the Manuafcturer1.

Theorem 1

Consider m^* be the optimum of (1), if $L^2\ge\frac{2k_2}{Dh_2}\left(\frac{P}{P-D}\right)$, then

$$m^*=min\left\{\left\lceil\sqrt{\frac{k_1h_2}{k_2h_1}+\frac{1}{4}}-\frac{1}{2}\right\rceil,\left\lceil\frac{L}{\sqrt{\frac{2k_2}{Dh_2}\left(\frac{P}{P-D}\right)}}\right\rceil\right\},\ (2)$$

here $\lceil x\rceil$ is the least integer greater than or equal to x, $L^2\ge\frac{2k_2}{Dh_2}\left(\frac{P}{P-D}\right)$ is to ensure that $m^*\ge 1$

Proof

$\frac{d^2TC_v(m)}{dm^2}=\frac{k_1}{m^3}\sqrt{\frac{2Dh_2}{k_2}\left(\frac{P-D}{P}\right)}>0$, $TC_{M2}(m)$ is strictly convex in m.

Considered m_1^* be the optimum of $\min TC_{M2}(m)$, then

$$m_1^*=\max\{\min\{m\,/\,TC_{M2}(m)\le TC_{M2}(m+1)\},1\}$$

$$=\max\{\min\{m\,/\,m(m+1)\ge\frac{2Dk_1}{Q_0^2\left(1-\frac{D}{P}\right)h_1}\},1\}$$

$$=\left\lceil\sqrt{\frac{h_2k_1}{h_1k_2}+\frac{1}{4}}-\frac{1}{2}\right\rceil\ge 1$$

Put the value of t_0 into the constraints in (1), then we have $m\sqrt{\frac{2k_2}{Dh_2}\left(\frac{P}{P-D}\right)}\le L$

Take $m_2^*=\frac{L}{\sqrt{\frac{2k_2}{Dh_2}\left(\frac{P}{P-D}\right)}}\ge 1$, is true since $L^2\ge\frac{2k_2}{Dh_2}\left(\frac{P}{P-D}\right)$.

$m^*=m_1^*$ where $m_1^*\le m_2^*$, otherwise $m^*=m_2^*$. Therefore $m^*=min\{m_{1>}^*,m_2^*\}$, if $L^2\ge\frac{2k_2}{Dh_2}\left(\frac{P}{P-D}\right)$

Remark 1: Without coordination, the Manufacturer2 optimum total cost is $TC_{M2}(m^*)$, order size is $m^*\sqrt{\frac{2Dk_2}{h_2}\left(\frac{P}{P-D}\right)}$ and place $\frac{D}{m^*\sqrt{\frac{2Dk_2}{h_2}\left(\frac{P}{P-D}\right)}}$ orders each year with an interval $\frac{m^*\sqrt{\frac{2Dk_2}{h_2}\left(\frac{P}{P-D}\right)}}{D}$.

2.3 Model development for system with coordination

In coordination scheme, Manufacturer1 stop their production and purchase the product of Manufacturer2. In this strategy, Manufacturer2 given quantity discount with the discount factor $d(K)$, if Manufacture1 change his lot size by KQ_0, $K>0$. Now the Manufacturer2 lot size is $nKQ_{0,}$ where n is a positive integer and KQ_0 is the Manufacturer1 new order quantity. The various parts of $TC_{M2}(n)$ is the ordering cost $\frac{Dk_1}{nKQ_0}$, the inventory holding cost $\frac{(n-1)\left(1-\frac{D}{P}\right)h_1KQ_0}{2}$ and the buyer's quantity discount $Dd(k)p_2$.

Therefore, Manufacturer2 total cost $\mathrm{TC_{M2}(n)} = \frac{\mathrm{Dk_1}}{\mathrm{nKQ_0}} + \frac{(\mathrm{n}-1)\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_1\,KQ_0}}{2} + \mathrm{p_2Dd(K)}$ (3)

In coordination discount strategy, the problem can be developed as follows

min $\mathrm{TC_{M2}(n)}$

subject to

$$\begin{cases} \mathrm{nKt_0} \le \mathrm{L}, \\ \frac{\mathrm{Dk_2}}{\mathrm{KQ_0}} + \frac{\mathrm{KQ_0}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_2}}{2} - \sqrt{2\mathrm{Dk_2\,h_2}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)} \le \mathrm{p_2Dd(K)}, \\ \mathrm{n} \ge 1, \end{cases} \quad (4)$$

Now $\mathrm{nKt_0} \le \mathrm{L}$ shows that the product is not overdue before they are sold up by the Manufacturer1. The second constraint shows that the Manufacturer1 cost under coordination cannot exceed that without coordination.

Theorem 2

$\mathrm{TC_{M2}(n^*)} \le \mathrm{TC_{M2}(m^*)}$ is true, if $m^*$is optimum of (1) and n^* be the optimum of (4).

Proof

If the second constraint must be an equation ,then $\mathrm{p_2Dd(K)}$ takes smallest value and $\mathrm{TC_{M2}(n)}$ is optimized.

i.e., $\frac{\mathrm{Dk_2}}{\mathrm{KQ_0}} + \frac{\mathrm{KQ_0}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_2}}{2} - \sqrt{2\mathrm{Dk_2\,h_2}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)} = \mathrm{p_2Dd(K)}$

$$d(K) = \frac{\frac{\mathrm{Dk_2}}{\mathrm{KQ_0}} + \frac{\mathrm{KQ_0}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_2}}{2} - \sqrt{2\mathrm{Dk_2}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_2}}}{\mathrm{p_2\,D}} \quad (5)$$

If $K = 1$, then

$$d(1) = \frac{\sqrt{2\mathrm{Dk_2h_2}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)} - \sqrt{2\mathrm{Dk_2h_2}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)}}{\mathrm{p_2\,D}} = 0.$$

So if $K = 1$, then (4) is equivalent to (1). Therefore, $\mathrm{TC_{M2}(n^*)} \le \mathrm{TC_{M2}(m^*)}$ is true.

Remark 2: Theorem (2), ensures that Manufacturer2 will get more benefit to compare with Manufacturer1 if the manufacturer1 order size is $KQ_0, K > 0$ because optimum total cost under coordination is not exceeding without coordination.

Put equation (5) into equation (3), we have

$$\mathrm{TC_{M2}(n)} = \frac{\mathrm{Dk_1}}{\mathrm{nKQ_0}} + \frac{(\mathrm{n}-1)\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_1\,KQ_0}}{2} + \mathrm{p_2\,D}\left(\frac{\frac{\mathrm{Dk_2}}{\mathrm{KQ_0}} + \frac{\mathrm{KQ_0}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_2}}{2} - \sqrt{2\mathrm{Dk_2}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_2}}}{\mathrm{p_2\,D}}\right) \quad (6)$$

Let K^* be the optimum of $\mathrm{TC_{M2}(n)}$, we have

$$K^*(\mathrm{n}) = \frac{1}{\mathrm{Q_0}}\sqrt{\frac{2\mathrm{D}\left(\frac{\mathrm{k_1}}{\mathrm{n}} + \mathrm{k_2}\right)}{\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)[(\mathrm{n}-1)\mathrm{h_1} + \mathrm{h_2}]}} \quad (7)$$

From first constraint of (4), we have

$$\left(\frac{\mathrm{k_1}}{\mathrm{n}} + \mathrm{k_2}\right)\mathrm{n}^2 \le \frac{\mathrm{L^2Q_0^2h_2}}{4\mathrm{k_2}}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)^2((\mathrm{n}-1)\mathrm{h_1} + \mathrm{h_2})$$

Take
$$g(n) = -\mathrm{k_2\,n^2} + \left(\frac{\mathrm{DL}^2}{2}\left(\frac{\mathrm{P}-\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_1} - \mathrm{k_1}\right)\mathrm{n} + \frac{\mathrm{DL}^2}{2}\left(\frac{\mathrm{P}-\mathrm{D}}{\mathrm{P}}\right)(\mathrm{h_2} - \mathrm{h_1}) \quad (8)$$

Substituting (7) and $\mathrm{t_0} = \sqrt{\frac{2k_2}{Dh_2}\left(\frac{\mathrm{P}}{\mathrm{P}-\mathrm{D}}\right)}$ into (3), we have

$$\mathrm{TC_{M2}(n)} = \sqrt{2D\left[\mathrm{k_1}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_1} + \frac{\mathrm{k_1}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)[h_2 - \mathrm{h_1}]}{n} + n\mathrm{k_2}\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{h_1} + \left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)\mathrm{k_2}[h_2 - \mathrm{h_1}]\right]} - \sqrt{2Dh_2k_2\left(1-\frac{\mathrm{D}}{\mathrm{P}}\right)} \quad (9)$$

Therefore, (4) becomes

min $\mathrm{TC_{M2}(n)}$

$$\text{subject to} \begin{cases} g(n) \ge 0, \\ n \ge 1, \end{cases} \quad (10)$$

for $x \ge 0$, $\sqrt{x}$ is a strictly increasing so the above equation is equivalent to

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$$min\,\widetilde{TC_{M2}}(n) = D\left[k_1\left(1-\frac{D}{P}\right)h_1 + \frac{k_1\left(1-\frac{D}{P}\right)[h_2-h_1]}{n} + nk_2\left(1-\frac{D}{P}\right)h_1 + \left(1-\frac{D}{P}\right)k_2[h_2-h_1]\right]$$

subject to $\begin{cases} g(n) \geq 0, \\ n \geq 1 \end{cases}$ (11)

Here, $\widetilde{TC_{M2}}(n)$ is convex when $h_2 \geq h_1$, since $\widetilde{TC_{M2}}''(n) = \frac{2Dk_1\left(1-\frac{D}{P}\right)[h_2-h_1]}{n^3} > 0$, otherwise it is concave. $g(n)$ is strictly concave because $g''(n) = -2k_2 < 0$.

Proposition 1

Let n_1^* be the optimum of $\widetilde{TC_{M2}}(n)$ for $n \geq 1$, then

$$n_1^* = \begin{cases} \left\lceil\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}+\frac{1}{4}}-\frac{1}{2}\right\rceil, & \frac{k_1[h_2-h_1]}{k_2 h_1} \geq 2 \\ 1 & ,otherwise \end{cases} \quad (12)$$

Proof

$\widetilde{TC_{M2}}(n_1^*) \leq min\{\widetilde{TC_{M2}}(n_1^*-1), \widetilde{TC_{M2}}(n_1^*+1)\}$ because n_1^* is the minimum of $\widetilde{TC_{M2}}(n), n \geq 1$

Now $\widetilde{TC_{M2}}(n_1^*) - \widetilde{TC_{M2}}(n_1^*-1) = \frac{-Dk_1\left(1-\frac{D}{P}\right)[h_2-h_1]}{n_1^*(n_1^*-1)} + Dk_2\left(1-\frac{D}{P}\right)h_1 \leq 0$

$$\left(n_1^*-\frac{1}{2}\right)^2 \leq \frac{k_1[h_2-h_1]}{k_2 h_1}+\frac{1}{4} \quad (13)$$

Similarly,

$\widetilde{TC_{M2}}(n_1^*) - \widetilde{TC_{M2}}(n_1^*+1) \leq 0$, we have

$$\left(n_1^*+\frac{1}{2}\right)^2 \geq \frac{k_1[h_2-h_1]}{k_2 h_1}+\frac{1}{4} \quad (14)$$

Hence, if $\frac{k_1[h_2-h_1]}{k_2 h_1}+\frac{1}{4} < 0$, $\widetilde{TC_{M2}}(n_1^*) \leq \widetilde{TC_{M2}}(n_1^*+1)$ for any given n, then $n_1^* = 1$.

If $\frac{k_1[h_2-h_1]}{k_2 h_1}+\frac{1}{4} \geq 0$ by (13) & (14), $\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}+\frac{1}{4}}-\frac{1}{2} \leq n_1^* \leq \sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}+\frac{1}{4}}+\frac{1}{2}$.

So $n_1^* = \left\lceil\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}+\frac{1}{4}}-\frac{1}{2}\right\rceil$. Also note that, if $0 < \frac{k_1[h_2-h_1]}{k_2 h_1} < 2$ then $n_1^* = 1$, so (12) holds.

Proposition 2

The solutions of $g(n)$ be $n_{2(1)}^*$ and $n_{2(2)}^*$ then

1) If $\left(\frac{DL^2}{2}\left(1-\frac{D}{P}\right)h_1 - k_1\right)^2 + 2DL^2k_2\left(1-\frac{D}{P}\right)[h_2-h_1] < 0$, or $\left(\frac{DL^2}{2}\left(1-\frac{D}{P}\right)h_1 - k_1\right)^2 + 2DL^2k_2\left(1-\frac{D}{P}\right)[h_2-h_1] \geq 0$ and $n_{2(1)}^* < 1$, then $g(n) < 0$ for $n \geq 1$.

2) If $\left(\frac{DL^2}{2}\left(1-\frac{D}{P}\right)h_1 - k_1\right)^2 + 2DL^2k_2\left(1-\frac{D}{P}\right)[h_2-h_1] \geq 0$ and $n_{2(1)}^* \geq 1$, then

i) If $n_{2(2)}^* \geq 1$, $g(n) \geq 0$ for $\lceil n_{2(2)}^*\rceil \leq n \leq \lceil n_{2(1)}^*\rceil$ and

ii) If $n_{2(2)}^* < 1$ and $n_{2(1)}^* \geq 1$, $g(n) \geq 0$ for $1 \leq n \leq \lceil n_{2(1)}^*\rceil$.

Proof

To solve (8) we have,

$$n_{2(1)}^* = \frac{\left(\frac{DL^2}{2}\left(1-\frac{D}{P}\right)h_1 - k_1\right) + \sqrt{\left(\frac{DL^2}{2}\left(1-\frac{D}{P}\right)h_1 - k_1\right)^2 + 2DL^2k_2\left(1-\frac{D}{P}\right)[h_2-h_1]}}{2k_2}$$

$$n_{2(2)}^* = \frac{\left(\frac{DL^2}{2}\left(1-\frac{D}{P}\right)h_1 - k_1\right) - \sqrt{\left(\frac{DL^2}{2}\left(1-\frac{D}{P}\right)h_1 - k_1\right)^2 + 2DL^2k_2\left(1-\frac{D}{P}\right)[h_2-h_1]}}{2k_2}$$

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Here (8) is an quadratic function, therefore

1) If $\left(\frac{DL^2}{2}\left(1-\frac{D}{P}\right)h_1 - k_1\right)^2 + 2DL^2k_2\left(1-\frac{D}{P}\right)[h_2 - h_1] < 0$, then $g(n) < 0$ for every n.

2) If $\left(\frac{DL^2}{2}\left(1-\frac{D}{P}\right)h_1 - k_1\right)^2 + 2DL^2k_2\left(1-\frac{D}{P}\right)[h_2 - h_1] \geq 0$ then $n^*_{2(1)}$ and $n^*_{2(2)}$ are real solutions of $g(n) = 0$ then

i) If $n^*_{2(1)} < 1$, then $g(n) < 0$ for $n \geq 1$

ii) If $n^*_{2(2)} \geq 1$ then $g(n) \geq 0$ for $\lceil n^*_{2(2)} \rceil \leq n \leq \lceil n^*_{2(1)} \rceil$ iii) If $n^*_{2(2)} < 1$ and $n^*_{2(1)} \geq 1$, then view of n is positive integer, $g(n) \geq 0$ for $1 \leq n \leq \lceil n^*_{2(1)} \rceil$.

Remark 3: If (i) of proposition 2 is true, and $nKt_0 \geq L$ for any $n \geq 1$, then the problem is meaningless. If proposition 2 is true, and $nKt_0 \leq L$, is true for $\lceil n^*_{2(2)} \rceil \leq n \leq \lceil n^*_{2(1)} \rceil$ or $1 \leq n \leq \lceil n^*_{2(1)} \rceil$.

Theorem 3

If $h_2 \geq h_1$, and $n^*_{2(2)} \geq 1$ then

i) If $1 \leq n_1^* \leq \lceil n^*_{2(1)} \rceil$, $n^* = n_1^*$

ii) ii) If $n_1^* > \lceil n^*_{2(1)} \rceil$, $n^* = \lceil n^*_{2(1)} \rceil$.

Proof

n_1^* is the minimum of $\widetilde{TC_{M2}}(n)$, then $\widetilde{TC_{M2}}(n)$ is a convex. If $1 \leq n_1^* \leq \lceil n^*_{2(1)} \rceil$, then $n^* = n_1^*$ else $n^* = \lceil n^*_{2(1)} \rceil$.

Remark 4: If the Manufacturer2 unit holding cost is higher than the manufacture, $\widetilde{TC_v}(n)$ is strictly concave in n. So we will not give further discussion about this.

Theorem 4

If $h_2 \geq h_1$, then $K^*(n^*) > 1$

Proof

$$K^*(n^*) = \frac{1}{Q_0}\sqrt{\frac{2D\left(\frac{k_1}{n}+k_2\right)}{\left(1-\frac{D}{P}\right)[(n-1)h_1+h_2]}}$$

$$= \sqrt{\frac{h_2\left(\frac{k_1}{n}+k_2\right)}{k_2\left[(n-1)\left(1-\frac{D}{P}\right)h_1+h_2\right]}}$$

I. If $n^* = n_1^* = \left\lceil\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}+\frac{1}{4}}-\frac{1}{2}\right\rceil$, where $\frac{k_1[h_2-h_1]}{k_2 h_1} \geq 2$.

Because $\left\lceil\sqrt{x+\frac{1}{4}}-\frac{1}{2}\right\rceil \leq \sqrt{x}+1$ is true for $x \geq 0$, and $K^*(n^*)$ is a decreasing function of n.

To prove $K^*\left[\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}}+1\right] > 1$

i.e., $\sqrt{\frac{h_2\left(\frac{k_1}{\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}}+1}+k_2\right)}{k_2\left[\left(\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}}\right)h_1+h_2\right]}}$

$$= \frac{h_2\left(k_1+k_2\left(\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}}+1\right)\right)}{k_2\left(\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}}+1\right)\left[\left(\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}}\right)h_1+h_2\right]} > 1 \quad (15)$$

$$h_2 k_1 + h_2 k_2\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}} + h_2 k_2 >$$

$$k_2 h_1\frac{k_1[h_2-h_1]}{k_2 h_1} + h_2 k_2\sqrt{\frac{k_1[h_2-h_1]}{k_2 h_1}}$$

$$h_2[k_1 + k_2] > k_1[h_2 - h_1] \quad (16)$$

Equation (16) holds, if k_1, k_2, h_1 *and* h_2 are all positive.

II. If $n^* = n_1^* = 1$, and $K^*(1) = \frac{k_1+k_2}{k_2}$, k_1 & k_2 are positive then $K^*(1) > 1$.

III. If $n_1^* > \lceil n^*_{2(1)} \rceil$, then $n^* = \lceil n^*_{2(1)} \rceil$. Here $K^*(n)$ is a decreasing function, so $K^*(\lceil n^*_{2(1)} \rceil) \geq K^*(n_1^*) > 1$. From (I) to (III), $K^*(n) > 1$ if $h_2 \geq h_1$.

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3. NUMERICAL EXAMPLES

In this section, numerical examples are presented to illustrate the performance of the developed model. The sensitivity analyses of cost savings on parameters have been given.

The savings percentage of Manufacturer1 is

$SP_{M1} = 100 \propto (TC_{M2}(m^*) - TC_{M2}(n^*))/TC_{M1}(m^*)$

and the savings percentage of Manufacturer2 is

$SP_{M2} = 100(1-\propto)(TC_{M2}(m^*) - TC_{M2}(n^*))/TC_{M2}(m^*)$.

Example 1

Given P = 20000 units per year, D = 10,000 units per year, p_2 = 30$ per unit, α = 0.5, L = 0.25 year, k_1 = 300$ per order, h_1= 3$ per year, h_2 = 5$ per year. The different values of k_2 computational results are as specified in Table 1.

Example 2

Given P = 20000 units per year, D = 10,000 units per year, p_2 = 30$ per unit, α = 0.5, L = 0.25 year, k_2 = 100$ per order, h_1= 3$ per year. The different values of k_1 computational results are as specified in Table 2.

Example 3

Given P = 20000 units per year, D = 10,000 units per year, p_2 = 30$ per unit, α = 0.5, L = 0.25 year, k_1 = 300$ per order, k_2 = 100$ per order, h_2= 6$ per year. The different values of h_1 computational results are as specified in Table 3.

Example 4

Given P = 20000 units per year, D = 10,000 units per year, p_2 = 30$ per unit, α = 0.5, L = 0.25 year, k_1 = 300$ per order, k_2 = 100$ per order, h_1= 5$ per year. The different values of h_2 computational results are as specified in Table 4.

Example 5

Given P = 20000 units per year, D = 10,000 units per year, p_2 = 30$ per unit, α = 0.5, L = 0.25 year, k_1 = 300$ per order, k_2 = 100$ per order. The different values of h_1, h_2 computational results are as specified in Table 5.

*** TABLES Given in bottom of Manuscript**

*** FIGURES Given in bottom of Manuscript**

Computational result indicates that

1) The optimum total cost of Manufacturer2 under coordination is not greater than that without coordination.
2) If the holding cost for Manufacturer2 increases then the saving percentage of Manufacturer1 and Manufacturer2 also increases. In this case Manufacturer1 and Manufacturer2 can have more benefit under coordination strategy.
3) If the holding cost for Manufacturer1 increases then the savings percentage of Manufacturer1 and Manufacturer2 is decreases. In this case Manufacturer1 and Manufacturer2 cannot get benefit under quantity discount coordination strategy.
4) The profit would be same even when the holding cost is increased by both manufacturers.
5) The set up cost for Manufacturer1 and Manufacturer2 increases automatically the total cost of Manufacturer1and Manufacturer2 gets increased and savings

percentage should be decreased. Hence it is understood that the setup cost should not be increased.

4. CONCLUSION

Single product - multiple manufacturers inventory model for fixed lifetime product with and without coordination is considered in this paper. The numerical example applied in this model reveals that the Manufacturer1 optimize his cost with coordination i.e., purchase from Manufacturer2 with quantity discount. This paper concludes that Manufacturer1 and Manufacturer2 are benefitted only when coordination strategy is adopted. Manufacturer2 is comparatively highly benefitted than Manufacturer1 in spite of giving quantity discount. It is proved that the quantity discount is the best strategy to achieve system optimization and win – win outcome. Numerical examples are also provided to illustrate the proposed model. The proposed model can further extended by taking more realistic assumptions such as stochastic demand patterns, multi products etc.

Table 1
Summary of solution of example 1

k_2	K*(n)	d(K)	TC_{M1}	TC_{M2} (m)	TC_{M2} (n)
50	1.7541	0.0009	1581.1	2529.8	2440.4
75	2.2361	0.0022	1936.5	2517.4	2393.6
100	2.0000	0.0019	2236.1	2347.9	2236.1
125	1.8439	0.0016	2500.0	2250.0	2109.8
150	1.7321	0.0014	2738.6	2738.6	2004.8

Table 2
Summary of solution of example 2

k_1	K*(n)	d(K)	TC_{M1}	TC_{M2} (m)	TC_{M2} (n)	SP_{M2}	SP_{M1}
200	1.7321	0.0012	2236.1	2236.1	1636.9	0.2679	0.1340
225	1.8028	0.0013	2236.1	1928.6	1795.1	0.0692	0.0299
250	1.8708	0.0015	2236.1	2068.4	1947.2	0.0586	0.0271
275	1.9365	0.0017	2236.1	2208.1	2094.1	0.0517	0.0255
300	2.0000	0.0019	2236.1	2347.9	2236.1	2.3810	2.5000

Table 3
Summary of solution of example 3

h_1	K*(n)	d(K)	TC_{M1}	TC_{M2} (m)	TC_{M2} (n)	SP_{M2}	SP_{M1}
3	2.0000	0.0020	2449.5	2449.5	2449.5	0.0000	0.0000
4	2.0000	0.0020	2449.5	2653.6	2449.5	0.0769	0.0417
5	2.0000	0.0020	2449.5	3674.2	2449.5	0.3333	0.2500
6	2.0000	0.0020	2449.5	3674.2	2449.5	0.3333	0.2500

Table 4
Summary of solution of example 4

h_2	K*(n)	d(K)	TC_{M1}	TC_{M2} (m)	TC_{M2} (n)	SP_{M2}	SP_{M1}
5	2.0000	0.0019	2236.1	3354.1	2236.1	0.3333	0.2500
6	2.0000	0.0020	2449.5	3674.2	2449.5	0.3333	0.2500
7	2.0000	0.0022	2645.8	2929.2	2645.8	0.0968	0.0536
8	2.0000	0.0024	2828.4	3005.2	2828.4	0.0588	0.0312

Table 5
Summary of solution of example 5

h_1	h_2	K*(n)	d(K)	TC_{M1}	TC_{M2} (m)	TC_{M2} (n)	SP_{M1}	SP_{M2}
5	5	2.0000	0.0019	2236.1	3354.1	2236.1	0.3333	0.2500
6	7	2.0000	0.0022	2645.8	3968.6	2645.8	0.3333	0.2500
7	8	2.0000	0.0024	2828.4	4242.6	2828.4	0.3333	0.2500
8	9	2.0000	0.0025	4500.0	3000.0	3000.0	0.3333	0.2500

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