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Analysis and Modelling of Capacitive Transducers Based on the Charge Transfer Method

KEYWORDS: capacitive sensor, charge transfer method, transducer, constant phase element.

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ABSTRACT *The steady-state response of a capacitive measuring transducer based on the charge transfer method was analysed. According to the new generalised equivalent circuit of a capacitive sensor which contains a constant phase element, the additional error due to this element was calculated. Specific recommendations were given concerning the control of the switches and the method of processing the measurement results obtained. Following them, a transducer was built that could measure the sensor capacity in the presence of a constant phase element. The simulation in PSpice and the experimental study of the transducer confirmed the analytical results obtained.*

INTRODUCTION

Using continuously operating charge transfer circuits in capacitive transducers is favoured due to their advantages as follows [9, 10]: they are simple and low-priced, have excellent characteristics and can be controlled digitally. On the other hand, the selection of a differential structure of measuring transducers can help avoid problems caused by bias currents, offset voltages and charge injection from analogue switches. Furthermore, the linearity of the transfer function is improved, and better signal\noise ratio and resistance to interference are achieved [11].

GENERALISED EQUIVALENT ELECTRICAL CIRCUIT OF A CAPACITIVE SENSOR

The equivalent electrical circuit of capacitive sensors can often be displayed by a passive three-terminal circuit as in fig.1, where $C = C(X, P_1, P_2, \ldots, P_l)$ shows the capacity which depends on the input quantity X and influence factors $P_1, P_2, \ldots, P_l$ such as temperature, humidity, environment, etc. [1]. The complex conductivities Y_S, Y_{S1}, Y_{S2}, are related to the transfer process $X \rightarrow C$ and the design features of the capacitive sensor.

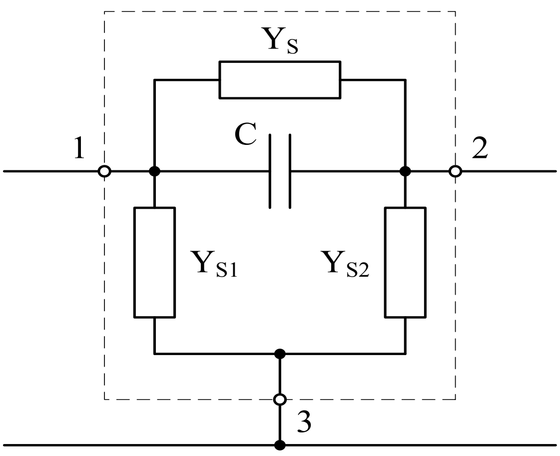

Figure 1: three-terminal equivalent electrical circuit of capacitive sensors

When Y_{S1} and Y_{S2} have no bearing to the definition of X (parasitic conductivities), their influence can be suspended through an 'active shield' or by using a measuring transducer with low input impedance [1]. In this way, and in all cases where $Y_{S1} \rightarrow 0$ and $Y_{S2} \rightarrow 0$, the three-terminal model can be simplified and transformed into a two-terminal one.

The nature of Y_S is a major factor in the choice of a measuring transducer. Typically, when the order of the electrical circuit between terminals 1 and 2 is higher, a transducer with a more complex structure should be used. Therefore, an equivalent circuit of the sensor is preferred that can achieve the required accuracy of presentation at the lowest possible order of conductivity $Y_{12} = j\omega C + Y_S$.

The most common and popular models of capacitive sensors are:

- C-model. Here $Y_S = 0$ and $Y_{12} = j\omega C$. The model has one parameter;

- CG-model with $Y_S = G$ and $Y_{12} = j\omega C + G$ in which $G = G(X, P_1, P_2, \ldots, P_l)$. The model has two parameters;

- CCPE-model (fig.2) with parallel connected capacitor C and constant phase element (CPE), with complex conductivity $Y_S = Y_{CPE} = A(j\omega)^\alpha$, where $0 \le \alpha < 1$. If α = const Y_{12} depends on two parameters $C = C(X, P_1, P_2, \ldots, P_l)$ and $A = A(X, P_1, P_2, \ldots, P_l)$, and is described as follows

$$Y_{12} = j\omega C + A(j\omega)^\alpha. \quad (1)$$

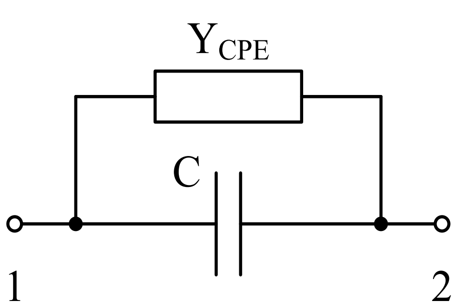

Figure 2: two-terminal equivalent circuit

The CG-model is the most popular one and it is normally used to take into account the conductivity and dielectric losses of the medium in which the electromagnetic field created by the electrodes is propagated. However, the CCPE-model has a lot of advantages [6, 7]:

- The CCPE-model is a generalisation of the other two models. When A = 0, the CCPE-model becomes a C-model, and when α = 0, the CCPE-model turns into a CG-model;

Volume : 4 | Issue : 2 | Feb 2014 | ISSN - 2249-555X

- The logarithmic plot (fig.3) of the CPE admittance is a straight line where $\alpha\pi/2$ is the slope of the line and $A^{-1/\alpha}$ is the horizontal coordinate of the location where the line crosses the horizontal axis. In a limited frequency interval [ω_1, ω_2], this allows for a linear approximation of complicated complex conductivities connected in parallel with the measured capacity and thereby leads to a more precise model of the capacitive sensor;

- The replacement of the CG model by a CCPE model does not increase the number of elements, parameters or the order of conductivity Y_{12}, so it could not complicate the transducer.

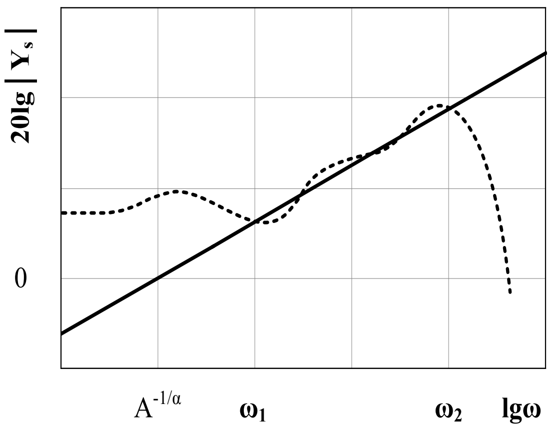

Figure 3: logarithmic plot of the CPE admittance

ANALYSIS OF THE STEADY-STAY RESPONSE OF A CAPACITIVE MEASURING TRANSDUCER BASED ON THE CHARGE TRANSFER METHOD

Fig.4 displays a charge transfer differential transducer with a capacitive sensor presented in a three-terminal CCPE-model, and fig.5 shows the clocks that control switches ($S_1 \div S_4$). The steady-state response of the transducer will be analysed under the following conditions: C(t) = C = const and A(t) = A = const [4, 5, 7].

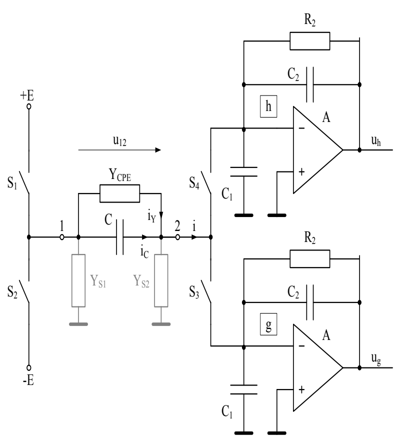

Figure 4: charge transfer differential transducer with a capacitive sensor presented in a three-terminal CCPE-model

Figure 5: control switches of the transducer

Representing voltage u_{12} of the capacitive sensor by a Fourier series (the potential of node 2 remains equal to zero, and $\omega_{cl} = 2\pi/T_{cl}$), it can be concluded that

$$u_{12}(t) = E\,\mathrm{sgn}(\sin(\omega_{cl} t)) \quad \text{or}$$

$$u_{12}(t) = E\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{2k-1}\sin\big((2k-1)\omega_{cl}\, t\big) \quad . \quad (2)$$

The charge transfer by currents i_C and i_Y using (1) as well is

$$q(t) = q_C(t) + q_Y(t) = Cu_{12}(t) + A\,{}_{-\infty}J^{1-\alpha}u_{12}(t) \quad , \quad (3)$$

where ${}_{-\infty}J^{1-\alpha}$ is the fractional integration operator of order (1-α). In equation (3), it is assumed that at a moment of time $t \to \infty$, CPE is not charged ($q_Y(t)|_{t\to-\infty} = 0$), but this condition is not critical for the analysis.

The solution of equation (3) is presented by an anti-periodic function [4]

$$q(t) = CE\,\mathrm{sgn}\big(\sin(\omega_{cl}\, t)\big) - \quad (4)$$

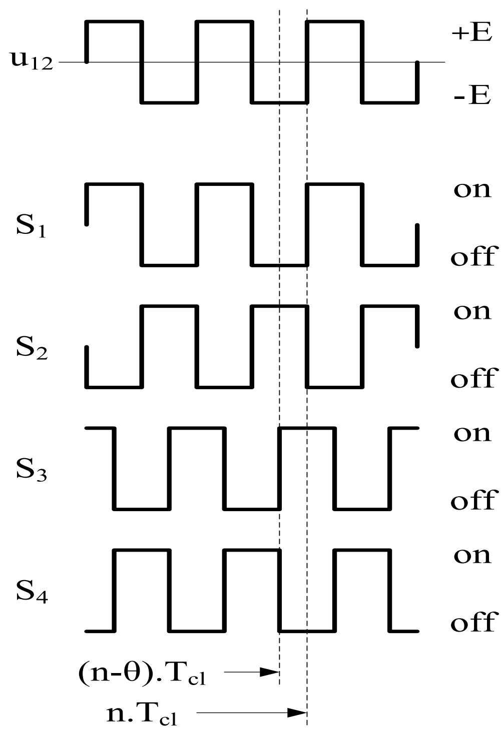

Figure 5: control switches of the transducer

$$- AE\omega_{cl}^{\alpha-1}\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^{2-\alpha}}\cos\left((2k-1)\omega_{cl}t + \alpha\frac{\pi}{2}\right)$$

Let us assume that n = 1, 2, 3, … is the number of the cycles when t > 0 and θT_{cl} is the time by which the control sequence of switches S_3 and S_4 outruns the control sequence of switches S_1 and S_2 ($0 < \theta < 1/2$). In the interval {$(n - 1 - \theta).T_{cl} \div (n - 1/2 - \theta).T_{cl}$}, S_3 is closed and the electrical charge passing trough capacitor C, CPE and the input of the trans-impedance amplifier g will be

$$\Delta q = q((n - ½ - \theta)T_{cl}) - q((n - 1 - \theta)T_{cl}) =$$

$$= 2q((n - 1 - \theta)T_{cl} \quad . \quad (5)$$

During the rest of this period {$(n - ½ - \theta)T_{cl} \div (n - \theta)T_{cl}$}, the S_3 switch is open and the electrical charge passing through the input of the amplifier g equals zero. Therefore the average value of the output voltage of the amplifier (dc component) is

$$U_g = -R_2 I_{av} = -R_2(\omega_{cl}\Delta q)/(2\pi) \quad , \quad (6)$$

Volume : 4 | Issue : 2 | Feb 2014 | ISSN - 2249-555X

where $I_{av} = \Delta q/T_{cl} = (\omega_{cl}\Delta q)/(2\pi)$ is the average value of the output current of the amplifier.

Likewise, for the other amplifier h

$$U_h = R_2 I_{av} = R_2(\omega_{cl}\Delta q)/(2\pi) \quad , \qquad (7)$$

and the difference $U = U_h - U_g$ can be derived from equations (6) and (7)

$$U = 2R_2 I_{av} = R_2(\omega_{cl}\Delta q)/\pi \quad . \qquad (8)$$

Considering equations (4)÷(8), the following can be written:

$$\Delta q = 2CE + \qquad (9)$$

$$+2AE\omega_{cl}^{\alpha-1}\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^{2-\alpha}}\cos\left((2k-1)2\pi\theta-\alpha\frac{\pi}{2}\right)$$

$$U = \frac{2R_2E}{\pi}\omega_{cl}C + \qquad (10)$$

$$+\frac{2R_2E}{\pi}\omega_{cl}^{\alpha}A\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^{2-\alpha}}\cos\left((2k-1)2\pi\theta-\alpha\frac{\pi}{2}\right)$$

$$U = \frac{2R_2E}{\pi}(\omega_{cl}C + \omega_{cl}^{\alpha}Ab) \qquad (11)$$

$$b = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^{2-\alpha}}\cos\left((2k-1)2\pi\theta-\alpha\frac{\pi}{2}\right) \qquad (12)$$

or $\quad b = b(\theta,\alpha) \quad .$

Equations (10) and (11) present a transfer function of the examined capacitive measuring transducer (fig.4, and fig.5). With the use of the same method, the transfer function of other similar continuously operating transducers can be determined.

DEFINING THE ERROR CAUSED BY A PHASE CONSTANT ELEMENT

Without a constant phase element (A = 0) and using equation (11), the value of C

$$C = \frac{\pi}{2R_2E\omega_{cl}}U \quad . \qquad (13)$$

Generally $A \neq 0$, so if equations (11), (12) and (13) are used to determine the capacity, the following will be achieved instead of C:

$$C' = C + \omega_{cl}^{\alpha-1}Ab \quad . \qquad (14)$$

Therefore the presence of CPE causes an error amounting to

$$\delta_{CPE} = \omega_{cl}^{\alpha-1}\frac{A}{C}b \quad . \qquad (15)$$

The conclusions drawn are [2]:

1. The terms $\omega_{cl}C$ and $\omega_{cl}^{\alpha}A$ are respectively equal to the magnitude of the complex conductivity of capacitor C and CPE at frequency ω_{cl}. The error δ_{CPE} depends on the variable b. If it is a finite and positive b > 0:

- the error is reduced at a higher frequency of the clocks, this effect being more evident at lower α values (1.41 times when α = 0.50 and only 1.18 times when α = 0.75);

- the error reaches zero if the conductivities of the two elements comply with the rule $\omega_{cl}^{\alpha}A << \omega_{cl}C$ and the CPE influence can be neglected.

2. In the presence of the CPE, the error (δ_{CPE}) depends on b = b(θ, α). In the $0<\theta\leq0.25$ interval, the quadrature control (θ = 0.25) is preferable to the classical control $\theta \to 0$, as the first control provides the minimum value of b = b(θ, α). According to equations (12) and (15):

- $b(1/4, \alpha) = \min b(\theta, \alpha)$ if $0 \leq \alpha < 1$;
- $b(1/4, \alpha) = 0$ if $\alpha = 0$;
- $0 < b(1/4, \alpha)$ if $0 < \alpha < 1$.

Therefore the quadrature control allows invariant determination of C (δ_{CPE} = 0) in one of the most basic models of capacitive sensors (the CG-model). In all other cases, the C capacity exceeds the true value and the error is positive (δ_{CPE} > 0).

MINIMISATION OF THE ERROR CAUSED BY THE PHASE CONSTANT ELEMENT THROUGH CONDUCTING OVERLAP MEASUREMENTS

It is possible that in the interval $0<\theta\leq0.25$ there will be no θ to reduce the error to zero so the designated δ_{CPE} value will not be reached using quadrature control.

Equation (11) may be used to determine capacity C:

$$U = \frac{2R_2E}{\pi}(\omega_{cl}C + \omega_{cl}^{\alpha}Ab) = U(\theta) = \beta_0 + \beta_1 b(\theta,\alpha) \quad , (16)$$

where β_0, β_1 and C are equal to

$$\beta_0 = \frac{2R_2E}{\pi}\omega_{cl}C \quad , \quad \beta_1 = \frac{2R_2E}{\pi}\omega_{cl}^{\alpha}A \quad , \qquad (17)$$

$$C = \frac{\pi}{2\omega_{cl}R_2E}\beta_0 \quad . \qquad (18)$$

The two overlap voltage measurements $U_1 = U(\theta_1)$ and $U_2 = U(\theta_2)$ are sufficient to obtain the following from equation (16)

$$\beta_0 = ½[(1-a)U_1 + (1+a)U_2] \quad , \qquad (19)$$

where

$$a = [b(\theta_1, \alpha) + b(\theta_2, \alpha)]/[b(\theta_1, \alpha) - b(\theta_2, \alpha)] \quad . \qquad (20)$$

The use of equations (18), (19), and (20) for capacity C results in:

$$C = \frac{\pi}{2R_2E\omega_{cl}}\left[\frac{(1-a)}{2}U_1 + \frac{(1+a)}{2}U_2\right] \quad . \qquad (21)$$

If the voltages measured at the output of the capacitive transducer represent a non-correlated quantity with dispersion σ^2, then in order to determine the dispersion of C using equations (13) and (21), the following can be written, respectively:

Volume : 4 | Issue : 2 | Feb 2014 | ISSN - 2249-555X

$$\sigma_I^2 = \frac{\pi}{2R_2 E\omega_{cl}}\sigma^2 \text{ , and}$$

$$\sigma_{II}^2 = \frac{\pi}{2R_2 E\omega_{cl}} \frac{(1+a^2)}{2}\sigma^2 \ . \qquad (22)$$

Dispersion σ_{II}^2 has the lowest value when |a| has the lowest value. The analysis of equation (19) shows that |a| reaches its minimum when

$\theta_1 \to 0$ and $\theta_2 = 0.25$ (23)

(or vice versa $\theta_1 = 0.25$ and $\theta_2 \to 0$) .

Therefore charge transfer capacitive measuring transducers can operate with capacitive sensors represented by a fractional differential model. However, the result achieved for the sensor capacity depends on the control of the switches.

When operating in the interval $0<\theta\leq 0.25$, the following recommendations need to be observed:

1. If A=0 (CPE is not present) or if the capacitance is dominant ($\omega_{cl}^{\alpha}A<<\omega_{cl}C$), the recommended control is $\theta \to 0$ since it provides the longest period of time for charge transfer from sensor to transducer.

2. If $A\neq 0$ and $\alpha=0$, the recommended control is $\theta=0.25$ as the presence of the CPE does not influence the output voltage of the transducer.

3. If $A\neq 0$ and $0<\alpha<1$, the increase in θ reduces the CPE effect on the output voltage. Then the quadrature control $\theta=0.25$ is the most efficient solution in the interval $0<\theta\leq 0.25$.

4. If $A\neq 0$ and $0<\alpha<1$, when quadrature control cannot provide reasonable error (caused by the CPE), the sensor capacity can be determined by conducting two consecutive measurements using different controls: $\theta \to 0$ and $\theta=0.25$.

SIMULATION AND EXPERIMENTAL INVESTIGATION OF THE CHARGE TRANSFER CAPACITIVE MEASURING TRANSDUCER

In order to evaluate the practical significance of the results achieved, the following data on the capacitive transducer from fig.4 (and fig.5) were provided:

- nominal range of measured C0÷1000 pF;
- clock frequency $1/T_{cl}$ = 100 kHz (ω_{cl} = 6.2832 s^{-1});
- nominal coefficient 15.04 mV/pF;
- nominal voltage to the sensor E = 2.5 V.

The values of its elements were as follows: R_2 = 15,039 kΩ; C_1 = 100 nF; C_2 = 10 nF. At outputs U_g and U_h, resistors R_3 = 1 kΩ were connected in series and capacitors C_3 = 100 nF were connected in parallel.

When Oustaloup's approximation was applied, a Foster circuit (tabl.1 and fig.6) was synthesised (consisting of 31 resistors and 30 capacitors) which modelled the CPE with an error less than 0.08 % [3, 5, 8].

TABLE – 1 VALUES OF THE RESISTORS AND CAPACITORS PRESENTED IN FIGURE 6

No	R, kΩ	C, pF
0	1.9974E+01	-
1	1.7148E+02	6.3234E+03
2	1.2594E+02	4.6595E+03
3	1.0200E+02	3.1132E+03
4	8.5204E+01	2.0169E+03
5	7.2117E+01	1.2896E+03
6	6.1429E+01	8.1931E+02
7	5.2495E+01	5.1885E+02
8	4.4936E+01	3.2801E+02
9	3.8501E+01	2.0718E+02
10	3.3003E+01	1.3080E+02
11	2.8298E+01	8.2552E+01
12	2.4267E+01	5.2096E+01
13	2.0812E+01	3.2873E+01
14	1.7849E+01	2.0743E+01
15	1.5309E+01	1.3088E+01
16	1.3130E+01	8.2583E+00
17	1.1261E+01	5.2109E+00
18	9.6579E+00	3.2880E+00
19	8.2827E+00	2.0748E+00
20	7.1026E+00	1.3094E+00
21	6.0897E+00	8.2646E-01
22	5.2197E+00	5.2181E-01
23	4.4714E+00	3.2964E-01
24	3.8263E+00	2.0847E-01
25	3.2678E+00	1.3210E-01
26	2.7801E+00	8.4028E-02
27	2.3474E+00	5.3856E-02
28	1.9504E+00	3.5077E-02
29	1.5579E+00	2.3765E-02
30	1.0747E+00	1.8643E-02

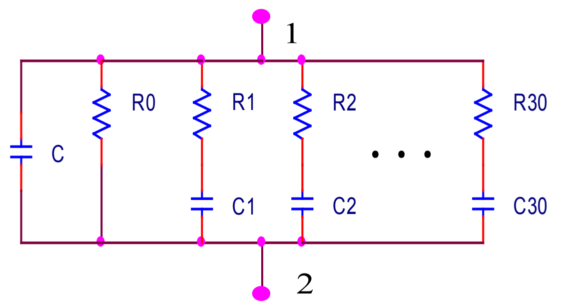

Figure 6: Oustaloup's approximation of CCPE-model

The model of the charge transfer capacitive measuring transducer in PSpice should reflect as fully as possible the conditions under which equation (10) is deducted. To this end, it is of topmost significance to take into account the ideal nature of the transducer and its elements.

In view of the limitations set by the PSpice simulator, the verification procedure was conducted using a model in which:

- the on/off time of the analogue switch was 1 ns;
- analogue switches with R_{ON} = 0.1 Ω and R_{OFF} = 1.10^{11} Ω were used;
- OPAMP PSpice models of an operational amplifier with open loop gain of 1.10^{6} were used.

The PSpice model of capacitive transducer thus obtained makes it possible to conduct simulations when parameters change within a wide range (clock frequency $1/T_{cl}$ = 50 ÷ 100 kHz, α = 0÷0.75 and θ = 0.05÷0.25).

In the cases examined, the maximum relative deviation of the output voltage obtained through PSpice simulation from those determined in accordance with (10) did not exceed 0.04%.

Evaluation was made of the differences between the transfer function of a real and an ideal capacitive measuring transducer. For this purpose, the transducer described above was implemented. Operational amplifiers of the LM6361 type and MAX4066 analogue switches were used. Combinations of Suntan NPO precise capacitors with nominal value of 150 pF and uncertainty of ±0.2 pF were applied. The output voltage was measured using a Mastech MS8218 voltmeter with maximum error of 0.04%.

As a result, a conclusion was drawn that the systematic error allowed did not exceed 0.49% when the experimental trans-

Volume : 4 | Issue : 2 | Feb 2014 | ISSN - 2249-555X

fer function was replaced by a theoretically defined transfer function.

While defining the capacity in the presence of a constant phase element through two overlap measurements, the maximum error after application of formula (21) did not exceed 0.49%.

CONCLUSIONS

A general equivalent scheme of a primary capacitive measuring transducer containing a constant phase element was presented. Compared to conventional schemes, this one allows a wider range of capacitive sensors to be described without leading to an increase in the number of elements or changes in the model arrangement. A continuously operating charge transfer differential capacitive measuring transducer running in a steady-state mode was analysed with a constant phase element in the capacitive sensor.

The additional error created by the constant phase element in an equivalent capacitive sensor scheme was also analysed . Recommendations were made concerning the application of the appropriate control of the measuring transducer thus minimising the error.

The simulations and experiments conducted proved the significance of the analytical results obtained.

REFERENCE [1]Baxter, L. (1996), "Capacitive Sensors." John Wiley and Sons, 320. | [2]Maslinkov, Iv., and Nikovski, Pl. (2012), "Application of the Charge/Dischage Method for Capacity Determination in the Presence of Resistance." 56-th Conference for Electronics, Telecommunications, Computers, Automation and Nuclear Engineering ETRAN 2012, 11÷14.06.2012, Zlatibor, Serbja, Vol. ML1.4-1.4, 4. | [3]Nikovski Pl. (2011), "Practical Synthesis Of Irrational Impedance Based On Solutions Of The Quadratic Equation." IU - Journal of Electrical & Electronics Engineering, Vol. 11, No 2, 1391-1398. | [4] Nikovski Pl. (2012) "Metrological analysis of charge - transfer capacitive transducer in the presence of resistance." International Journal of Electronics, Mechanical and Mechatronics Engineering (IJEMME), Vol 2, No 3, 288-292. | [5]Nikovski Pl. (2012), "Practical synthesis and analysis of one RC model of the half - order constant phase element." University of Pitesty Scientific bulletin „Electronics and computers science", Vol. 12, Issue 1, 19-24. | [6]Nikovski, Pl., and Maslinkov, Iv. (2007), "Transient Response and Time Characteristic of One Capacitive Interface Circuit in Case of Fractality." 42-th International Scientific and Applied Science Conference "Electronics ET`2007", book 3, 19÷21.09.2007, Sozopol, Bulgarien, 65-70. | [7]Nikovski Pl., and Maslinkov, Iv. (2009), "Analysis of an Equivalent CCPE Connection Diagram of the One-port Circuit by Square-wave Voltage." 44-th International Scientific and Applied Science Conference Electronics ET`2009, book 1, Sozopol, Bulgarien, 197-199. | [8]Oustaloup, A. (1983) "Systems asservis lineaires d'ordre fractionnaire: Theorie et pratique". Editions Masson, Paris, 296. | [9]Philipp H. (1999), "Charge transfer sensing." Sensor Review, Vol.19, No2, 96-105 . | [10]Yang, W. (1996), "Hardware Design of Electrical Capacitance Tomography Systems." Meas. Sci. Technol. Vol. 7, 225-232. | [11]Yang, W. (1996), "Charge Injection Compensation for Charge/Discharge Capacitance Measuring Circuits Used in Tomography Systems." Meas. Sci. Technol. Vol. 7, 1073–1078.